Development Economics

Slides 4

Debraj Ray, NYU

Theory of Economic Growth

Combines production function with consumption-savings choices.

- A constant fraction of income is saved, and the rest consumed:

$$S(t) = sY(t)$$

- Savings equals investment: $S(t) = I(t)$.

- Investment adds to capital stock:

$$K(t+1) = (1-\delta)K(t) + I(t) = (1-\delta)K(t) + sY(t)$$

where δ is the rate of depreciation.

- This is the **accumulation equation**.

Theory of Economic Growth

$$K(t+1) = (1-\delta)K(t) + sY(t)$$

- Convert to per-capita magnitudes: $k = K/L$, $y = Y/L$:

$$(1+n)k(t+1) = (1-\delta)k(t) + sy(t)$$

  where n is the rate of population growth.

  - Combine with per-capita production function $y = f(k)$:

$$(1+n)k(t+1) = (1-\delta)k(t) + sf(k(t))$$

  -

$$\Rightarrow \quad \frac{k(t+1)}{k(t)} = \frac{(1-\delta) + s\Gamma(k(t))}{1+n}$$

- where $\Gamma(k) \equiv y/k = f(k)/k$ is the output-capital ratio.

Theory of Economic Growth

$$\frac{k(t+1)}{k(t)} = \frac{(1-\delta) + s\Gamma(k(t))}{1+n}$$

- This is the **fundamental equation for the growth model**.

  - A lot now hangs on what we view as exogenous or endogenous.

  - $(s, \delta, n, \Gamma) \ldots$

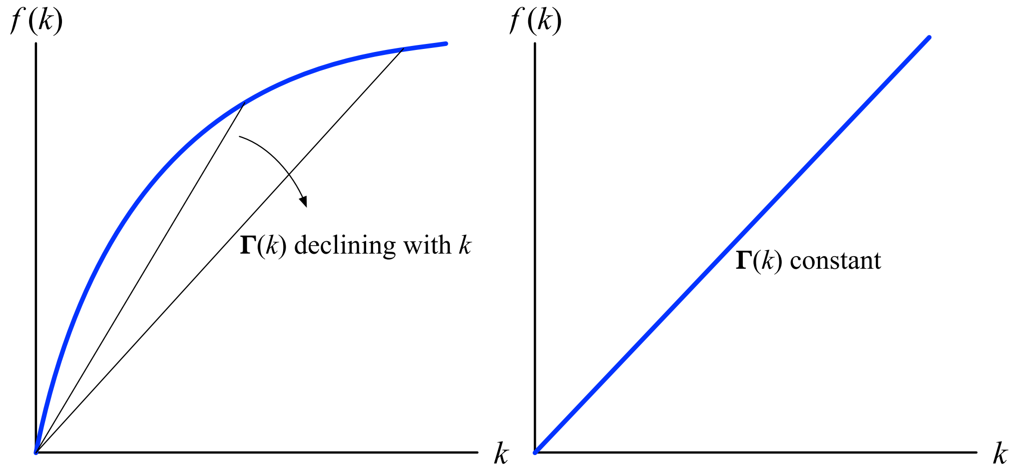
Solow and Harrod-Domar: Introduction

- In the **Solow model**, the output-capital ratio Γ is **endogenous**:

  It falls as k rises, because of diminishing returns to capital.

- In the **Harrod-Domar** model, the output-capital ratio Γ is **exogenous**:

  The distinction with Solow comes from the implicit role played by labor.



