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ECON-UA 323

Development Economics

Answers to Problem Set 5

(1) Are the following statements true or false? It is *not* enough to just guess one or the other. You need to provide an argument for or against, and only then will any credit be awarded.

[a] In the Harris-Todaro model, a policy that creates additional jobs in the formal urban sector must reduce the number of people working in the urban informal sector.

False. In the short run the new jobs are filled from the informal pool, so the probability of a formal job rises and so does the expected urban wage. But that attracts fresh migration from agriculture, which refills the informal sector. The Harris-Todaro condition $p\bar{w}_m + (1-p)w_i = w_a$, with $p = \bar{L}_m/(\bar{L}_m + L_i)$, tells us how far this goes. If the agricultural wage does not change at all (the Lewis surplus-labor phase), then p must return to its old value, so L_i rises in exact proportion to $\bar{L}_m$: the informal sector *expands*. Only if the agricultural wage rises as workers leave can the informal sector shrink. What is always true is that the informal sector shrinks as a *share of the urban workforce*; it is the absolute size that can go either way (the Todaro paradox). Question (3) works this out with numbers.

[b] A potential entrant to a new market with superior costs of production may not enter if there are imperfect credit markets *and* increasing returns in production, but will not find it a problem if only one of these restrictions holds.

True. Even if the better product has a lower cost curve, its initial average cost (at relatively small levels of production) may be higher. It will take time for the product to build up to volumes which will lower cost to the point at which it can make a profit. If capital markets are imperfect, the entrant may not have enough funds to cover this interim period.

If only one of these restrictions holds, then there is no problem. If credit markets are positive, then the entrepreneur can ride out the intervening period of losses by getting a loan. Even if credit markets are imperfect or missing, she is fine if the products are produced using decreasing or constant returns to scale. The better product can then make money from day 1, starting at small scales of production.

Credit will be given for the following central points: (i) it takes time for the product quality to spread via word of mouth, and so profits will be negative in the short or medium run, and (ii) it will take access to deep pockets or good capital markets to ride out this short-run period of losses. You will also get credit for drawing diagrams and using them in a clean, careful way to illustrate these points.

[c] Consider a complementarity map $x(n)$ and an equilibrium n^* at which the map crosses the 45° line from *below* (that is, $x(n) < n$ just to the left of n^* and $x(n) > n$ just to the right). This equilibrium is stable, because a small increase in n makes even more people want to take the action.

False. The second half of the statement is exactly why the equilibrium is *unstable*. Perturb n slightly above n^*: then $x(n) > n$, so more people want to take the action than are expected to, the fraction taking it rises further, and the economy moves *away* from n^* towards the next equilibrium above. Perturb it slightly below: $x(n) < n$, fewer want to act than expected, and the economy slides away downwards. A stable equilibrium is one where the map crosses the 45° line from *above*, so that deviations in either direction are reversed. Crossings from below are the tipping points that separate the basins of attraction of the stable equilibria.

[d] If the savings rate is not fixed but rises with per-capita income (because richer people save a larger fraction of their incomes), then the Solow model can have more than one steady state, so that two economies with identical technology and identical savings *behavior* in the sense just described, can end up at different long-run levels of per-capita income.

True. In the Solow model the steady state per-capita income y^* is an increasing function of the savings rate s. If s in turn is an increasing function of y, we have two increasing relationships between the same two variables, and two increasing curves can cross more than once. Each crossing is a steady state: a low-income steady state with a low savings rate that sustains the low income, and a high-income steady state with a high savings rate that sustains the high income. Which one an economy ends up in depends on where it starts, not on its parameters, which are the same in both. This is the macro-feedback version of a development trap: history matters because an endogenous variable (here s) feeds back on the outcome that determines it. (Draw the diagram from the slides: the s-as-a-function-of-y^* curve and the y^*-as-a-function-of-s curve, crossing twice or three times.)

[e] A project for a risk-neutral population of 100, that yields \$5 to a winner and \$-3 to a loser, with 60 winners overall, where it is also known that exactly 40 of the winners fully know their identity, but no one else does, must fail a majority vote for adoption.

