

Professor Debraj Ray
 19 West 4th Street, Room 608
 email: debraj.ray@nyu.edu, homepage: <https://debrajray.com/>
 Webpage for course: <https://pages.nyu.edu/debraj/Courses/26UGDev/index.html>

ECON-UA 323
Development Economics

Problem Set 4

(1) OneGood is an even simpler place than TwoGood — it produces just *one* good but with two different techniques of production in the a and m sectors. (So now there is no relative price p .) Here are the two production functions in OneGood:

$$Y_a = 100 \ln(1 + L_a) \text{ and } Y_m = 200 \ln(1 + L_m).$$

To make your life easier, you can assume that both the survival subsistence and the work subsistence are equal to 0. Also don't worry about employment numbers coming out as fractions.

(a) Prove that the two demand functions for labor are given by $L_a = (100/w_a) - 1$ and $L_m = (200/w_m) - 1$, as long as the relevant wage for the former is no higher than 100 and the latter no higher than 200, and otherwise labor demand is zero.

(b) Assume that there are 298 workers in all, who can costlessly switch between the two sectors, and that wages are completely flexible. Show that the equilibrium wage must equal 1 and find the allocation of labor across the two sectors.

(c) Now suppose that the wage rate in sector m has a floor of 2, below which it cannot go. Suppose also that workers make an irreversible decision to migrate between the two sectors, and if they fail to get the m -job, they can become informal vendors at a *fixed, exogenous* income of $1/2$. Using the Harris-Todaro model, find the new allocation of labor across the a , m and informal sectors. (Again, don't worry if some answers don't turn out to be whole numbers.) Also record the fraction of labor in the informal sector.

(d) What is the amount of migration restriction that completely eliminates the informal sector? At that level, what is the wage w_a ?

(e) What is the minimum subsidy on m -wages that allows for an equilibrium with no informal sector? At that level, how is the labor force allocated across a and m ?

(2) In 2007, the Kenyan mobile-phone company Safaricom launched *M-Pesa*, a service that lets people send and receive money by text message. Mobile money is a good example of complementarities: an account is useful only if the people you want to pay, or be paid by, have one too. (It is also an example of "leapfrogging": for many Kenyans, a mobile-money account was their first financial account of any kind.)

Suppose that a mobile-money service is launched in the land of Pesaland. Households are deciding whether or not to open an account. This decision partly depends on how many *other* households are expected to open accounts. Think of a complementarity map that describes

the fraction x of households that *will* open an account as a function of the fraction n of households that are *expected* to open one:

$$x(n) = \begin{cases} A + (0.8)n^2 & \text{for a constant } A, \text{ whenever this value is between 0 and 1.} \\ 0 & \text{if the above value is negative.} \\ 1 & \text{if the above value exceeds 1.} \end{cases}$$

(a) Draw and describe this graph for four values of A : $A = -0.5$, $A = 0.1$, $A = 0.3$, and $A = 0.5$. Pay particular attention to the value of $x(n)$ at the edges 0 and 1, and to the points where it hits the diagonal of the graph. Also, in the region where $x(n)$ lies strictly between 0 and 1, make sure you explain intuitively why an increase in n is increasing $x(n)$.

(b) Analyze the equilibrium adoption of mobile money in Pesaland as A varies. For which ranges of A does a *unique* equilibrium exist? When multiple equilibria exist, what do they look like? Provide some intuition for your answer.

(3) Multiplania is a community that wants to recycle its glass and plastic, and encourages its residents to do so. But you have to take the trash pretty far to find recycling bins, so it is inconvenient to do so. Everyone has a benefit $b > 0$ from just throwing all glass and plastic into the garbage.

On the other hand, anybody who doesn't recycle can be *seen* not doing so, and there is an individual stigma or shame attached to it, a cost $s(n)$, where n is the overall fraction of the community who actually recycle. Assume that $s(n) = a + cn$, where $a \geq 0$ and $c > 0$.

The payoff from not recycling is the benefit minus the shame. The payoff from recycling is just taken to be zero.

(i) Interpret the function $s(n)$. In particular, why should c be positive? What about a ?

(ii) Describe the range of values of (a, b, c) for which there is a unique equilibrium.

(iii) Describe the range of values of (a, b, c) for which there is are multiple equilibria, and describe all the equilibria, including the unstable one.

(iv) In the world of part (iii), consider a policy that imposes a fine $F > 0$ on every garbage thrower. Show that the threshold value for the population share of recyclers that tips the society over into the good equilibrium (i.e., the unstable equilibrium value) must fall with the fine.

(v) Using part (iv), can you discuss a situation where we begin with a bad equilibrium (i.e., one in which no one recycles), impose a fine, then remove the fine after a temporary imposition, with the society now in a good equilibrium with recycling?

(4) Suppose that each of N citizens in the country of Taxland needs to pay a tax of T every year to the government. Each citizen may decide to pay or to evade the tax. Assume that each citizen is risk-neutral and simply seeks to maximize expected income net of any taxes or fines.