Theory of Economic Growth: The Solow Model

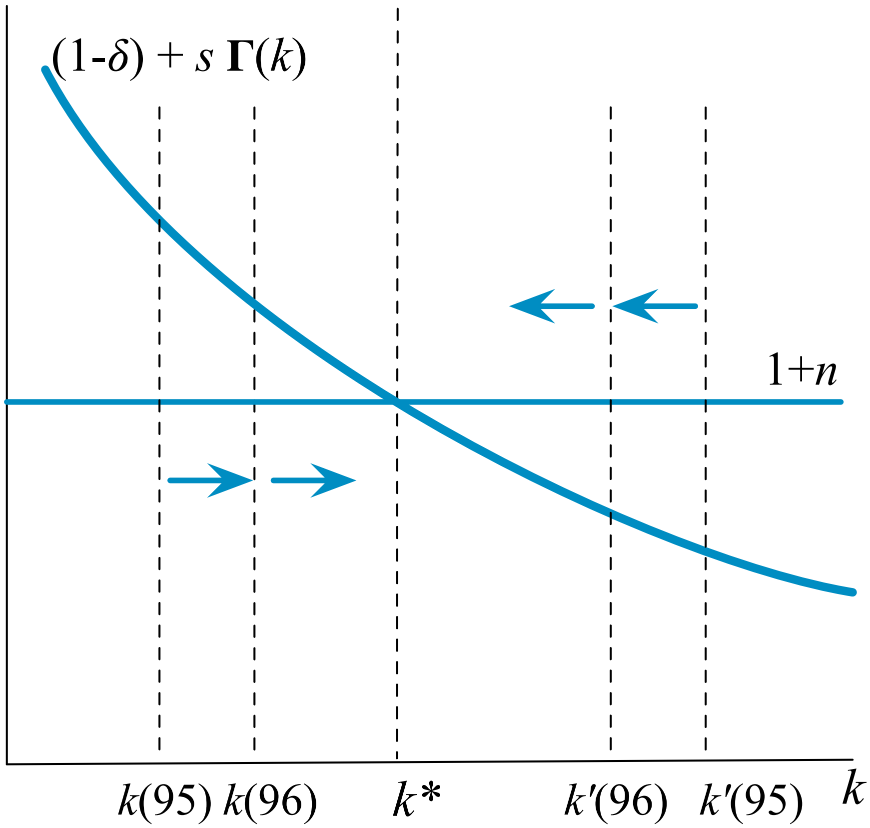
- Solow first, as it is the central growth model, then just a short remark on Harrod-Domar.

$$\frac{k(t+1)}{k(t)} = \frac{(1-\delta) + s\Gamma(k(t))}{1+n}$$

- **A central prediction:** $k(t+1)/k(t)$ declines as $k(t)$ goes up.

  - Reason: As $k(t)$ goes up, $\Gamma(k(t))$ comes down.
  - Implication: "Convergence."

Theory of Economic Growth: The Solow Model



The Steady State

- **Steady state capital-labor ratio** of the economy is k^*, which solves:

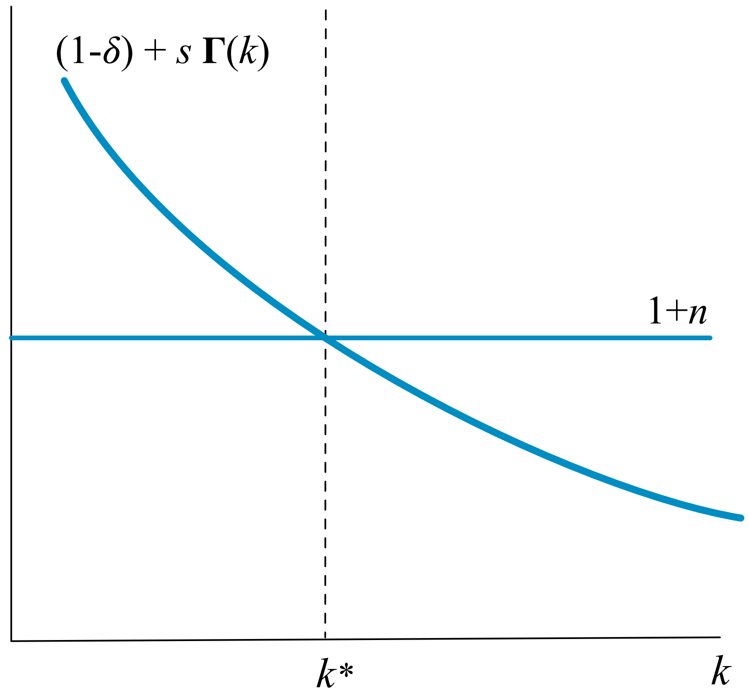
$$\underbrace{\Gamma(k^*) = \frac{y^*}{k^*} = \frac{f(k^*)}{k^*}}_{\text{all the same}} = \frac{n+\delta}{s}$$

