

Development Economics

Slides 14

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The Return of Inequality

- **The financial crisis sparked a new interest in inequality.**

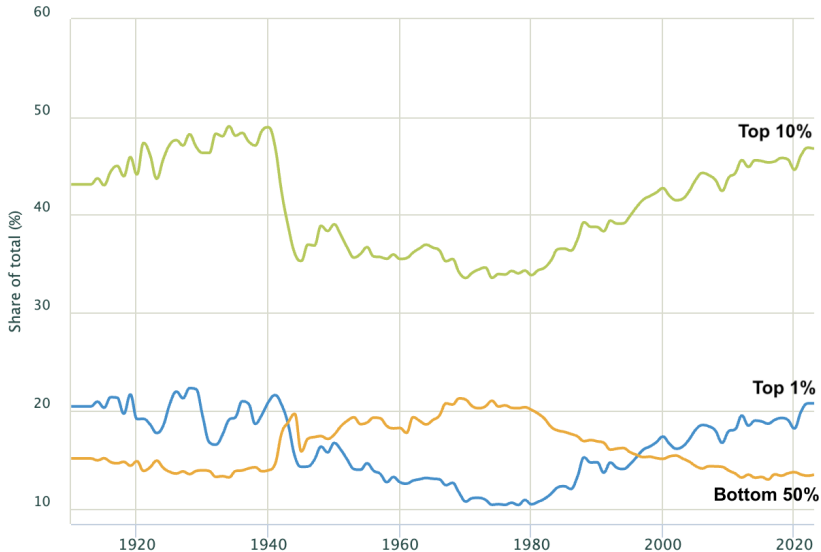
- But inequality has been historically high
- Growing steadily through late 20th century

Wolff, Piketty, Saez, Atkinson, many others

- **A recent book by Piketty 2014**

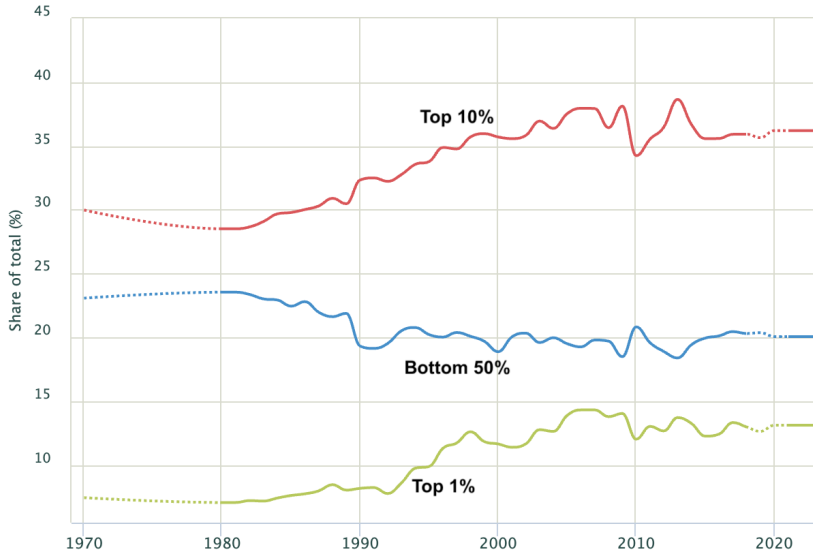
- brought rising inequality to the public consciousness
- summarized the evidence (compelling and useful)
- was a runaway hit in the United States, touching a raw nerve

