

Development Economics

Slides 4

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Theory of Economic Growth

Combines production function with consumption-savings choices.

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- Savings equals investment: $S(t) = I(t)$.
- Investment adds to capital stock:

$$K(t+1) = (1 - \delta)K(t) + I(t) = (1 - \delta)K(t) + sY(t)$$

where δ is the rate of depreciation.

- This is the **accumulation equation**.

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Theory of Economic Growth

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- Convert to per-capita magnitudes: $k = K/L$, $y = Y/L$:

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- Combine with per-capita production function $y = f(k)$:

$$(1 + n)k(t+1) = (1 - \delta)k(t) + sf(k(t))$$

■

$$\Rightarrow \frac{k(t+1)}{k(t)} = \frac{(1 - \delta) + s\Gamma(k(t))}{1 + n}$$

- where $\Gamma(k) \equiv y/k = f(k)/k$ is the **output-capital ratio**.

$$\frac{k(t+1)}{k(t)} = \frac{(1-\delta) + s\Gamma(k(t))}{1+n}$$

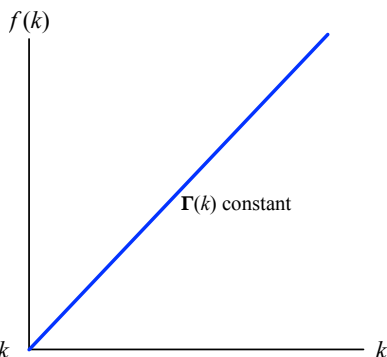
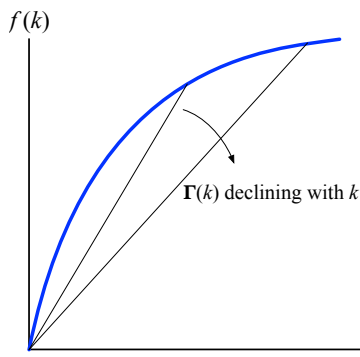
- This is the **fundamental equation for the growth model**.
- A lot now hangs on what we view as exogenous or endogenous.
- $(s, \delta, n, \Gamma) \dots$

Solow and Harrod-Domar: Introduction

- In the **Solow model**, the output-capital ratio Γ is **endogenous**:
It falls as k rises, because of diminishing returns to capital.
- In the **Harrod-Domar** model, the output-capital ratio Γ is **exogenous**:
The distinction with Solow comes from the implicit role played by labor.

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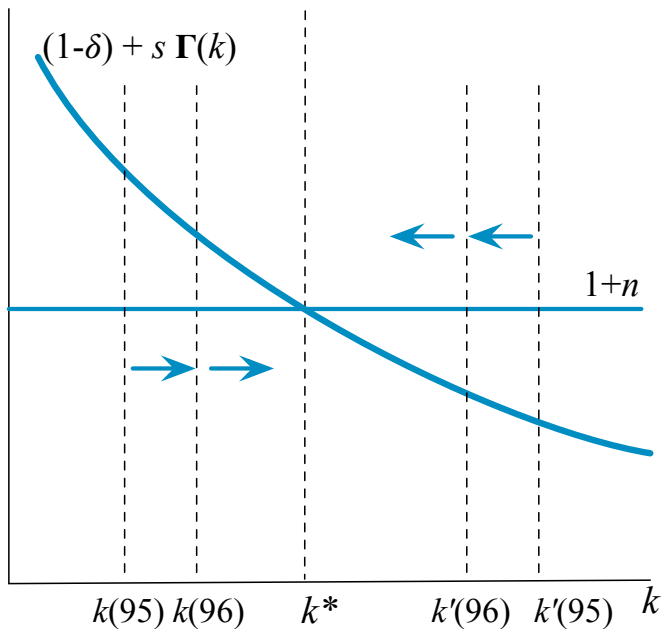
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- **A central prediction:** $k(t+1)/k(t)$ declines as $k(t)$ goes up.
 - **Reason:** As $k(t)$ goes up, $\Gamma(k(t))$ comes down.
 - **Implication:** “Convergence.”

Theory of Economic Growth: The Solow Model



The Steady State

- **Steady state capital-labor ratio** of the economy is k^* , which solves:

$$\underbrace{\Gamma(k^*) = \frac{y^*}{k^*} = \frac{f(k^*)}{k^*}}_{\text{all the same}} = \frac{n + \delta}{s}$$

- $k^* = K(t)/L(t)$, but $K(t)$ and $L(t)$ keep rising at rate n .
- Argument crucially depends on diminishing returns to inputs.

