Development Economics

Slides 14

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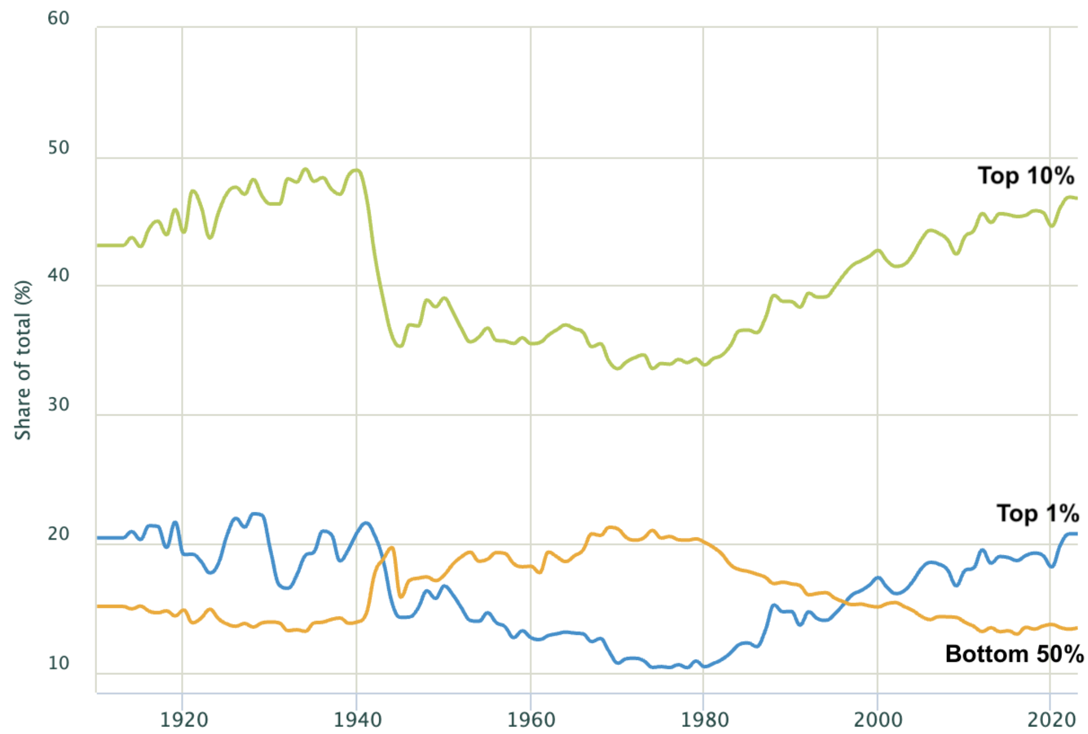
The Return of Inequality

- **The financial crisis sparked a new interest in inequality.**
  - But inequality has been historically high
  - Growing steadily through late 20th century

  Wolff, Piketty, Saez, Atkinson, many others

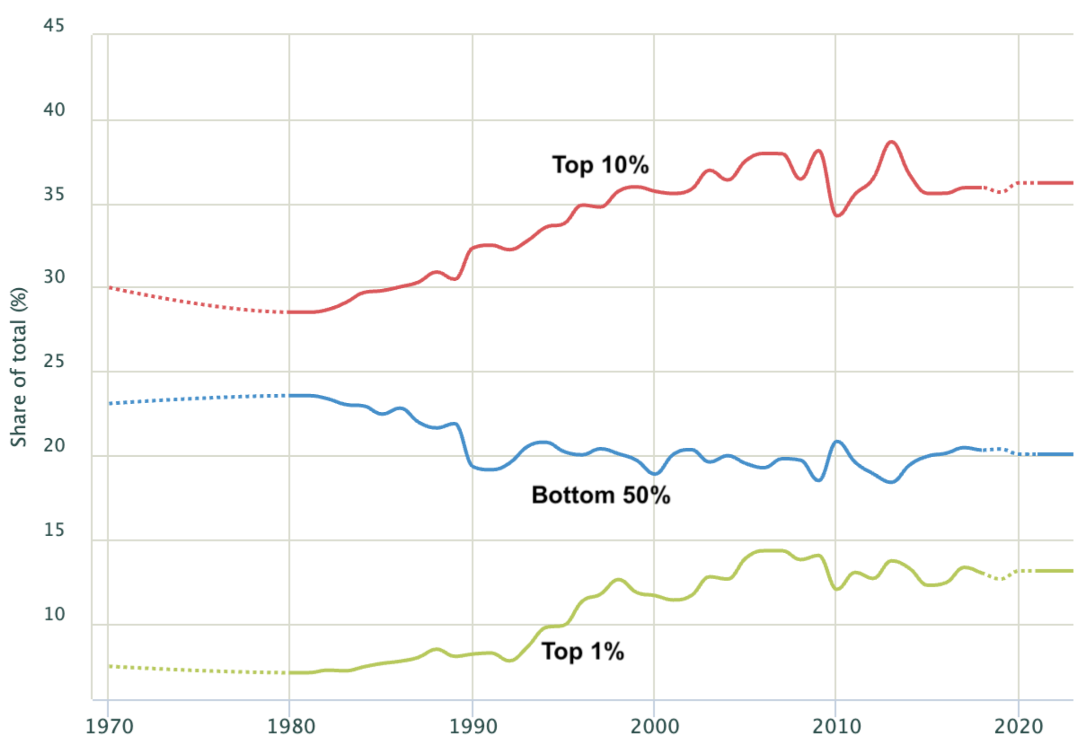
- A recent book by Piketty 2014
  - brought rising inequality to the public consciousness
  - summarized the evidence (compelling and useful)
  - was a runaway hit in the United States, touching a raw nerve