Income Inequality, United States, 1910-2023



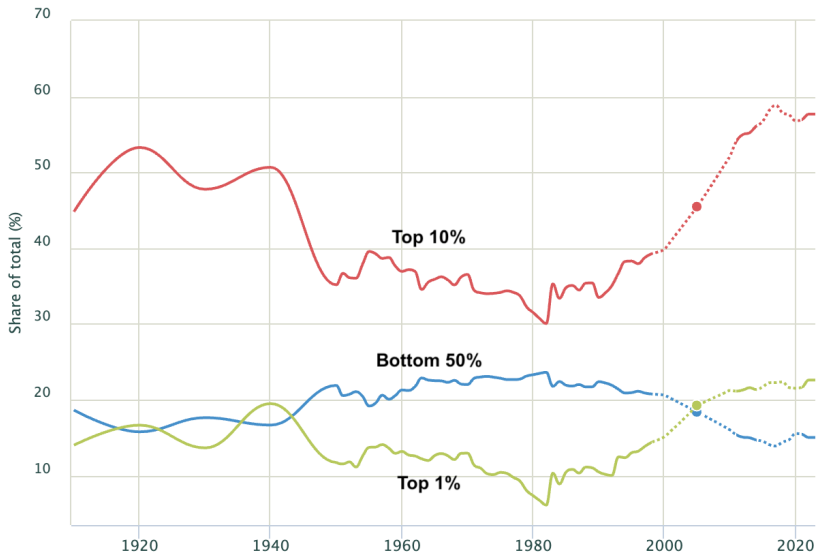
Source: World Inequality Database

Income Inequality, United Kingdom, 1970-2023



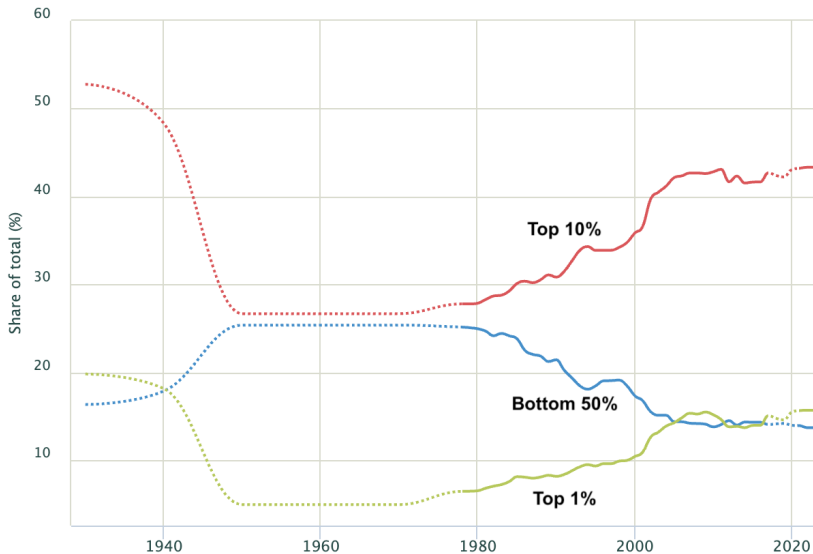
Source: World Inequality Database

Income Inequality, India, 1910-2023



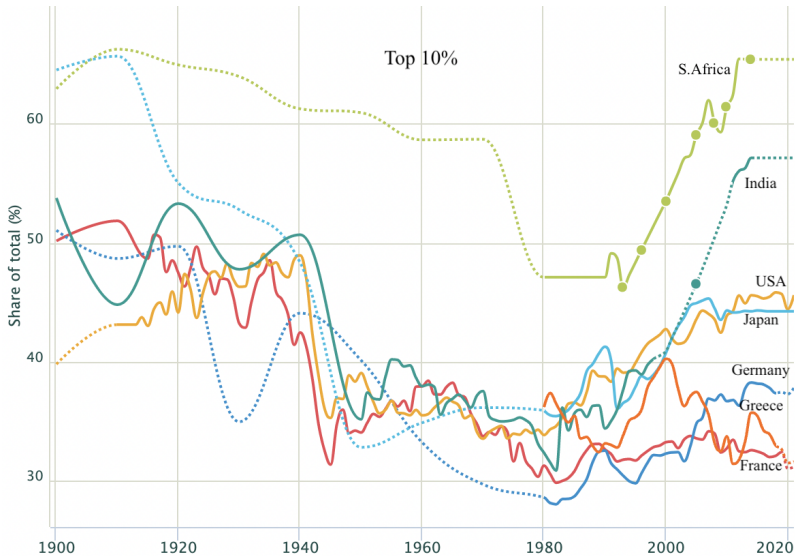
Source: World Inequality Database

Income Inequality, China, 1929-2023



Source: World Inequality Database

Top 10% Share in Selected Countries



Source: World Inequality Database

Intrinsic Versus Instrumental

- Inequality is of **intrinsic** as well as **instrumental** interest
- **Intrinsic:**
 - inequality measurement: evaluate and compare distributions
 - evolution of inequality in societies (e.g., Piketty)
- **Instrumental:**
 - inequality and various outcomes: growth, nutrition, employment
 - inequality and history-dependence
 - Inequality and incentives ...