The Steady State

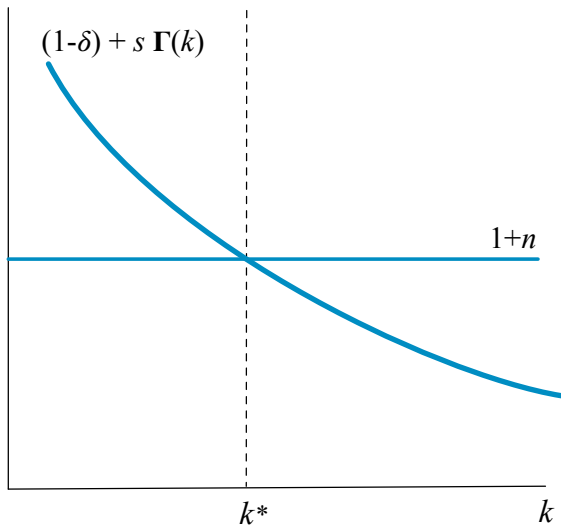
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- $k^* = K(t)/L(t)$, but $K(t)$ and $L(t)$ keep rising at rate n .
- Argument crucially depends on diminishing returns to inputs.
- **No long-run growth over and above population growth:**
- Any extra growth only comes from technical progress, as we shall see.

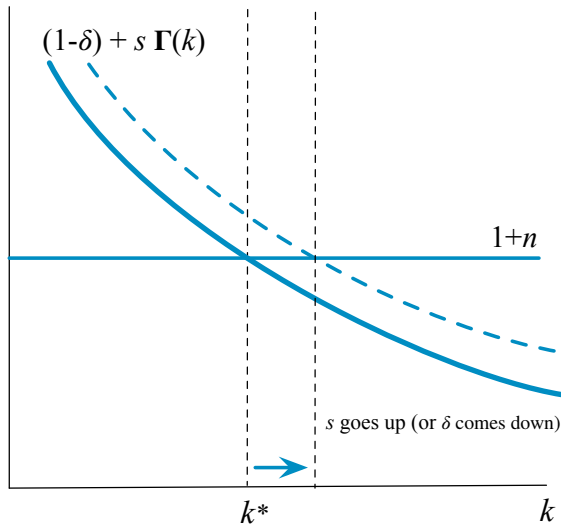
The Steady State

- How s , n and δ affect k^* :



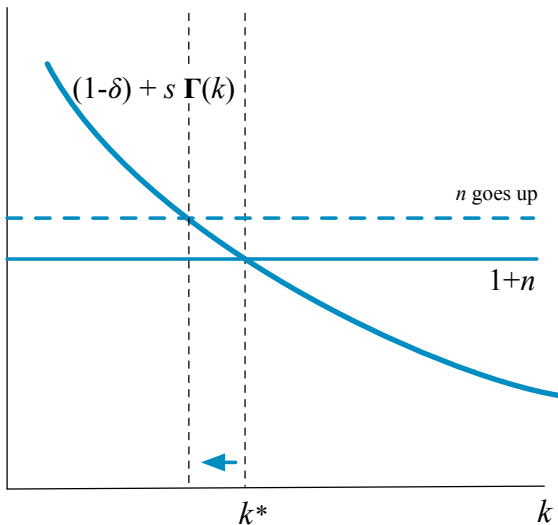
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The Steady State for Cobb-Douglas Production

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The Steady State for Cobb-Douglas Production

- Steady state k^* solves $\frac{f(k^*)}{k^*} = \frac{n + \delta}{s}$.
- With Cobb-Douglas production, $f(k) = Ak^a$, so:

$$\frac{f(k^*)}{k^*} = Ak^{*a-1} = \frac{n + \delta}{s}$$

$\Rightarrow$

$$k^* = \left(\frac{sA}{n + \delta} \right)^{1/(1-a)} \quad \text{and} \quad y^* = A^{1/(1-a)} \left(\frac{s}{n + \delta} \right)^{a/(1-a)}$$

- Can directly verify the properties we established geometrically earlier.

Two Sources of Per-Capita Growth

Exogenous versus endogenous growth

Two Sources of Per-Capita Growth

Exogenous versus endogenous growth

- **Exogenous growth:** comes from “outside” the model.

ongoing technical progress

- **Endogenous growth:** comes from “inside” the model.

induced technical progress

absence of diminishing returns

Exogenous Growth: Technical Progress

- $Y(t) = F(K(t), e(t)L(t))$, so $e(t)L(t)$ is “effective” labor.
- Exogenous technical progress: $e(t+1) = e(t)(1 + \pi)$, $\pi > 0$.