Income Inequality, United States, 1910-2023



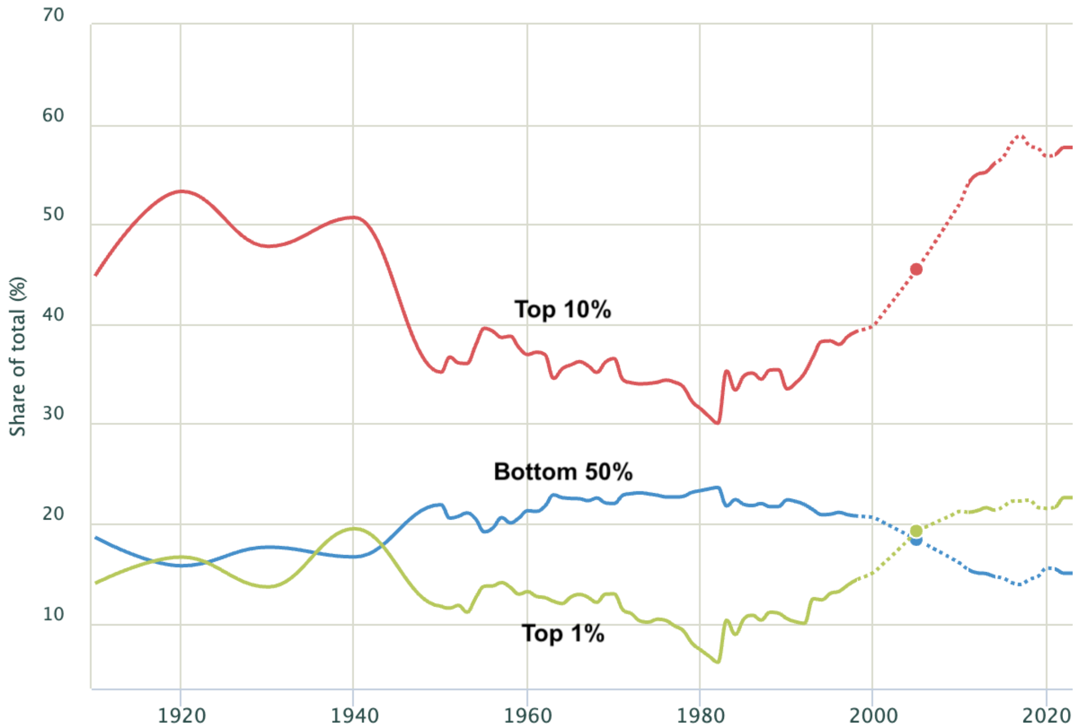
Source: World Inequality Database

Income Inequality, United Kingdom, 1970-2023



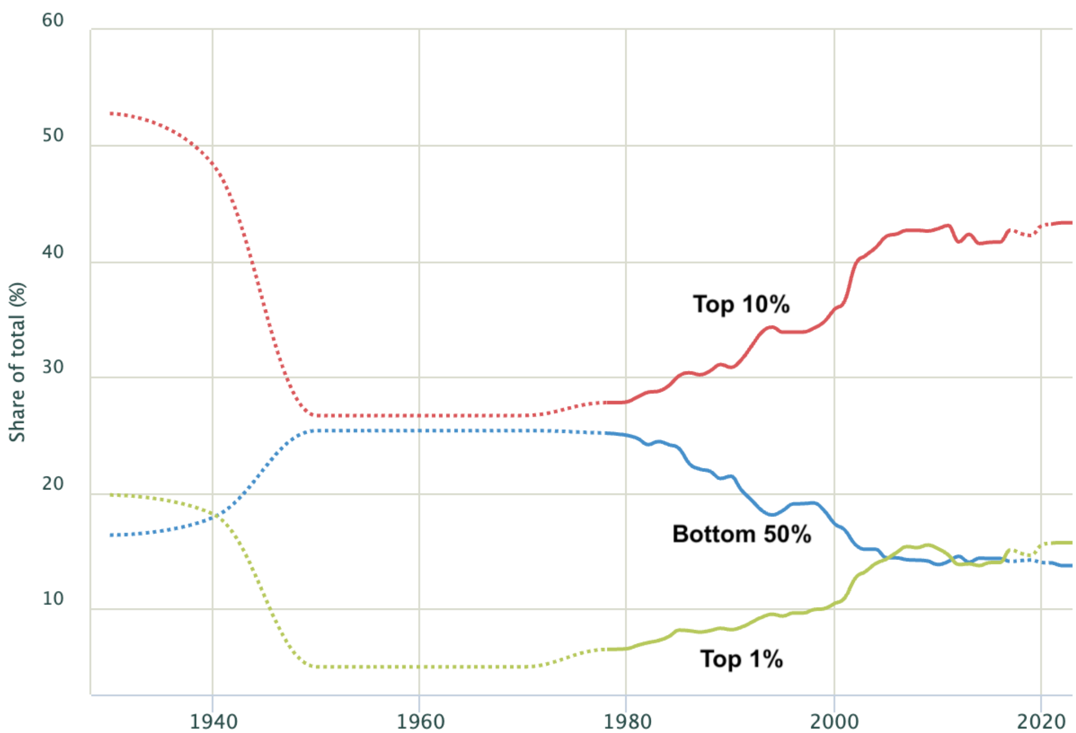
Source: World Inequality Database

Income Inequality, India, 1910-2023



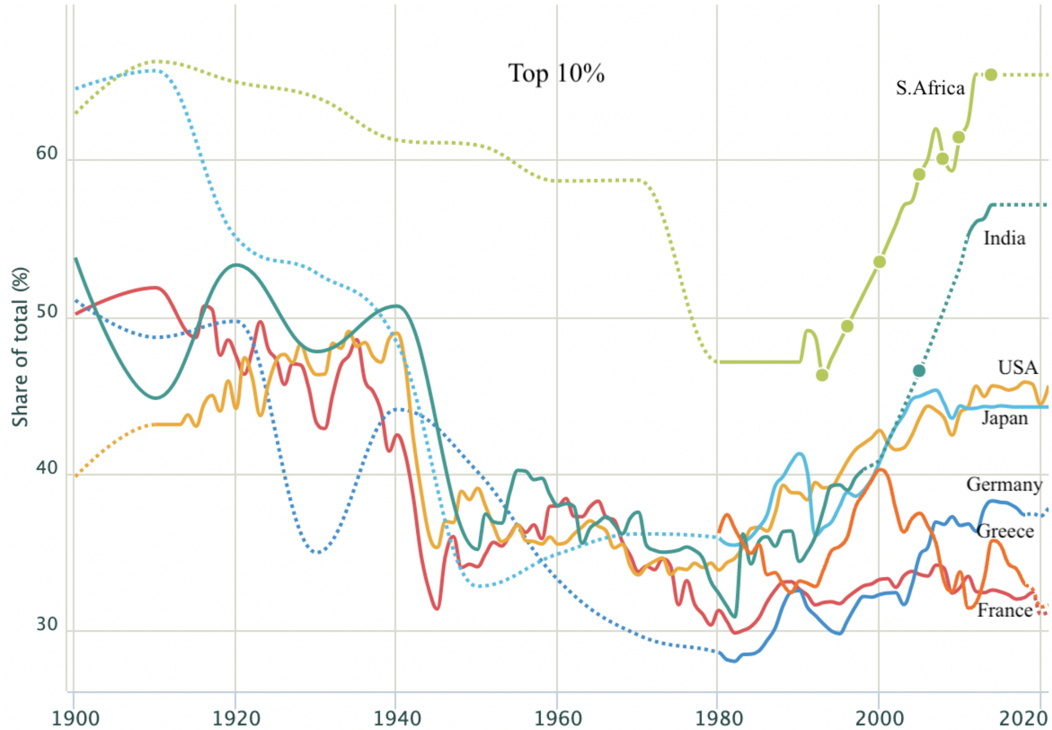
Source: World Inequality Database

Income Inequality, China, 1929-2023



Source: World Inequality Database

Top 10% Share in Selected Countries



Source: World Inequality Database

Intrinsic Versus Instrumental

- Inequality is of intrinsic as well as instrumental interest
- **Intrinsic**:
- inequality measurement: evaluate and compare distributions
- evolution of inequality in societies (e.g., Piketty)
- **Instrumental**:
- inequality and various outcomes: growth, nutrition, employment
- inequality and history-dependence
- Inequality and incentives . . .

Conceptualizing and Measuring Inequality

- **Income distribution** $(y_1, \ldots, y_N)$.
  - inequality measure maps each vector $(y_1, \ldots, y_N)$ to a number.