Conceptualizing and Measuring Inequality

- **Income distribution** $(y_1, \dots, y_N)$.
- inequality measure maps each vector $(y_1, \dots, y_N)$ to a number.
- Rankings matter, not the exact numbers.
- Axiomatic approach.

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- **Income distribution** $(y_1, \dots, y_N)$.
 - inequality measure maps each vector $(y_1, \dots, y_N)$ to a number.
 - Rankings matter, not the exact numbers.
 - Axiomatic approach.
- **Anonymity principle**: Names do not matter.
 - So rewrite as $(y_1, N_1; y_2, N_2; \dots; y_m, N_m)$,
 - where m is number of distinct incomes,
 - N_i is population with income y_i ,
 - $y_1 < y_2 < \dots < y_m$.

Conceptualizing and Measuring Inequality

- **Population principle:**
 - Cloning entire population (and incomes) doesn't alter inequality.
 - So only the population *share* $n_i = \frac{N_i}{N}$ is relevant: **share**: $\sum_{i=1}^m n_i = 1$.

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■ A critical look:

- 2-person society where one person has income? Perfect inequality?
- **Read the debate here** [not needed for course]

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- **A critical look:**

- presumes linear link between income and “utility”.
- For instance, think of subsistence constraint.

Conceptualizing and Measuring Inequality

- **Pigou-Dalton Transfers principle:**
 - An income transfer from relatively poor to relatively rich must increase inequality.
 - Postpone critical look to later.

Conceptualizing and Measuring Inequality

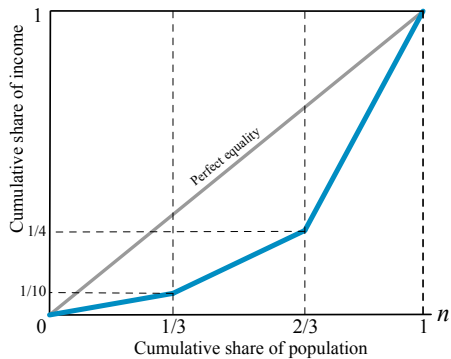
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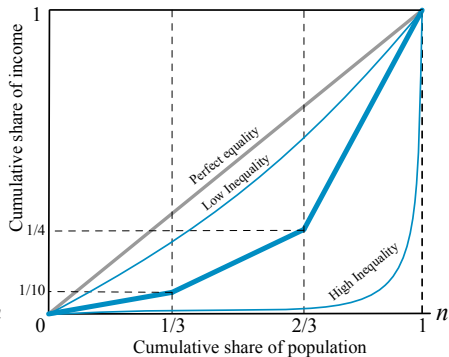
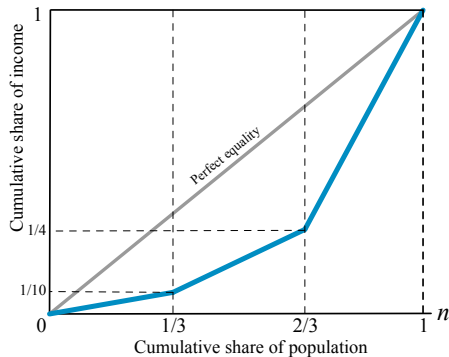
■ **Summary:** $I(\mathbf{y})$, where $\mathbf{y} = (y_1, \dots, y_N)$.

- **Anonymity:** $I(\mathbf{y}) = I(\sigma\mathbf{y})$, where σ is a permutation.
- **Population:** $I(\mathbf{y}) = I(\mathbf{y}, \mathbf{y})$.
- **Relative Income:** $I(\mathbf{y}) = I(\lambda\mathbf{y})$ for all $\lambda > 0$.
- **Transfers:** $I(\mathbf{y}) < I(y_1, \dots, y_i - \delta, \dots, y_j + \delta, \dots, y_N)$ for $\delta > 0$.