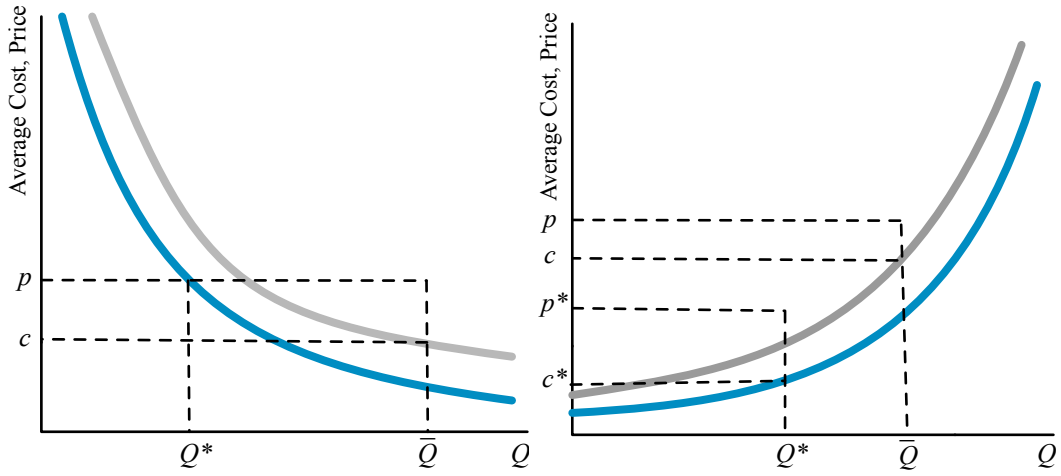
True. If 40 winners know their identity they will all vote yes. But for each of the remaining 60 (the 20 winners who don't know it, and the 40 losers), the probability of being a winner is

$$p = \frac{60 - 40}{100 - 40} = 1/3,$$

and so the expected value for the other 60 voters is negative: $(1/3)(5) + (2/3)(-3) = -1/3 < 0$, so all 60 vote no, and the project fails by 40 votes to 60.

[f] All other things being equal, it is easier to displace a product using a more efficiently produced newer version if the new product is produced under conditions of *decreasing* rather than *increasing* returns to scale.

True. This requires you to draw diagrams at the very least. The idea would be to show that under increasing returns to scale, you might initially need to bear a loss even if the other firm has a worse technology:



If the old firm is charging p, the first diagram shows that for every current quantity of the rival that is *below* Q^*, the rival must make a loss in the current period. Of course, as news of the rival's product spreads then the rival will ultimately make a profit, but must bear the initial losses. If credit markets are imperfect, that may not be possible. In contrast, the second diagram shows that the rival can make profits from day one if there is decreasing returns to scale.

(2) A new train route will be planned to service a population of 100 people. They have different willingnesses to pay for the route. Array them in decreasing order of willingness to pay, and suppose that person i has a willingness to pay equal to $100 - i$.

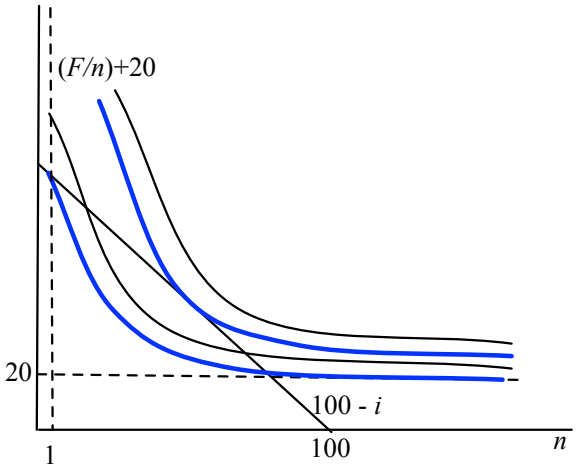
The new route will have a fixed cost of F and a variable cost of 20 per person. The route will be operative only if enough people use the route to cover the average costs of running the train.

(a) Explain intuitively why this is a situation of complementarities.

As more people use the route, the average cost comes down. After all, average cost is given by $(F/n) + 20$, where n is the anticipated number of users, and this is a declining function of n. But if the average cost comes down, more people will be willing to use the route. So we have a case of complementarities.

(b) Prove that there exist two thresholds $F_1 > F_2$ such that if F is either above F_1 or below F_2, there is a unique equilibrium. Discuss how you would go about solving for F_1 and F_2.

Draw a diagram as shown here.



