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ECON-UA 323
Development Economics

Problem Set 5

(1) Are the following statements true or false? It is *not* enough to just guess one or the other. You need to provide an argument for or against, and only then will any credit be awarded.

[a] In the Harris-Todaro model, a policy that creates additional jobs in the formal urban sector must reduce the number of people working in the urban informal sector.

[b] A potential entrant to a new market with superior costs of production may not enter if there are imperfect credit markets *and* increasing returns in production, but will not find it a problem if only one of these restrictions holds.

[c] Consider a complementarity map $x(n)$ and an equilibrium n^* at which the map crosses the 45° line from *below* (that is, $x(n) < n$ just to the left of n^* and $x(n) > n$ just to the right). This equilibrium is stable, because a small increase in n makes even more people want to take the action.

[d] If the savings rate is not fixed but rises with per-capita income (because richer people save a larger fraction of their incomes), then the Solow model can have more than one steady state, so that two economies with identical technology and identical savings *behavior* in the sense just described, can end up at different long-run levels of per-capita income.

[e] A project for a risk-neutral population of 100, that yields \$5 to a winner and \$-3 to a loser, with 60 winners overall, where it is also known that exactly 40 of the winners fully know their identity, but no one else does, must fail a majority vote for adoption.

[f] All other things being equal, it is easier to displace a product using a more efficiently produced newer version if the new product is produced under conditions of *decreasing* rather than *increasing* returns to scale.

(2) A new train route will be planned to service a population of 100 people. They have different willingnesses to pay for the route. Array them in decreasing order of willingness to pay, and suppose that person i has a willingness to pay equal to $100 - i$.

The new route will have a fixed cost of F and a variable cost of 20 per person. The route will be operative only if enough people use the route to cover the average costs of running the train.

(a) Explain intuitively why this is a situation of complementarities.

(b) Prove that there exist two thresholds $F_1 > F_2$ such that if F is either above F_1 or below F_2 , there is a unique equilibrium. Discuss how you would go about solving for F_1 and F_2 .

(c) Describe the equilibria if F lies between F_1 and F_2 .

(d) Prove that no more than 80 people will ever use the train.

(3) Migrandia has a labor force of 1000 people and three sectors: agriculture (a), a formal urban sector (m), and an urban informal sector (i). The formal sector pays a wage that is fixed at $\bar{w}_m = 4$ by law and union contracts. Formal employers hire labor to maximize profits, and their demand for labor is

$$L_m = \frac{800}{w},$$

where w is the wage that the employer pays per worker; at $\bar{w}_m = 4$ they therefore hire exactly $\bar{L}_m = 200$ workers. Anyone who moves to the city but fails to find a formal job works in the informal sector, where income is $w_i = 1$. Formal jobs are allocated among all urban residents by lottery, so that the probability of landing one is $p = \bar{L}_m / (\bar{L}_m + L_i)$. Migration is costless, people are risk-neutral, and nobody can hold a formal job while living in the countryside.

(a) Suppose that the agricultural wage is below 4. Explain why neither of the following can be an equilibrium: (i) everyone without a formal job works in agriculture; (ii) the agricultural wage equals 4 and there is no informal sector. Then write down the Harris-Todaro equilibrium condition for Migrandia and explain in words what it says.

(b) Suppose that agriculture is in the Lewis surplus-labor phase: output is shared on family farms, and over the relevant range the income of an agricultural worker is $w_a = 2$ no matter how many people work there. Find the Harris-Todaro equilibrium: the probability p , informal employment L_i , and agricultural employment L_a .

(c) The government, worried about the size of the informal sector, offers tax breaks to formal firms, which raise their demand for labor at the fixed wage from 200 to 250 workers. Find the new equilibrium. What happens to the number of people in the informal sector, and to the informal sector's share of the urban workforce? Explain the result in words.

(d) Now suppose instead that agriculture is past the surplus-labor phase, with a downward-sloping demand for labor: $w_a = 10 - L_a/100$. Show that with $\bar{L}_m$ formal jobs, equilibrium agricultural employment solves

$$(900 - L_a)(1000 - L_a) = 300 \bar{L}_m,$$

and verify that with $\bar{L}_m = 200$ the equilibrium is $L_a = 700$, $L_i = 100$ and $w_a = 3$. Now the government creates 112 more formal jobs, so $\bar{L}_m = 312$. Verify that the new equilibrium is $L_a = 640$ and $L_i = 48$. Why does the informal sector shrink here, when it grew in part (c)?

(e) Keep the agricultural demand curve of part (d), and forget the tax breaks. Instead, the government pays formal employers a subsidy of σ per worker hired, so that an employer who pays its workers the legal wage of 4 effectively pays only $4 - \sigma$. Find the value of σ at which the informal sector disappears entirely, and the resulting allocation of labor. Compare the marginal product of labor in the two sectors at that allocation, and use the comparison to

explain why wage subsidies and migration restrictions err in *opposite* directions relative to the efficient allocation of labor.

(4) The urban areas around the village of Bhanipur have jobs to offer, and people are thinking of migrating out from the village. But what's nice about Bhanipur is that it has a tradition of reciprocity, people help each other out. Think of it as a community benefit. This problem tries to understand this situation by using a complementarity map.

Suppose that migration is costless and an outside job has a value of 20.

Within Bhanipur, the community benefit has a value of $b = 20(1 - n)$ for *each* resident of the village, where n is the share of migrants leaving Bhanipur.

Each resident in Bhanipur receives a total payoff of $y + b$, where b is the community benefit and y is her village income. They compare this payoff to the payoff of the outside job (which is 20), and decide whether or not to migrate.

Finally, there are three types of people in Bhanipur, of equal sizes in the population. The first receive an income of $y = 5$, the second an income of $y = 10$ and the third an income of $y = 15$.

(a) Explain intuitively (using examples if you like) why the community benefit per person goes down when there are fewer people living in Bhanipur. Also explain in words why this can give rise to a complementarity.

(b) Draw the complementarity map. Don't worry about drawing everything to scale, but do indicate any relevant values on the axes, such as points where the map might jump upwards.

(c) Show that there are multiple equilibria and describe all of them, including the unstable one(s).