  - Rankings matter, not the exact numbers.
  - Axiomatic approach.
- **Anonymity principle**: Names do not matter.
  - So rewrite as $(y_1, N_1; y_2, N_2; \ldots; y_m, N_m)$,
  - where m is number of distinct incomes,
  - N_i is population with income y_i,
  - $y_1 < y_2 < \cdots < y_m$.

Conceptualizing and Measuring Inequality

- **Population principle**:
  - Cloning entire population (and incomes) doesn't alter inequality.
  - So only the population *share* $n_i = \frac{N_i}{N}$ is relevant: share: $\sum_{i=1}^{m} n_i = 1$.
- A critical look:
  - 2-person society where one person has income? Perfect inequality?
  - **Read the debate here** [not needed for course]

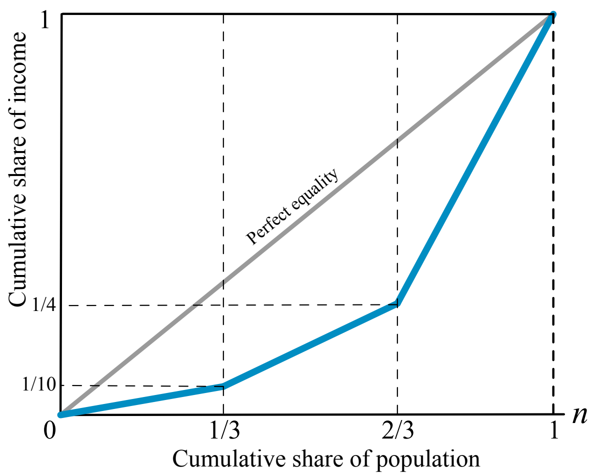
Conceptualizing and Measuring Inequality

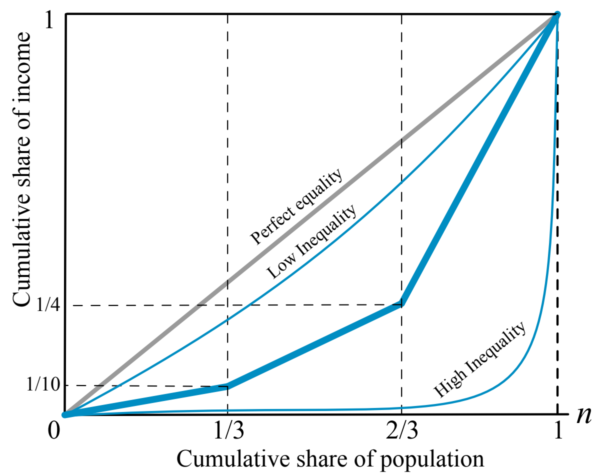
- **Relative Income principle**:
  - Scaling incomes up or down does not affect inequality.
- A critical look:
  - presumes linear link between income and “utility”.
  - For instance, think of subsistence constraint.