  - $k^* = K(t)/L(t)$, but $K(t)$ and $L(t)$ keep rising at rate n.
  - Argument crucially depends on diminishing returns to inputs.

- No long-run growth over and above population growth:
  - Any extra growth only comes from technical progress, as we shall see.

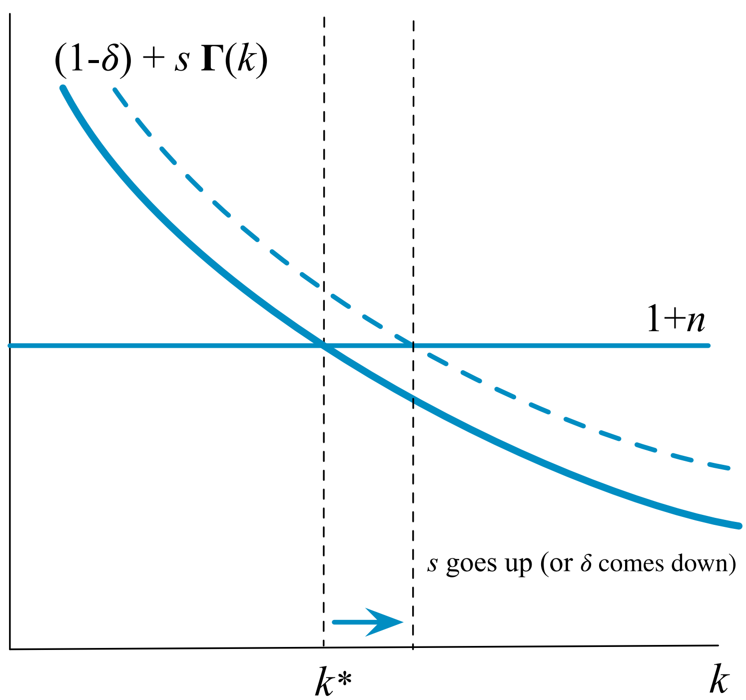
The Steady State

- **How s, n and δ affect k^*:**



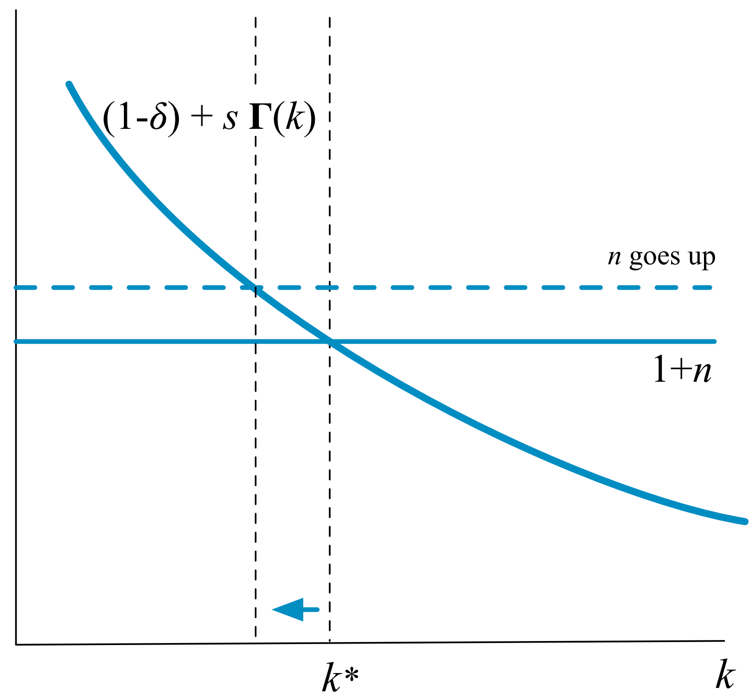
The Steady State

- **How s, n and δ affect k^*:**



The Steady State

- **How s, n and δ affect k^*:**



The Steady State for Cobb-Douglas Production

- Steady state k^* solves $\frac{f(k^*)}{k^*} = \frac{n+\delta}{s}$.