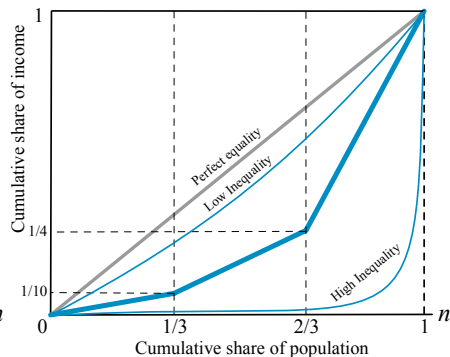
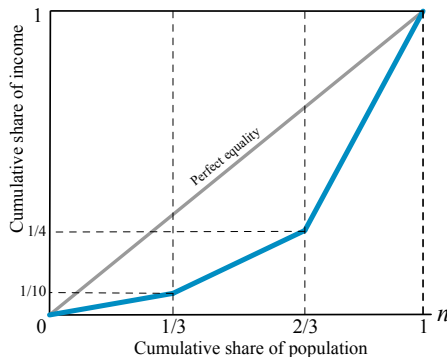
The Lorenz Curve



The Lorenz Curve



The Lorenz Curve



Properties:

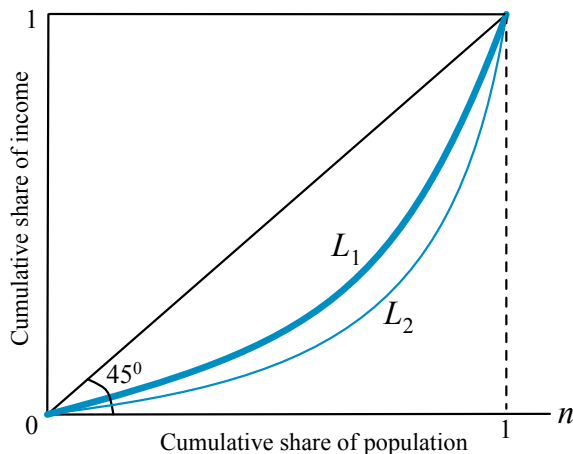
1. L is increasing, with $L(0) = 0$ and $L(1) = 1$.
2. L is convex, and always lies “below” 45° line.

The Lorenz Criterion

- If Lorenz curve $L(2)$ lies **everywhere below** (or “**to the right of**”) $L(1)$, then inequality under $L(2)$ is higher than inequality under $L(1)$:

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- **Illustration.** 3 groups of the same size:
 - **Situation 1 incomes:** (10, 20, 30). **Situation 2 incomes:** (14, 44, 62).

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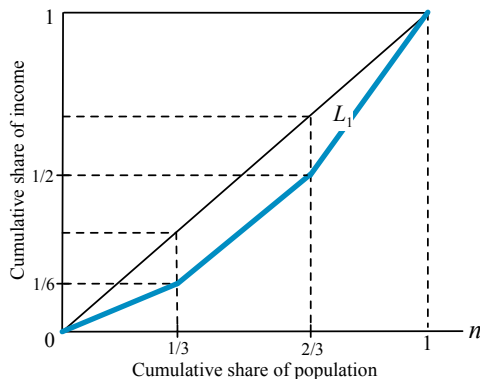
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- **Illustration.** 3 groups of the same size:
 - **Situation 1 incomes:** (10, 20, 30). **Situation 2 incomes:** (14, 44, 62).
 - Rescale and **note that the transfers principle can be applied here.**

The Lorenz Criterion

An example

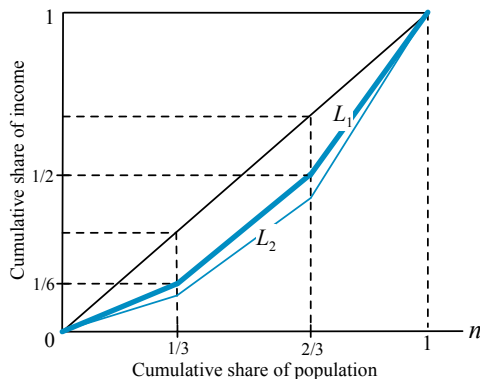
- 3 groups, $n_1 = n_2 = n_3 = 1/3$
- Situation 1 incomes: (10, 20, 30)
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- Rescale: $(14, 44, 62) \sim (7, 22, 31)$