Conceptualizing and Measuring Inequality

- **Pigou-Dalton Transfers principle**:
  - An income transfer from relatively poor to relatively rich must increase inequality.
  - Postpone critical look to later.
- **Summary:** $I(\mathbf{y})$, where $\mathbf{y} = (y_1, \ldots, y_N)$.
  - Anonymity: $I(\mathbf{y}) = I(\sigma\mathbf{y})$, where σ is a permutation.
  - Population: $I(\mathbf{y}) = I(\mathbf{y}, \mathbf{y})$.
  - Relative Income: $I(\mathbf{y}) = I(\lambda\mathbf{y})$ for all $\lambda > 0$.
  - Transfers: $I(\mathbf{y}) < I(y_1, \ldots, y_i - \delta, \ldots, y_j + \delta, \ldots, y_N)$ for $\delta > 0$.

The Lorenz Curve



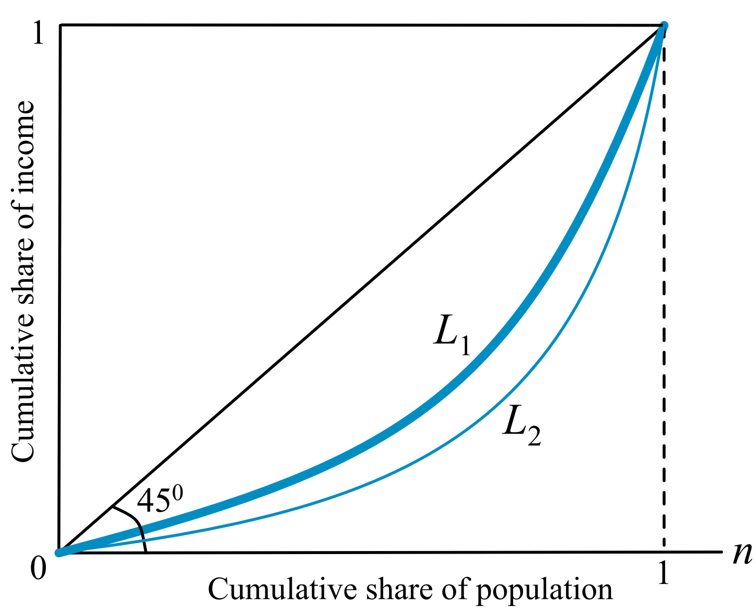


Properties:

1. L is increasing, with $L(0) = 0$ and $L(1) = 1$.
2. L is convex, and always lies "below" 45° line.

The Lorenz Criterion

- If Lorenz curve $L(2)$ lies **everywhere below** (or "**to the right of**") $L(1)$, then inequality under $L(2)$ is higher than inequality under $L(1)$:



The Lorenz Criterion

- An inequality measure I is **Lorenz-consistent** if it agrees with the Lorenz criterion whenever one Lorenz curve lies completely below the other (it could touch in parts).

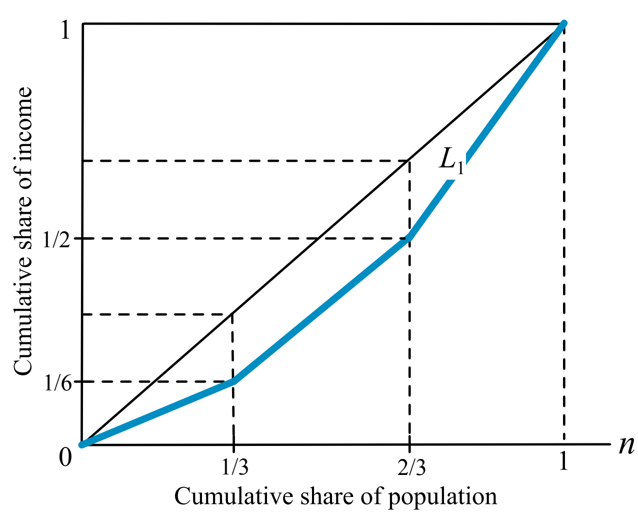
Theorem 1

Inequality measure I is Lorenz consistent if and only if it satisfies Anonymity, the Population principle, the Relative Income principle and the Pigou-Dalton Transfers principle.

- **Illustration.** 3 groups of the same size:
  - Situation 1 incomes: $(10, 20, 30)$. Situation 2 incomes: $(14, 44, 62)$.
  - Rescale and **note that the transfers principle can be applied here**.

The Lorenz Criterion

- **An example**
  - 3 groups, $n_1 = n_2 = n_3 = 1/3$
  - Situation 1 incomes: $(10, 20, 30)$
  - Situation 2 incomes: $(14, 44, 62)$
  - Rescale: $(14, 44, 62) \sim (7, 22, 31)$