- With Cobb-Douglas production, $f(k) = Ak^a$, so:

$$\frac{f(k^*)}{k^*} = Ak^{*a-1} = \frac{n+\delta}{s}$$

$\Rightarrow$

$$k^* = \left(\frac{sA}{n+\delta}\right)^{1/(1-a)} \text{ and } y^* = A^{1/(1-a)}\left(\frac{s}{n+\delta}\right)^{a/(1-a)}$$

- Can directly verify the properties we established geometrically earlier.

Two Sources of Per-Capita Growth

Exogenous versus endogenous growth

- **Exogenous growth:** comes from "outside" the model.

  ongoing technical progress

- **Endogenous growth:** comes from "inside" the model.

  induced technical progress

  absence of diminishing returns

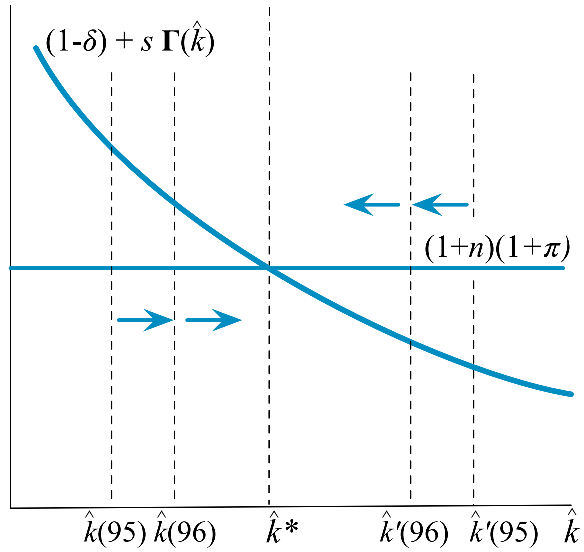
Exogenous Growth: Technical Progress

- $Y(t) = F(K(t), e(t)L(t))$, so $e(t)L(t)$ is "effective" labor.
- Exogenous technical progress: $e(t+1) = e(t)(1+\pi)$, $\pi > 0$.
- Divide through in the accumulation equation by $e(t)L(t)$ instead of $L(t)$:

$$(1+\pi)(1+n)\hat{k}(t+1) = (1-\delta)\hat{k}(t) + sf(\hat{k}(t)),$$

- where $\hat{k} = K/eL$ is in "effective per-capita" units.
- And now do exactly what we did before ...

Steady State in Effective Units With Technical Progress



- **Steady state:** $\dfrac{f(\hat{k}^*)}{\hat{k}^*} \approx \dfrac{(1+n)(1+\pi)-(1-\delta)}{s}$
- **Per-capita long run growth rate** = π.

Steady State in Effective Units With Technical Progress

$$\frac{f(\hat{k}^*)}{\hat{k}^*} = \frac{(1+n)(1+\pi)-(1-\delta)}{s} \approx \frac{n+\delta+\pi}{s},$$

- where we approximate: $(1+\pi)(1+n) = 1+\pi+n+n\pi \approx 1+\pi+n$

  ($n\pi$ is tiny relative to n or π).