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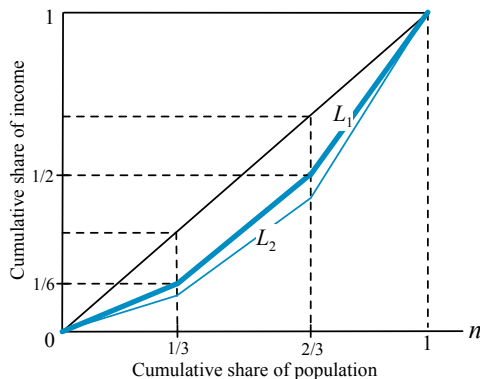
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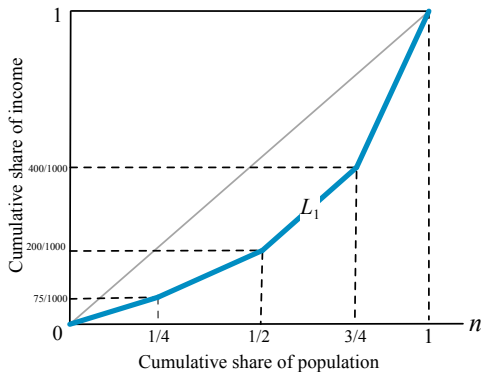


- Nice: the axioms and the Lorenz criterion are the same thing..
- But also tells us that the axioms don't cover all cases, **because Lorenz curves can cross.**

The Lorenz Criterion

■ Another example

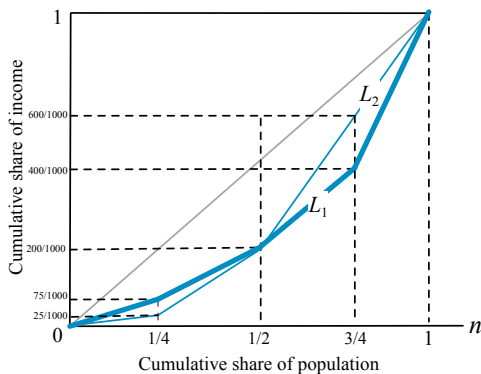
- $n_1 = n_2 = n_3 = n_4 = 1/4$
- Situation 1: (75, 125, 200, 600)
- Situation 2: (25, 175, 400, 400)