If an evader is caught, Taxland imposes a fine of amount F , where $F > T$. However, the Government of Taxland has limited resources to detect evaders. Assume that out of all the people who evade taxes, *the government has the capacity to audit a maximum of m of them, where $1 \leq m < N$.*

Distinguish between two cases: In Case 1, m persons are randomly chosen from the population to be audited. If any of these is an evader, then conditional on being audited, he will be fined for sure. In Case 2, each evader — and no one else — has an anomaly on his tax return which alerts the government. Up to m of them will be randomly audited — and fined.

(a) Write down the expected payoffs in each case from evasion, and show that there are no complementarities in Case 1, while there are complementarities in Case 2.

(b) Contrast Case 1 and Case 2. In Case 1 show that there is typically a unique equilibrium. In Case 2, show that it is always an equilibrium for nobody in society to evade taxes. Are there another equilibria as well? Describe parametric configurations of (m, n, T, F) for which such equilibria will exist, and also describe the equilibria.

(5) Suppose that people's attitudes can take three possible positions: L , M , and R , where you can think of L as leftist, R as rightist, and M as middle-of-the-road. Consider a society in which *it is known that* a fraction α are M types, and the remaining fraction $1 - \alpha$ are divided equally between L and R , but no individual is known to be L , M , or R at first sight. Suppose that each individual gets satisfaction S from expressing his or her own true views, but feels a loss ("social disapproval") in not conforming to a middle-of-the-road position. The amount of the loss depends on the fraction α of M types: suppose that it equals the amount $\alpha/(1 - \alpha)$.

(a) Show that there is a threshold value of α such that everybody in society will express their own view if α is less than the threshold, but will all express M -views if α exceeds the threshold.

(b) What happens if we change the specification somewhat to say that the "social disapproval loss" equals $\beta/(1 - \beta)$, where β is the expected fraction of people who *choose* to express M views (and not necessarily the true fraction of M types)?

(c) Indicate how you would extend the analysis to a case in which there are potential conformist urges attached to each of the views L , M , and R , and not just M .

(6) The country of Linkland has no steel industry and no car industry. Two firms are each considering a large investment: Steelco would build a steel mill, and Autoco would build a car plant. The two investments are *linked*. Autoco would be the steel mill's main customer: without a local car plant, Steelco would have to export its steel at a low price. And without local steel, Autoco would have to import steel at a high price. The two firms decide simultaneously whether to Invest or Not Invest, with the following payoffs (in millions of dollars, Steelco's payoff first):

	Autoco Invests	Autoco Does Not
Steelco Invests	40, 50	-30, 0
Steelco Does Not	0, -20	0, 0

- (a) Using the idea of a complementarity map, find all the equilibria of this interaction. Which one do both firms prefer? Explain why the other one can nevertheless persist.
- (b) The government wants both firms to invest. It offers Steelco a *guarantee*: if Steelco invests and Autoco does not, the government will pay Steelco an amount G . What is the smallest G that makes “both invest” the only equilibrium? How much does the guarantee actually cost the government?
- (c) Compare this with a *subsidy* S that is paid to Steelco whenever it invests. How large must S be to achieve the same result, and what does it cost? Would it be better for the government to offer its guarantee to Autoco instead of Steelco?
- (d) Now suppose that Steelco can build its mill first, and Autoco decides after seeing what Steelco has done. What happens? What does this suggest about the value of being able to commit to an investment early?
- (e) A real economy has many linked sectors, not two. Use this example to explain why a “big push” of the kind proposed by Rosenstein-Rodan is hard to carry out in practice.

Optional Questions: These questions do not need to be handed in.

- (7) This is a question about an abstract situation in which people have the option to “be honest” or to “cheat.” We define payoffs for these two actions.

Let us suppose that people are arrayed in order of decreasing honesty, and that the “honesty payoff” to person i , where i is an index between 0 and 1, is given by $h(i) = 1 - ai$, where i runs from 0 to 1, and a is a positive parameter. This is the direct payoff person i gets from being honest in a particular situation. But she also gets a cheating payoff equal to bn , where n is the fraction of individuals being dishonest in that situation, and $b > 0$ is a positive parameter. Each person has just two choices, to be honest or dishonest, receiving the honesty payoff if she takes the honest action and the cheating payoff if she takes the dishonest action.

Describe the equilibria of this model for different values of the parameters a and b .

- (8) In the question that follows, n refers to the *number* of people rather than a fraction of the population. In the land of Pampa, living in the countryside gives you a fixed payoff of 100 (Pampa has lots of land), while living in a city gives you a payoff that first increases with the number of people living in the city (agglomeration), and then declines after the number of people goes above a certain threshold (congestion). Let us write this payoff as

$$r = 20n - n^2/2,$$

where n is the number of city dwellers in that particular city.

(a) Let N be the total population in Pampa. If only one city can exist in the entire country, trace out the set of equilibria (i.e., population allocations between countryside and city) as N varies from 0 to infinity.