- In the Cobb-Douglas case, $\hat{y} = f(k^*) = A\hat{k}^a$, so

$$\hat{k}^* \approx \left(\frac{sA}{n+\delta+\pi}\right)^{1/(1-a)},$$

- ...and steady state output in effective labor units is:

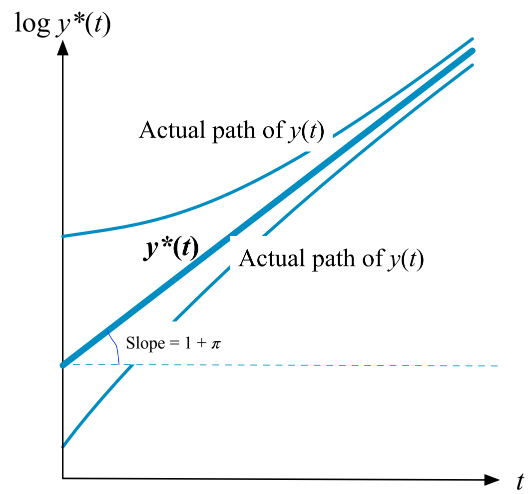
$$\hat{y}^* \approx A^{1/(1-a)} \left(\frac{s}{n+\delta+\pi}\right)^{a/(1-a)}.$$

- Actual per-capita output and capital grow at the rate of π.

Steady State Growth With Technical Progress

- **Path of steady state output per capita:**

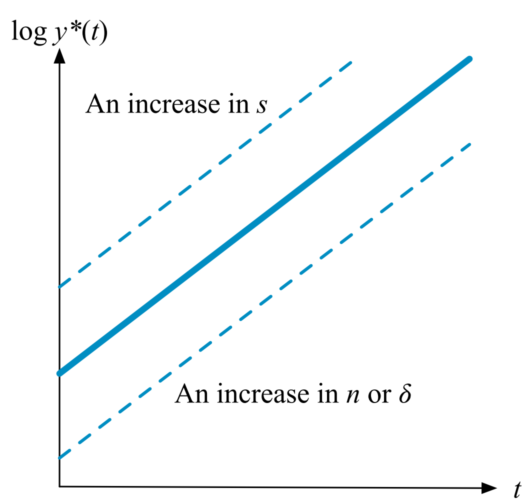
$$y^*(t) = \hat{y}^*(1+\pi)^t \approx A^{1/(1-a)} \left(\frac{s}{n+\delta+\pi}\right)^{a/(1-a)} (1+\pi)^t.$$

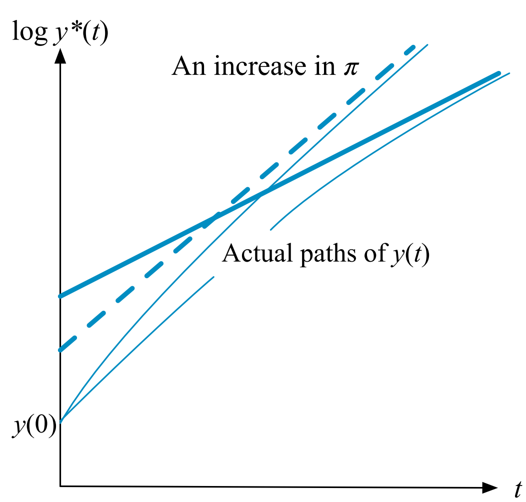


Steady State Growth: Parametric Changes

- **Path of steady state output per capita:**

$$y^*(t) = \hat{y}^*(1+\pi)^t \approx A^{1/(1-a)} \left(\frac{s}{n+\delta+\pi}\right)^{a/(1-a)} (1+\pi)^t.$$





Endogenous Growth: The Harrod-Domar or AK Model

- In the Harrod-Domar model, output-capital ratio is exogenous:
  - $\Gamma(k)$ is unchanging in k.
  - In the Cobb-Douglas model $y = Ak^a$, that happens when $a = 1$.
  - When a hits 1, the model behaves *very* differently:

$$(1+n)k(t+1) = (1-\delta)k(t) + sAk(t).$$

  - Moving terms around:

$$\text{Rate of growth } = \frac{k(t+1)-k(t)}{k(t)} = \frac{sA-(n+\delta)}{1+n}.$$

  - **Now parameters have growth effects, unlike Solow model.**