The Lorenz Criterion

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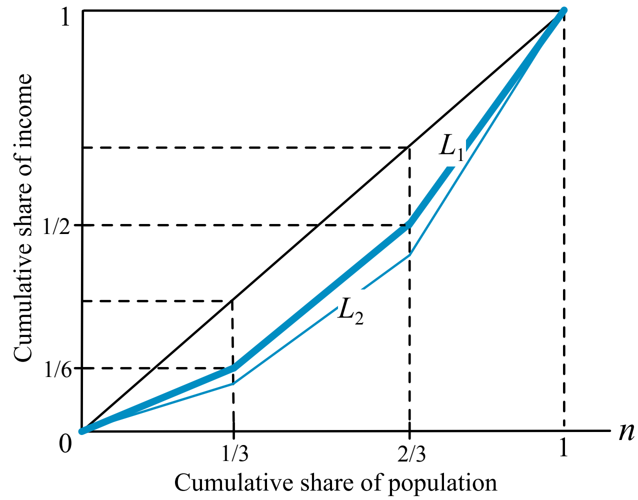
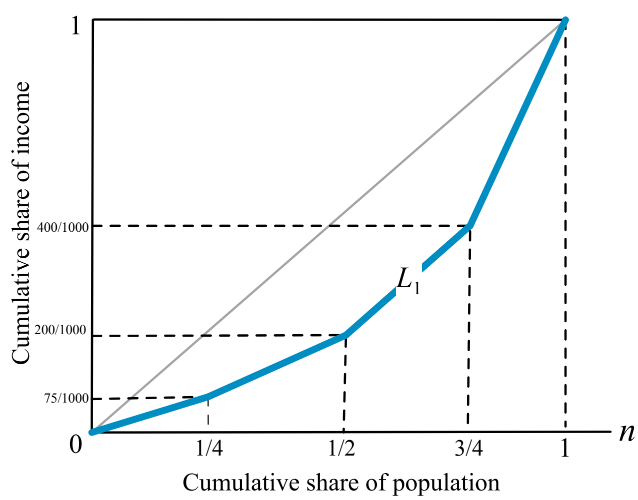


- Nice: the axioms and the Lorenz criterion are the same thing..
  - But also tells us that the axioms don't cover all cases, **because Lorenz curves can cross**.

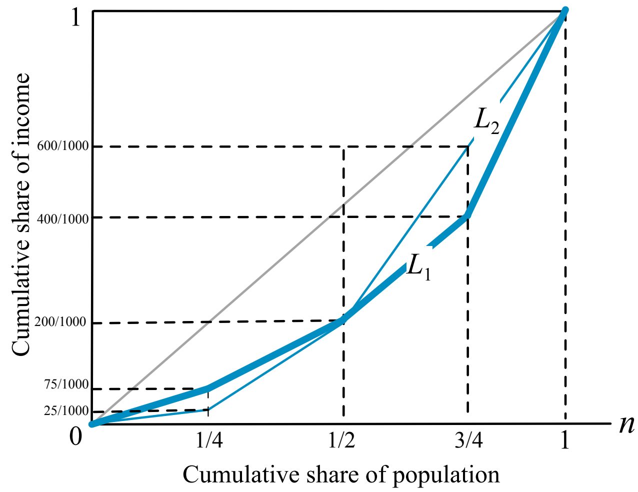
The Lorenz Criterion

- **Another example**
  - $n_1 = n_2 = n_3 = n_4 = 1/4$
  - Situation 1: $(75, 125, 200, 600)$
  - Situation 2: $(25, 175, 400, 400)$



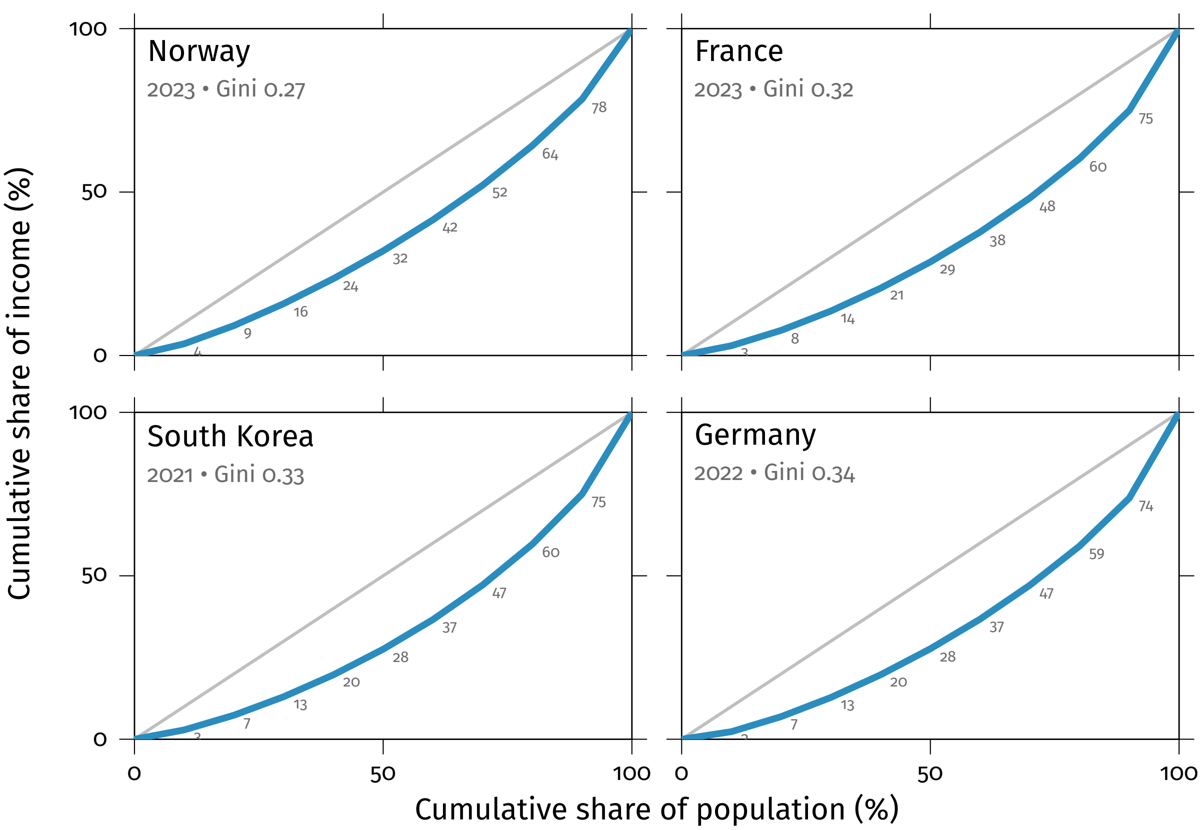
The Lorenz Criterion

- **Another example**
  - $n_1 = n_2 = n_3 = n_4 = 1/4$
  - Situation 1: $(75, 125, 200, 600)$
  - Situation 2: $(25, 175, 400, 400)$
  - Lorenz curves cross.

