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ECON-UA 323
Development Economics

Problem Set 3

(1) (a) Review how the demand for labor is determined by a profit-maximizing firm with a given capital stock. To do, start with the modern-sector production function

$$Y = MK_m^\mu L_m^{1-\mu},$$

along with output price p and wage w_m . Carefully work out every step leading to equation (6.14). For this problem, show both algebraically and intuitively how the TFP parameter M and the wage rate w_m affect labor demand, all other things equal.

(b) Now let's change notation to the archaic sector, where the production function is:

$$Y = AK_a^\alpha L_a^{1-\alpha}.$$

Of course, if this sector is capitalistic, then the answer is the same as in part (a), only with different notation. Otherwise, find a formula for the wage, or per-person payment in the family farm, if all output is divided equally.

(c) If we provisionally think of L_a as a fixed labor supply to the a -sector and reduce it exogenously (for a given K_a), show that under either method of determining wages ((a) or (b)), wages must go up in the a -sector.

(2) Let's study the Lewis open-economy model in which capital is accumulated by saving a fraction of industrial profits, generating the accumulation equation:

$$K_m(t+1) = (1 - \delta)K_m(t) + s \times \text{Profits}.$$

Now suppose a tax of τ is placed on profits, which is transferred lump sum to all workers in the economy irrespective of where they are located. (a) Write down the new accumulation equation between time t and time $t+1$.

Suppose that workers get a chance to vote on whether to place the tax of τ or not.

(b) Would workers unambiguously support the tax? Why or why not? Does your answer vary depending whether or not the economy is currently in the Lewis surplus labor phase?

(3) TwoGood is a simple economy with sectors a and b , which produces just two commodities. The a -good is food, and each person needs to consume 5 units of it if they are to be able to work. After 5, the a -good is not needed any more in consumption, and every person then switches to the m -good, which is a manufactured consumption good, which they don't mind consuming in ever-increasing quantities up to whatever their budget is.

Here are the production functions for TwoGood:

$$Y_a = 200\sqrt{L_a} \text{ and } Y_m = 100\sqrt{L_m}.$$

These are operated by profit-maximizing entrepreneurs who *only* consume the m -good, not the a -good.

The a -good is numeraire, and the price of the m -good is denoted by p , while there is a common wage w — no extra education is needed to move from one sector to the other.

(a) Prove that if the wage is w and price is p , then $L_a = 10,000/w^2$ and $L_m = 2500(p/w)^2$.

Assume first that p is fixed on the international market, and that TwoGood can trade either good all it likes as long as it maintains a balance of payments. Assume that there are 2,000 laborers in the economy, so that $L_a + L_m \leq 2,000$ (if “ $<$ ” holds, the rest are unemployed).

However, the wage w is *endogenous*, though it must be at least 5.

(b) Prove that if $p < 4$, then the wage must settle at 5, and there is unemployment. On the other hand, if $p > 4$, prove that

$$w = \sqrt{5 + \frac{5p^2}{4}} > 5,$$

and there is full employment.

Now suppose that there is *no international trade*; then, the domestic price p becomes endogenous. To make our calculations easy in what follows, we will assume that the labor force is very large, so that the wage rate w is always at work subsistence, which is 5.

(c) Assume that the wage rate settles at 5. Use the first formula in part (a) and the Kalecki-Rao constraint to argue that $L_a = 400$ and $L_m = 400$.

(d) With $w = 5$, what must the price p be to coax m -employers into voluntarily hiring $L_m = 400$?

(e) Continue to assume $w = 5$. Suppose we make the production function in the m -sector twice as productive as before, so that $Y_m = 200\sqrt{L_m}$. What happens to L_m and L_a ? Contrast your finding with the open economy in which p is exogenously given and either good can be freely exported or imported.

(f) Continuing our assumption that $w = 5$, suppose we make the production function in the a -sector twice as productive as before, so that $Y_a = 400\sqrt{L_a}$, while the production function for L_m remains as it originally was. What happens to L_m and L_a ?

Additional Questions for Practice: I won’t hand out answers to these (except for Question 6) as you can check them easily by looking at the slides and the chapters.

(4) Review the sources of change that drive the structural transformation. For each of the following transformations, discuss the sources that might be most relevant:

(a) the agriculture-industry transformation during the Industrial Revolution;

- (b) the agriculture-industry transformation in China;
- (c) the IT revolution and the transformation from manual to digital work;
- (d) the ongoing replacement of labor by AI.
- (5) Describe the two fundamental resource flows — labor and basic needs — that link the agricultural sector with the industrial sector. Discuss the market forces that are relevant to the magnitudes of these flows. (This is an open-ended question that makes you think about just how economic forces operate in an open vs a closed economy.)
- (6) A farm household in rural Mexico consists of five adult sisters and no other dependents. Total annual income depends on the number of sisters working on the farm through the year and is given by the following schedule:

<i>Number of sisters</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>
Total farm output (in \$)	1,000	1,800	2,400	2,800	3,000

Each sister can decide to migrate to Mexico City, where a typical job, commensurate with her skills, pays \$650. A person who migrates to the city cannot come back and work on the farm that year. Furthermore, all city jobs are temporary one-year appointments. Also assume, for all questions that follow, that there is no difference in the cost of living between the city and the countryside. Now consider three different scenarios.

- (a) Suppose the family is completely individualistic: those sisters who work on the farm share the farm income equally among themselves. There are no remittances to or from any family member who goes to the city. Find the number of sisters who will migrate.
- (b) Now suppose the family is completely altruistic: the total family income, whether from a city job or from the farm, is pooled and shared equally, and everyone seeks to maximize this total. How many sisters will this family send off to Mexico City to look for jobs?
- (c) Here is a third possibility. Those sisters who migrate to the city do so selfishly and never send home any remittances, even if they are employed (maybe those in the countryside have no way to verify whether their sisters in the city have a job). However, the family sits down and makes the following arrangement: those sisters who try out their luck in the city will each be sent \$50 a year from the farm income. Find how many sisters will decide to migrate. (Assume that if a sister is indifferent between migrating and not migrating, she does migrate.)
- (d) Compare the numbers in parts (a)–(c). In light of the comparison, discuss the following assertion. “The extent of rural urban migration depends, *ceteris paribus*, on the nature and degree of altruistic links within families.”
- (7) Are the following statements true or false? It is *not* enough to just guess one or the other. You need to provide an argument for or against, and only then will any credit be awarded.

- (a) If some particular wage equates the overall supply and demand for labor in a two-sector model, *no higher wage* can do that as well for the same capital endowments.
- (b) Wages are equal across sectors in the Lewis model but not so in the Kuznets variant of the model.
- (c) If employment is restricted by the Kalecki-Rao constraint, producers can't be maximizing profits at any prices and wages.
- (d) In the open economy version of the structural transformation model, the international price of food adjusts to equate supply and demand in a single country.
- (e) The services sector declines relative to manufacturing over the course of economic development, because the informal sector mainly produces services — and we know that informality declines with development.
- (f) In the closed economy version of the structural transformation model, it is investment in sector m that raises L_m , while an investment in sector a reduces L_m .