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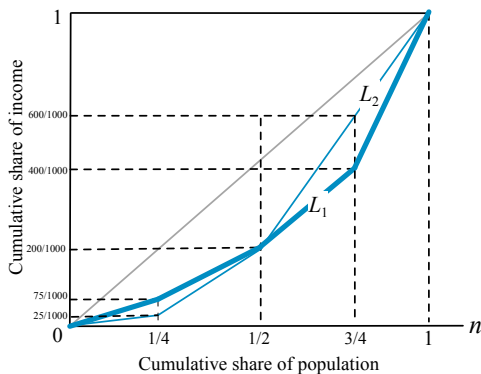
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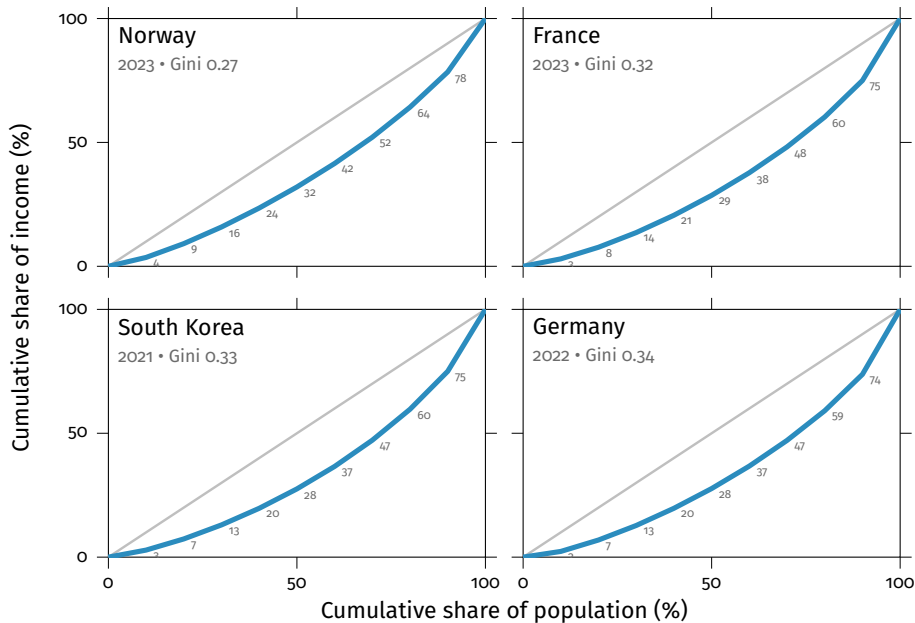
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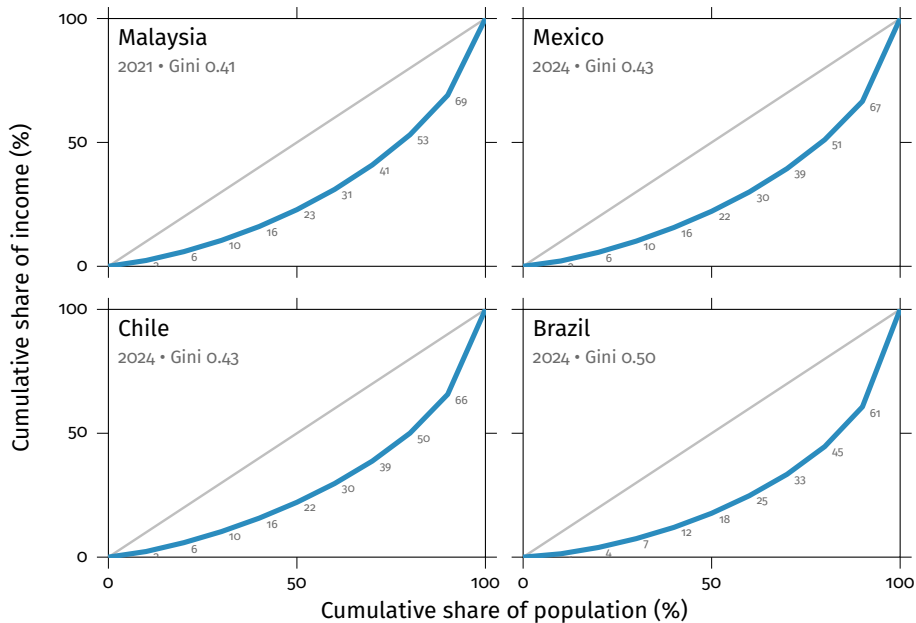
■ A third example. Modern sector enlargement (from Fields, 1980).