(b) Now suppose that new cities can come up, each yielding exactly the same payoff function as above, and that people can move freely between the countryside and any city, and from one city to another. Describe a particular equilibrium, call it the *maximal city equilibrium*, one in which each city is as large as it can possibly be in equilibrium. Explain how this equilibrium changes as the overall population N varies.

(c) In contrast, a *maximal city dweller* equilibrium is one in which *as many people as possible* live in cities. Again describe this equilibrium, and explain how it changes as the overall population N varies.

(d) Which of these two equilibria would the citizens of Pampa prefer to be in?

(e) Are there still other equilibria in Pampa?

(9) The country of Skillover has the good fortune of generating spillovers in skills! Each citizen in Skillover simultaneously decides whether to acquire “high skill” at cost $c > 0$, or remain low-skilled. Let y_h and y_ℓ denote the incomes earned by high- and low-skilled workers. Now here comes the spillover:

$$y_h = y_h^0 + nH \text{ and } y_\ell = y_\ell^0 + nL,$$

where $y_h^0 > y_\ell^0$ are baseline values for the two incomes, H and L are positive constants, and n is the fraction of the population that chooses to become high skilled. Thus in Skillover, a person’s productivity in both kinds of jobs is positively linked not only to her own skills, but also to the skills of her fellow workers.

(a) There are always positive spillovers ($H > 0$ and $L > 0$) but when exactly is there a *complementarity*?

For the next question, define $\Delta \equiv y_h^0 - y_\ell^0$ to be the “baseline difference” between skilled and unskilled incomes.

(b) Show that if $\Delta < c < \Delta + (H - L)$, there are three possible equilibria: one in which everybody acquires skills, one in which nobody does, and a third in which only a fraction of the population becomes high-skilled. Give an algebraic expression for this fraction in the last case. Explain why this equilibrium is “unstable” and is likely to give way to one of the two extreme cases.

(c) Describe the set of equilibria when $H < L$.

(d) Consider another variation, this time with no skill spillover onto incomes. Specifically, suppose that $y_h = y_h^0$ and $y_\ell = y_\ell^0$ are both fixed and independent of n (that is, $H = L = 0$). However, suppose that the *cost of individual education* is indeed affected by the presence of skilled people $c(n) = (1 - n)/n$. (The idea here is that it is easier to learn if there are more educated people around). Describe the set of equilibria.

(10) In the 1960s, facing massive unemployment in the cities (much of it disguised in the informal sector), the Kenyan government embarked on a “Keynesian” policy of creating new urban jobs through public investment. By many accounts, the size of the informal sector in Kenya went up instead of dropping in the months that followed. Give an economic explanation of this phenomenon, using the Harris-Todaro model.

(11) Carefully review the different migration policies studied in the chapter on Structural Transformation. Explain under what circumstances the flexible equilibrium allocation is the efficient allocation and how different policies situate themselves relative to the efficient allocation.

(12) Here is another two-sector problem. Sector A has the production function

$$Y_a = AL_a^\alpha$$

and there is no profit-maximizing production: output is shared equally.

Sector M has the production function

$$Y_m = ML_m^\mu$$

and all production is carried by profit-maximizing firms. There is no unemployment; all wages are fully flexible.

(a) Find formulae for average incomes or wages in each sector under any *given allocation* of labor L_a and L_m that add up to L .

(b) Prove that people could want to migrate from agriculture to industry even when average incomes in agriculture exceed the wage rate in industry.

(13) This question is about the “hot money” discussed in the box “Monkeying With the Rupee.” Foreign investors have put money into the country of Pecunia. Think of a large number of investors (a continuum of total size 1), each with one dollar invested. Each investor decides whether to keep her money in Pecunia or pull it out. An investor who pulls out gets her dollar back. An investor who stays earns a return $r > 0$ and ends up with $1 + r$ *unless* Pecunia’s currency collapses. Pecunia’s central bank holds dollar reserves θ , where $0 < \theta < 1$: it can pay out up to a fraction θ of the foreign investment at the current exchange rate. If the fraction w of investors who pull out exceeds θ , the reserves run out, the currency is devalued by a fraction d (where $0 < d < 1$), and those who stayed end up with only $(1 + r)(1 - d)$. For simplicity, assume that those who pull out always get their dollar.

(a) Show that if $(1 + r)(1 - d) < 1$, there are two equilibria: one in which everyone stays and one in which everyone pulls out. Why can a crisis happen even though nothing about Pecunia’s economy has changed?

(b) Suppose that $r = 10\%$ and $d = 20\%$. Is a run possible? How large would Pecunia’s reserves have to be to rule out a run altogether?

(c) Pecunia considers a capital control: a tax τ on every dollar taken out of the country. With the numbers in part (b), how large must τ be to rule out the run? What are the costs of such a policy?

(d) Now suppose that the size of the devaluation depends on how large the run is: $d = \delta(w - \theta)$ whenever $w > \theta$, and $d = 0$ otherwise, where $\delta > 0$. Find the condition under which a run is an equilibrium, and show that larger reserves make a run less likely.