The curved line represents average cost as a function of n; it asymptotes down to 20 as n grows large. The straight line represents willingness to pay. If the curved line lies everywhere above the straight line, then the only equilibrium will be where no one uses transportation. That will happen for any $F > F_1$, where F_1 is the threshold level for which the curved line is just tangent to the straight line (upper blue curve in diagram). You don't have to actually solve out for F_1.

On the other hand, if F is small enough so that the first person is always willing to pay even if he rides alone, there will also be a unique equilibrium, one in which the number of people who use transportation is given by the unique intersection of the curved line with the straight line (think of any curve below the lower blue line). The willingness to pay of the first person is $100 - 1 = 99$, which exceeds $(F/1) + 20$ exactly when F is smaller than 79; so for any F smaller than $F_2 = 79$, person 1 rides even if alone, the no-use equilibrium disappears, and there is a unique equilibrium.

(c) Describe the equilibria if F lies between F_1 and F_2.

Between F_1 and F_2 there are three equilibria. One in which no one uses the train, and two more in which a positive number use the train, given by the solution to the equation

$$\frac{F}{n} + 20 = 100 - n.$$

(This is a quadratic equation. Again, you do not have to solve it, just show it diagrammatically.) If you do solve it: $n^2 - 80n + F = 0$, with roots $n = 40 \pm \sqrt{1600 - F}$. The two roots merge at $n = 40$ when $F = 1600$, which is therefore F_1: at that point average cost and willingness to pay are both equal to 60. You should argue that the lower solution is "unstable" as discussed in class and the upper solution is stable.

(d) Prove that no more than 80 people will ever use the train.

The price that covers cost cannot fall below the variable cost, which is 20. So the largest index for an user cannot exceed 80, recall the formula $100 - i$ for willingness to pay.

(3) Migrandia has a labor force of 1000 people and three sectors: agriculture (a), a formal urban sector (m), and an urban informal sector (i). The formal sector pays a wage that is fixed at $\bar{w}_m = 4$ by law and union contracts. Formal employers hire labor to maximize profits, and their demand for labor is

$$L_m = \frac{800}{w},$$

where w is the wage that the employer pays per worker; at $\bar{w}_m = 4$ they therefore hire exactly $\bar{L}_m = 200$ workers. Anyone who moves to the city but fails to find a formal job works in the informal sector, where income is $w_i = 1$. Formal jobs are allocated among all urban residents by lottery, so that the probability of landing one is $p = \bar{L}_m/(\bar{L}_m + L_i)$. Migration is costless, people are risk-neutral, and nobody can hold a formal job while living in the countryside.

(a) Suppose that the agricultural wage is below 4. Explain why neither of the following can be an equilibrium: (i) everyone without a formal job works in agriculture; (ii) the agricultural wage equals 4 and there is no informal sector. Then write down the Harris-Todaro equilibrium condition for Migrandia and explain in words what it says.

(i) If everyone without a formal job were in agriculture, there would be full employment in both sectors but a wage of 4 in one and a wage below 4 in the other. With costless migration, agricultural workers would leave for the city to try for a formal job, so this cannot be an equilibrium. (ii) If the agricultural wage were 4, agriculture would absorb only as many workers as its demand curve allows at that wage; together with the 200 formal jobs this leaves people who are employed in neither sector. They cannot stay in agriculture (they would bid its wage down below 4) and they cannot be in the formal sector (there are only 200 jobs), so there must be a third place for them: the informal sector. The Harris-Todaro condition is

$$p\,\bar{w}_m + (1-p)\,w_i = w_a, \qquad \text{with } p = \frac{\bar{L}_m}{\bar{L}_m + L_i},$$

that is, $4p + (1-p) = w_a$. It says that a potential migrant compares the sure agricultural wage with the *expected* urban wage: a formal job with probability p, informal work otherwise. In equilibrium the two are equal, so nobody wants to move in either direction. The probability p is itself determined by how many people have already migrated, which is what pins down the allocation of labor across the three sectors.

(b) Suppose that agriculture is in the Lewis surplus-labor phase: output is shared on family farms, and over the relevant range the income of an agricultural worker is $w_a = 2$ no matter how many people work there. Find the Harris-Todaro equilibrium: the probability p, informal employment L_i, and agricultural employment L_a.