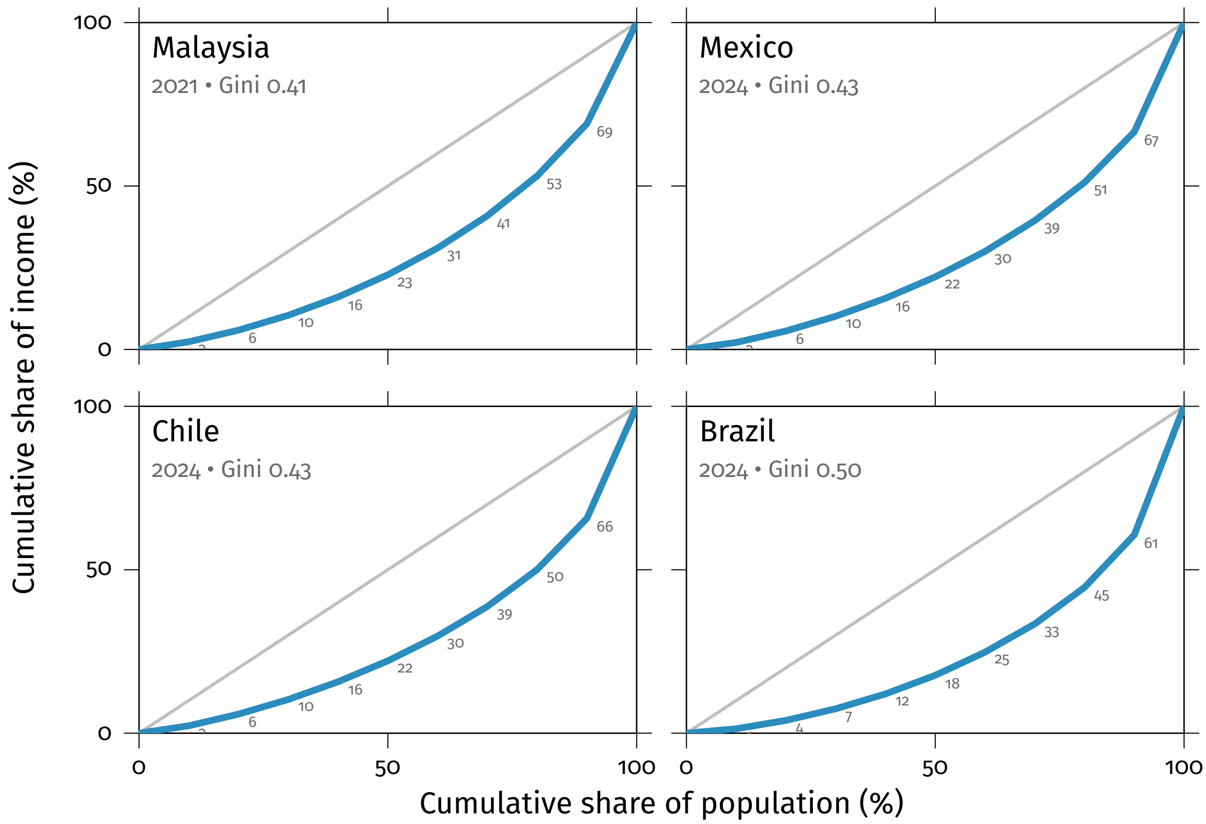


- **A third example**. Modern sector enlargement (from Fields, 1980).
  - 10 people, rural wage equals 50, urban wage equals 100
  - rural-urban migration