- 10 people, rural wage equals 50, urban wage equals 100
- rural-urban migration

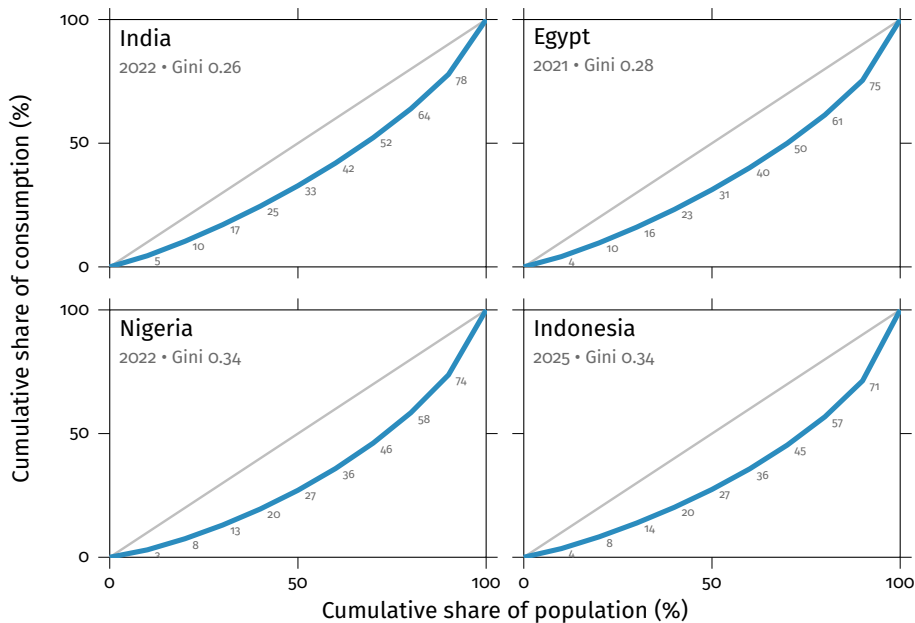
Some Lorenz Curves



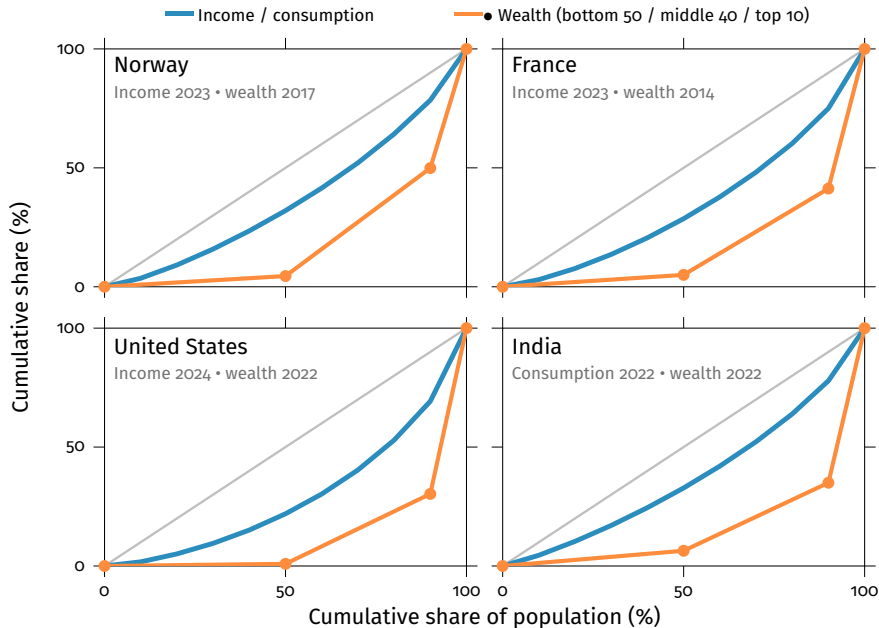
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Some Lorenz Curves: Consumption Data



Lorenz Curves for Income and Wealth



Complete Measures of Inequality

■ The Lorenz ordering is partial:

- Lorenz-consistent measures complete this ordering in different ways.
- We also want measures that take full advantage of available information:
- m groups, $\mathbf{y} = (y_1, \dots, y_m), (N_1, \dots, N_m)$,
- $n_i = \frac{N_i}{N}$ is pop share, $\mu = (\sum_{i=1}^m N_i y_i)/N$ is mean income.

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■ Examples of popular crude measures:

- **Range:** $(y_m - y_1) / \mu$.
- **Kuznets ratios:** (richest $x\%$) / (poorest $y\%$); e.g., 20%/80%.

Complete Measures of Inequality

- More seriously:

- **Mean absolute deviation:**
$$\frac{1}{\mu} \sum_{j=1}^m \frac{N_j}{N} |y_j - \mu| = \frac{1}{\mu} \sum_{j=1}^m n_j |y_j - \mu|.$$

- Is this Lorenz-consistent? Check axioms.

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- **Coefficient of variation:**
$$\frac{1}{\mu} \sqrt{\sum_{j=1}^m \frac{N_j}{N} (y_j - \mu)^2} = \frac{1}{\mu} \sqrt{\sum_{j=1}^m n_j (y_j - \mu)^2}.$$

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Complete Measures of Inequality

- **Gini:**
$$\frac{1}{2\mu} \sum_{j=1}^m \sum_{k=1}^m \frac{N_j N_k}{N^2} |y_j - y_k| = \frac{1}{2\mu} \sum_{j=1}^m \sum_{k=1}^m n_j n_k |y_j - y_k|.$$
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- A **variant** of the Gini (he had 13 of them!):

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- Declares that a two-person society can exhibit perfect inequality.