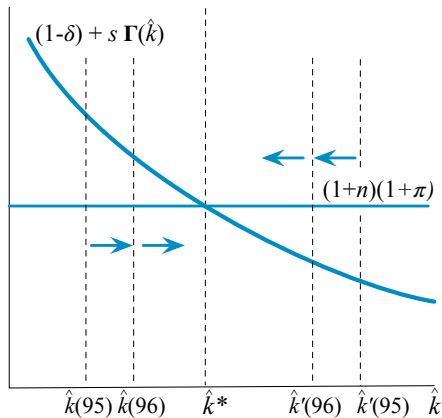
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- **Exogenous technical progress:** $e(t+1) = e(t)(1 + \pi)$, $\pi > 0$.
- Divide through in the accumulation equation by $e(t)L(t)$ instead of $L(t)$:

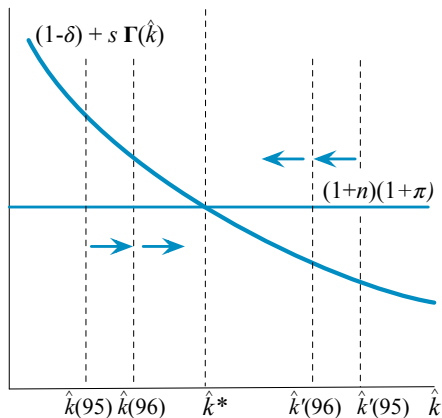
$$(1 + \pi)(1 + n)\hat{k}(t+1) = (1 - \delta)\hat{k}(t) + sf(\hat{k}(t)),$$

- where $\hat{k} = K/eL$ is in “effective per-capita” units.
- And now do exactly what we did before ...

Steady State in Effective Units With Technical Progress



Steady State in Effective Units With Technical Progress



- **Steady state:** $\frac{f(\hat{k}^*)}{\hat{k}^*} \approx \frac{(1+n)(1+\pi) - (1-\delta)}{s}$
- **Per-capita long run growth rate** = π .

Steady State in Effective Units With Technical Progress

$$\frac{f(\hat{k}^*)}{\hat{k}^*} = \frac{(1+n)(1+\pi) - (1-\delta)}{s} \approx \frac{n + \delta + \pi}{s},$$

- where we approximate: $(1+\pi)(1+n) = 1 + \pi + n + n\pi \approx 1 + \pi + n$
($n\pi$ is tiny relative to n or π).

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- where we approximate: $(1+\pi)(1+n) = 1 + \pi + n + n\pi \approx 1 + \pi + n$
($n\pi$ is tiny relative to n or π).
- In the Cobb-Douglas case, $\hat{y} = f(k^*) = A\hat{k}^a$, so

$$\hat{k}^* \approx \left(\frac{sA}{n + \delta + \pi} \right)^{1/(1-a)},$$

- ...and **steady state output in effective labor units** is:

$$\hat{y}^* \approx A^{1/(1-a)} \left(\frac{s}{n + \delta + \pi} \right)^{a/(1-a)}.$$

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- Actual per-capita output and capital grow at the rate of π .

Steady State Growth With Technical Progress

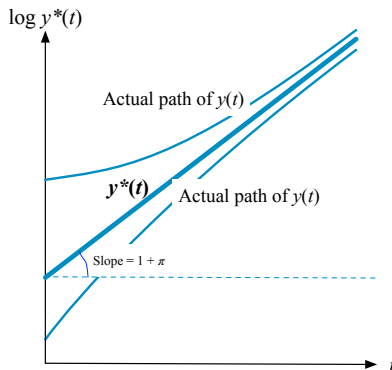
■ Path of steady state output per capita:

$$y^*(t) = \hat{y}^*(1 + \pi)^t \approx A^{1/(1-a)} \left(\frac{s}{n + \delta + \pi} \right)^{a/(1-a)} (1 + \pi)^t.$$

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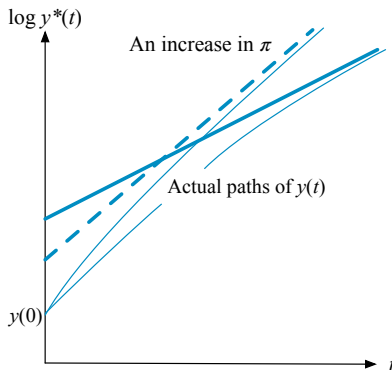
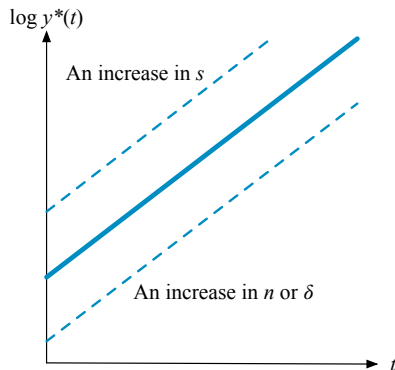
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Steady State Growth: Parametric Changes

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Endogenous Growth: The Harrod-Domar or AK Model

- In the Harrod-Domar model, output-capital ratio is exogenous:
 - $\Gamma(k)$ is unchanging in k .
 - In the Cobb-Douglas model $y = Ak^a$, that happens when $a = 1$.
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 - When a hits 1, the model behaves *very* differently:

$$(1 + n)k(t + 1) = (1 - \delta)k(t) + sAk(t).$$

- Moving terms around:

$$\text{Rate of growth} = \frac{k(t + 1) - k(t)}{k(t)} = \frac{sA - (n + \delta)}{1 + n}.$$

- **Now parameters have growth effects, unlike Solow model.**