With $w_a = 2$, the condition reads $4p + (1-p) = 2$, so $1 + 3p = 2$ and $p = 1/3$. Then $\bar{L}_m/(\bar{L}_m + L_i) = 1/3$ gives $L_i = 2\bar{L}_m = 400$: two informal workers for every formal job. Agricultural employment is the remainder, $L_a = 1000 - 200 - 400 = 400$. The urban sector holds 600 people, of whom only a third have formal jobs.

(c) The government, worried about the size of the informal sector, offers tax breaks to formal firms, which raise their demand for labor at the fixed wage from 200 to 250 workers. Find

the new equilibrium. What happens to the number of people in the informal sector, and to the informal sector's share of the urban workforce? Explain the result in words.

Nothing in part (b) depended on the *level* of $\bar{L}_m$: as long as the agricultural wage stays at 2, the equilibrium probability must still be $1/3$, and so $L_i = 2\bar{L}_m = 500$ and $L_a = 1000 - 250 - 500 = 250$. The informal sector has *grown*, from 400 to 500, although the policy created 50 formal jobs. Its share of the urban workforce is unchanged at $2/3$ (500 out of 750); what has changed is the size of the urban sector, from 600 to 750. In words: the 50 new jobs are initially filled from the informal pool, which raises the chance of a formal job and so the expected urban wage above 2. That draws migrants from agriculture, where the wage does not rise at all because labor is in surplus. Migration continues until the odds are back to one-in-three, which takes 150 migrants for the 50 jobs: 50 into the formal sector and 100 into the informal sector. This is the Todaro paradox: a policy aimed at the informal sector enlarges it, because the migration response dominates the initial soak-up effect.

(d) Now suppose instead that agriculture is past the surplus-labor phase, with a downward-sloping demand for labor: $w_a = 10 - L_a/100$. Show that with $\bar{L}_m$ formal jobs, equilibrium agricultural employment solves

$$(900 - L_a)(1000 - L_a) = 300\,\bar{L}_m,$$

and verify that with $\bar{L}_m = 200$ the equilibrium is $L_a = 700$, $L_i = 100$ and $w_a = 3$. Now the government creates 112 more formal jobs, so $\bar{L}_m = 312$. Verify that the new equilibrium is $L_a = 640$ and $L_i = 48$. Why does the informal sector shrink here, when it grew in part (c)?

The condition is $10 - L_a/100 = 1 + 3p$ with $p = \bar{L}_m/(1000 - L_a)$, since everyone not in agriculture is in the city. So $9 - L_a/100 = 3\bar{L}_m/(1000 - L_a)$, and multiplying through by $100(1000 - L_a)$ gives $(900 - L_a)(1000 - L_a) = 300\bar{L}_m$. With $\bar{L}_m = 200$ the right-hand side is 60,000, so $L_a^2 - 1900L_a + 840{,}000 = 0$, or $(L_a - 700)(L_a - 1200) = 0$. The feasible root is $L_a = 700$; then $L_i = 1000 - 700 - 200 = 100$, $w_a = 3$, and $p = 2/3$: check that $4(2/3)+1(1/3) = 3$. With $\bar{L}_m = 312$ the right-hand side is 93,600, so $L_a^2 - 1900L_a + 806{,}400 = 0$, or $(L_a - 640)(L_a - 1260) = 0$, giving $L_a = 640$, $L_i = 1000 - 640 - 312 = 48$, $w_a = 3.6$ and $p = 312/360 = 13/15$: check that $4(13/15) + 1(2/15) = 54/15 = 3.6$.

This time the informal sector shrinks, from 100 to 48, because the agricultural wage rises as workers leave, from 3 to 3.6. Each migrant makes agriculture a little more attractive for those who remain, which raises the bar that the expected urban wage has to clear; so the 112 new jobs attract only 60 migrants, fewer than the jobs created, and the informal pool falls. In part (c) the agricultural wage was flat, the bar never moved, and migration had to continue until the odds of a formal job were back where they started. The lesson: whether job creation shrinks or expands the informal sector depends on the elasticity of the agricultural labor demand curve.

(e) Keep the agricultural demand curve of part (d), and forget the tax breaks. Instead, the government pays formal employers a subsidy of σ per worker hired, so that an employer who pays its workers the legal wage of 4 effectively pays only $4 - \sigma$. Find the value of σ at which the informal sector disappears entirely, and the resulting allocation of labor. Compare the marginal product of labor in the two sectors at that allocation, and use the comparison to

explain why wage subsidies and migration restrictions err in *opposite* directions relative to the efficient allocation of labor.