Recall debate.

- But fails the population principle.

■ Gini (G) compared with coefficient of variation (COV)

- They may disagree when Lorenz curves cross.

■ **Example.** Compare $A = (3, 12, 12)$ and $B = (4, 9, 14)$.

- $\text{COV}(A): \frac{1}{9} \sqrt{\frac{1}{3}(3-9)^2 + \frac{2}{3}(12-9)^2} = \frac{1}{9} \sqrt{18}.$

- $\text{COV}(B): \frac{1}{9} \sqrt{\frac{1}{3}(4-9)^2 + \frac{1}{3}(14-9)^2} = \frac{1}{9} \sqrt{\frac{50}{3}}.$

- So the **COV falls** going from A to B .

- $G(A): \frac{[2|3-12|+2|12-3|]}{2 \cdot 9^2} = \frac{18}{81}.$

- $G(B): \frac{[|4-9|+|4-14|+|9-4|+|9-14|+|14-4|+|14-9|]}{2 \cdot 9^2} = \frac{20}{81}.$

- So the **Gini rises** going from A to B .

Gini and the COV: Real-Life Disagreement

Country	Gini	COV	Richest 5%	Poorest 40%
Puerto Rico				
1953	0.415	1.152	23.4	15.5
1963	↑0.449	↓1.035	↓22.0	↑13.7
Argentina				
1953	0.412	1.612	27.2	18.1
1959	↑0.463	↑1.887	↑31.8	↑16.4
1961	↑0.434	↓1.605	↑29.4	↑17.4
Mexico				
1950	0.526	2.500	40.0	14.3
1957	↑0.551	↓1.652	↓37.0	↑11.3
1963	↑0.543	↓1.380	↓28.8	↑10.1

■ **Inequality in three countries** (Weisskoff (1970), Fields (1980)).

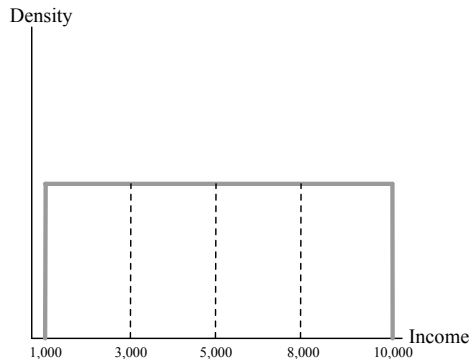
- Arrows ↑ and ↓ show direction of inequality relative to base year.

Beyond Economic Inequality

- “As the struggle proceeds, the whole society breaks up more and more into two hostile camps, two great, directly antagonistic classes ... The classes *polarize*, so that they become internally more homogeneous and more and more sharply distinguished from one another in wealth and power.” (Deutsch 1971, p.44)

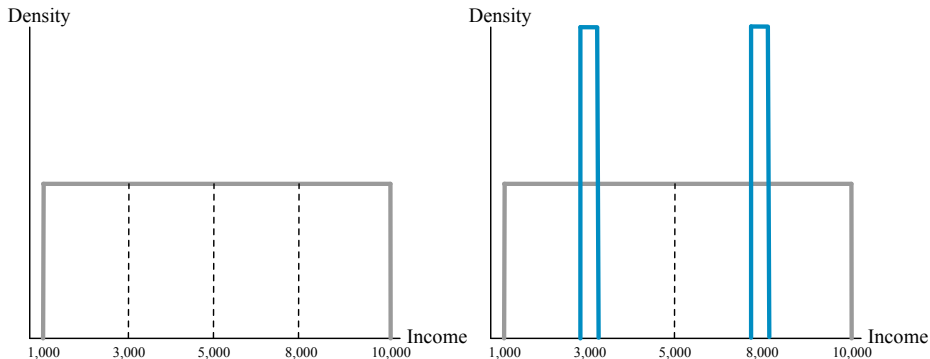
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Polarization

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- **Add these:**

$$P = \frac{1}{\mu} \sum_{i=1}^m \sum_{j=1}^m n_i^2 n_j |y_i - y_j|.$$

- Looks very much like the Gini coefficient, but different.
- Can be applied to study the connection between distribution and conflict.