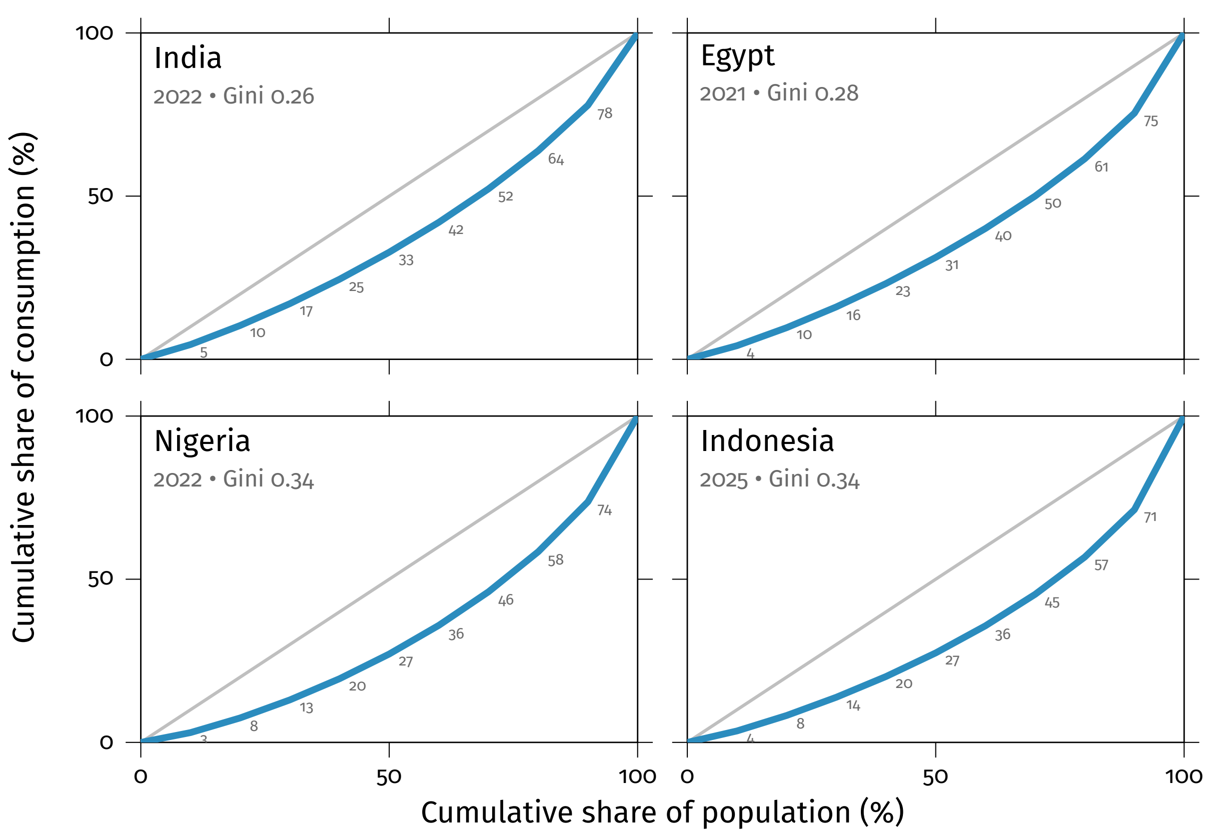
Some Lorenz Curves



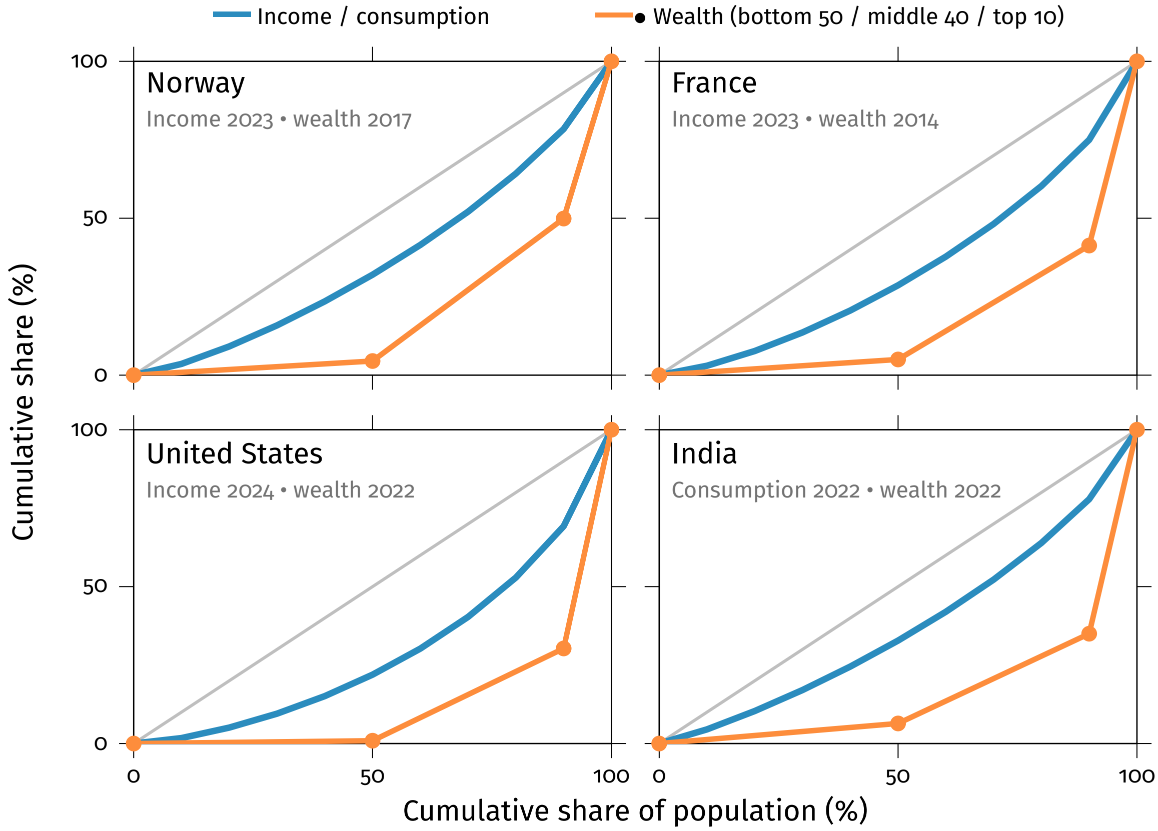
Some Lorenz Curves



Some Lorenz Curves: Consumption Data



Lorenz Curves for Income and Wealth



Complete Measures of Inequality

- **The Lorenz ordering is partial:**
  - Lorenz-consistent measures complete this ordering in different ways.
  - We also want measures that take full advantage of available information:
  - m groups, $\mathbf{y} = (y_1, \dots y_m)$, $(N_1, \dots, N_m)$,
  - $n_i = \frac{N_i}{N}$ is pop share, $\mu = (\sum_{i=1}^{m} N_i y_i)/N$ is mean income.
- Examples of popular crude measures:
  - Range: $(y_m - y_1)/\mu$.
  - Kuznets ratios: (richest x%)/(poorest y%); e.g., 20%/80%.

Complete Measures of Inequality

- More seriously:

- **Mean absolute deviation:** $\frac{1}{\mu}\sum_{j=1}^{m}\frac{N_j}{N}|y_j-\mu| = \frac{1}{\mu}\sum_{j=1}^{m} n_j|y_j-\mu|$.