With the subsidy, formal employers pay $4-\sigma$ per worker and hire $L_m = 800/(4-\sigma)$, while workers still receive 4. The informal sector disappears when the expected urban wage with $p = 1$, namely 4, equals the agricultural wage, so that nobody has any reason to migrate: $w_a = 10 - L_a/100 = 4$ gives $L_a = 600$, and then $L_m = 400$ with $L_i = 0$. The required subsidy solves $800/(4-\sigma) = 400$, so $4-\sigma = 2$ and $\sigma = 2$: the government pays half the formal wage.

At this allocation the marginal product of labor in agriculture is 4, but the marginal product in the formal sector is only $800/400 = 2$ (that is what employers equate to the net wage they pay). Moving a worker from the formal sector back to agriculture would raise national output by 2, so there is *too much* labor in the formal sector. Compare this with migration restrictions, which keep formal employment at 200 and force the other 800 into agriculture: there the agricultural marginal product is $10 - 8 = 2$ while the formal marginal product is 4, so there is *too little* labor in the formal sector. The efficient allocation equates the two marginal products and lies in between. Both policies abolish the informal sector, but each misses efficiency from the opposite side, which is why the slides consider combining them, or a *uniform* subsidy to both sectors.

(4) The urban areas around the village of Bhanipur have jobs to offer, and people are thinking of migrating out from the village. But what's nice about Bhanipur is that it has a tradition of reciprocity, people help each other out. Think of it as a community benefit. This problem tries to understand this situation by using a complementarity map.

Suppose that migration is costless and an outside job has a value of 20.

Within Bhanipur, the community benefit has a value of $b = 20(1-n)$ for *each* resident of the village, where n is the share of migrants leaving Bhanipur.

Each resident in Bhanipur receives a total payoff of $y + b$, where b is the community benefit and y is her village income. They compare this payoff to the payoff of the outside job (which is 20), and decide whether or not to migrate.

Finally, there are three types of people in Bhanipur, of equal sizes in the population. The first receive an income of $y = 5$, the second an income of $y = 10$ and the third an income of $y = 15$.

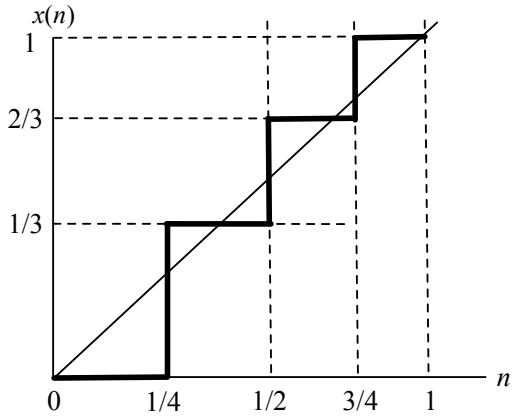
(a) Explain intuitively (using examples if you like) why the community benefit per person goes down when there are fewer people living in Bhanipur. Also explain in words why this can give rise to a complementarity.

Fewer people could imply less scope for reciprocal transactions. There is a smaller variety of people to seek help from. Imagine that you suddenly need, not just a friend, but a friend who can help you in choosing a particular type of seed, or with a specific knowledge about your tractor that's broken down, or for advice about which crop to plant. A larger community also means a more diverse community so the expected benefit is higher.

When the community starts to disintegrate, and people leave, these benefits drop. So at the margin there is a greater incentive for you to migrate as well. This is the defining feature of a complementarity.

(b) Draw the complementarity map. Don't worry about drawing everything to scale, but do indicate any relevant values on the axes, such as points where the map might jump upwards.

Starts at zero. Should jump up to 1/3 at the point $n = 1/4$ because the net payoff now falls exactly to 20 for the first third. Should jump up to 2/3 at the point $n = 1/2$ because the net payoff now falls exactly to 20 for the second third. Should jump up to 1 at the point $n = 3/4$ because the net payoff now falls exactly to 20 for the final third.



(c) Show that there are multiple equilibria and describe all of them, including the unstable one(s).

There are stable equilibria at $n = 0$, $n = 1/3$, $n = 2/3$ and $n = 1$. There are knife-edge unstable equilibria at $n = 1/4$, $n = 1/2$ and $n = 3/4$. I would expect you to show these with a bit of careful discussion.