- Is this Lorenz-consistent? Check axioms.

- **Coefficient of variation:** $\frac{1}{\mu}\sqrt{\sum_{j=1}^{m}\frac{N_j}{N}(y_j-\mu)^2} = \frac{1}{\mu}\sqrt{\sum_{j=1}^{m} n_j(y_j-\mu)^2}$.

- Is this Lorenz-consistent? Check axioms.

Complete Measures of Inequality

- **Gini**: $\frac{1}{2\mu}\sum_{j=1}^{m}\sum_{k=1}^{m}\frac{N_jN_k}{N^2}|y_j-y_k| = \frac{1}{2\mu}\sum_{j=1}^{m}\sum_{k=1}^{m} n_jn_k|y_j-y_k|$.

- Is this Lorenz-consistent? Check axioms.

- A **variant** of the Gini (he had 13 of them!):

- **Gini′**: $\frac{1}{2\mu}\sum_{j=1}^{m}\sum_{k=1}^{m}\frac{N_jN_k}{N(N-1)}|y_j-y_k|$.

- Declares that a two-person society can exhibit perfect inequality.

  Recall debate.

- But fails the population principle.

- **Gini (G) compared with coefficient of variation (COV)**
  - They may disagree when Lorenz curves cross.
- Example. Compare $A = (3, 12, 12)$ and $B = (4, 9, 14)$.
  - COV(A): $\frac{1}{9}\sqrt{\frac{1}{3}(3-9)^2 + \frac{2}{3}(12-9)^2} = \frac{1}{9}\sqrt{18}$.
  - COV(B): $\frac{1}{9}\sqrt{\frac{1}{3}(4-9)^2 + \frac{1}{3}(14-9)^2} = \frac{1}{9}\sqrt{\frac{50}{3}}$.
  - So the COV falls going from A to B.
  - $G(A)$: $\frac{[2|3-12|+2|12-3|]}{2.9^2} = \frac{18}{81}$.
  - $G(B)$: $\frac{[|4-9|+|4-14|+|9-4|+|9-14|+|14-4|+|14-9|]}{2.9^2} = \frac{20}{81}$.
  - So the Gini rises going from A to B.

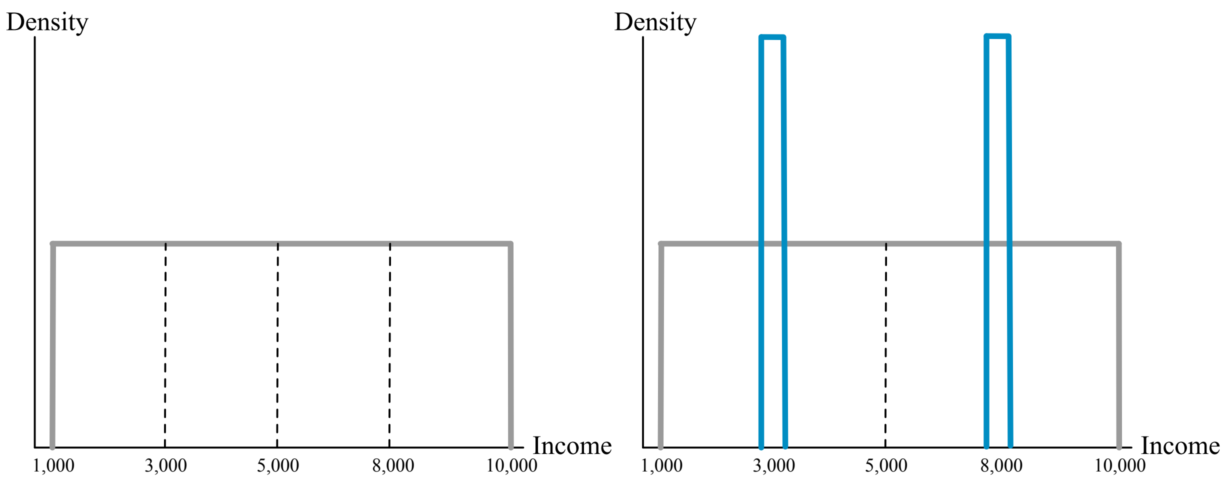
Gini and the COV: Real-Life Disagreement

Country	Gini	COV	Richest 5%	Poorest 40%
Puerto Rico				
1953	0.415	1.152	23.4	15.5
1963	↑0.449	↓1.035	↓22.0	↑13.7
Argentina				
1953	0.412	1.612	27.2	18.1
1959	↑0.463	↑1.887	↑31.8	↑16.4
1961	↑0.434	↓1.605	↑29.4	↑17.4
Mexico				
1950	0.526	2.500	40.0	14.3
1957	↑0.551	↓1.652	↓37.0	↑11.3
1963	↑0.543	↓1.380	↓28.8	↑10.1

- **Inequality in three countries** (Weisskoff (1970), Fields (1980)).
  - Arrows ↑ and ↓ show direction of inequality relative to base year.

Beyond Economic Inequality

- "As the struggle proceeds, the whole society breaks up more and more into two hostile camps, two great, directly antagonistic classes ... The classes *polarize*, so that they become internally more homogeneous and more and more sharply distinguished from one another in wealth and power." (Deutsch 1971, p.44)



Polarization

- **Polarization adds up alienations, like the Gini**
- But individual alienations are themselves determined by group size:
- $n_i|y_i - y_j|$ instead of $|y_i - y_j|$.
- **Add these**:

$$P = \frac{1}{\mu}\sum_{i=1}^{m}\sum_{j=1}^{m} n_i^2 n_j |y_i - y_j|.$$

- Looks very much like the Gini coefficient, but different.
- Can be applied to study the connection between distribution and conflict.