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ECON-UA 323

Development Economics

Problem Set 4

(1) OneGood is an even simpler place than TwoGood — it produces just *one* good but with two different techniques of production in the a and m sectors. (So now there is no relative price p .) Here are the two production functions in OneGood:

$$Y_a = 100 \ln(1 + L_a) \text{ and } Y_m = 200 \ln(1 + L_m).$$

To make your life easier, you can assume that both the survival subsistence and the work subsistence are equal to 0. Also don't worry about employment numbers coming out as fractions.

(a) Prove that the two demand functions for labor are given by $L_a = (100/w_a) - 1$ and $L_m = (200/w_m) - 1$, as long as the relevant wage for the former is no higher than 100 and the latter no higher than 200, and otherwise labor demand is zero.

This is another “marginal product equals wage” calculation with a small twist that you might be familiar with from intermediate micro. Yes, the marginal product of labor equals the wage, but if it is always lower than the wage, labor demand is zero. In the Cobb-Douglas case that never happens because marginal products are initially close to infinite and then descend to zero, so the solution is always an “interior” one. But with these production functions, that's not the case, as we shall see below. That is what determines the implicit exceptions (“as long as”) in the question above. The rest is easy: just take derivatives to find the MPL and work from there. For instance, in sector a ,

$$\text{MPL}_a = \frac{100}{1 + L_a}.$$

This has a maximum value of 100, and if the wage exceeds 100, it is not worth hiring any labor at all. Otherwise, we set MPL equal to w_a , and so

$$w_a = \frac{100}{1 + L_a}.$$

Rearranging terms, we get the desired answer. Likewise for the m -sector.

(b) Assume that there are 298 workers in all, who can costlessly switch between the two sectors, and that wages are completely flexible. Show that the equilibrium wage must equal 1 and find the allocation of labor across the two sectors.

Equilibrium is achieved at a common wage $w = w_a = w_m$ where the two demand curves for labor add up to 298, or under the condition:¹

$$\frac{100}{w} - 1 + \frac{200}{w} - 1 = 298,$$

from which it follows easily that $w = 1$. Substituting w into the demand curves for labor, we see that $L_a = 99$ and $L_m = 199$.

(c) Now suppose that the wage rate in sector m has a floor of 2, below which it cannot go. Suppose also that workers make an irreversible decision to migrate between the two sectors, and if they fail to get the m -job, they can become informal vendors at a *fixed, exogenous* income of $1/2$. Using the Harris-Todaro model, find the new allocation of labor across the a , m and informal sectors. (Again, don't worry if some answers don't turn out to be whole numbers.) Also record the fraction of labor in the informal sector.

To find the Harris-Todaro equilibrium, let's set up the relevant equations. First, we have the condition that the expected wage after migration should equal the wage in agriculture, or in other words:

$$(1) \quad \frac{L_m}{L_m + L_I} \bar{w}_m + \frac{L_I}{L_m + L_I} w_I = \frac{2L_m}{L_m + L_I} + \frac{1}{2} \frac{L_I}{L_m + L_I} = w_a$$

Next, we note, reading from the labor demand functions, that

$$(2) \quad L_a = (100/w_a) - 1 \text{ and } L_m = (200/w_m) - 1 = (200/2) - 1 = 99,$$

and finally, that

$$(3) \quad L_I = 298 - L_a - L_m.$$

These are four equations in four unknowns and so we can solve them. Since $L_m = 99$, we have $L_I = 199 - L_a$ and $L_m + L_I = 298 - L_a$. Substituting (??) and (??) into (??), we see that

$$\frac{198}{298 - L_a} + \frac{1}{2} \left[\frac{199 - L_a}{298 - L_a} \right] = \frac{100}{1 + L_a},$$

or simplifying,

$$L_a^2 - 794L_a + 59005 = 0.$$

This has two positive roots but only one of them is smaller than 298, which it needs to be. That value is given by $L_a \approx 83.0$. So we have

$$L_a \simeq 83.0, \quad L_m = 99, \text{ and } L_I \simeq 116.0,$$

with an agricultural wage of $w_a = 100/(1 + L_a) \simeq 1.19$. The fraction of the labor force in the informal sector is $116.0/298 \simeq 38.9\%$.

(d) What is the amount of migration restriction that completely eliminates the informal sector? At that level, what is the wage w_a ?

¹If you're reading carefully, you should note that an easy but extra step is missing: show that in equilibrium, both demands for labor are strictly positive.

To have enough migration restriction that completely eliminates the informal sector, we must put all of $298 - 99 = 199$ people in the a -sector. To soak that many people up, the wage rate must be given by the solution to

$$w_a = \frac{100}{1 + L_a} \text{ evaluated at } L_a = 199,$$

or $w_a = 1/2$. (Notice that this is exactly the informal-sector income: at any lower agricultural wage, people would rather migrate and take their chances, and the informal sector would reappear.)

(e) What is the minimum subsidy on m -wages that allows for an equilibrium with no informal sector? At that level, how is the labor force allocated across a and m ?

Let the subsidy be given by s per unit of hired labor; then using the demand curve for labor,

$$(4) \quad L_m = \frac{200}{2 - s} - 1.$$

But two other conditions need to be met. First, the a -wage needs to rise to 2 for a migration equilibrium in the absence of any informal sector. But that means that

$$L_a = \frac{100}{2} - 1 = 49.$$

The other condition is that the informal sector is eliminated, so L_m must be $298 - 49 = 249$. Using this information in (4), we can solve out for the size of the subsidy:

$$\frac{200}{2 - s} - 1 = 249, \text{ or } s = 1.2.$$

(2) In 2007, the Kenyan mobile-phone company Safaricom launched *M-Pesa*, a service that lets people send and receive money by text message. Mobile money is a good example of complementarities: an account is useful only if the people you want to pay, or be paid by, have one too. (It is also an example of “leapfrogging”: for many Kenyans, a mobile-money account was their first financial account of any kind.)

Suppose that a mobile-money service is launched in the land of Pesaland. Households are deciding whether or not to open an account. This decision partly depends on how many *other* households are expected to open accounts. Think of a complementarity map that describes the fraction x of households that *will* open an account as a function of the fraction n of households that are *expected* to open one:

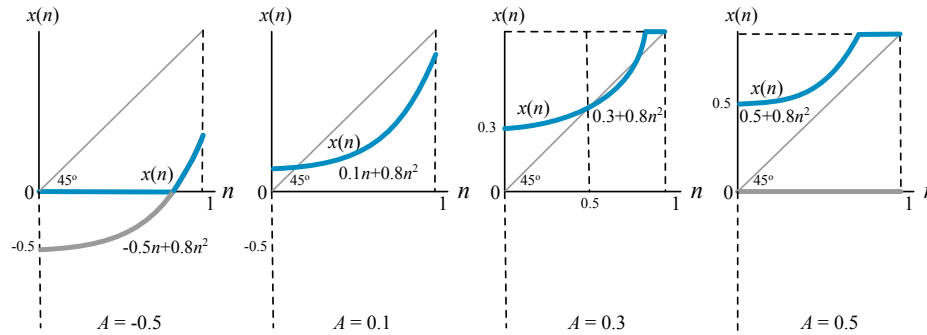
$$x(n) = \begin{cases} A + (0.8)n^2 & \text{for a constant } A, \text{ whenever this value is between 0 and 1.} \\ 0 & \text{if the above value is negative.} \\ 1 & \text{if the above value exceeds 1.} \end{cases}$$

(a) Draw and describe this graph for four values of A : $A = -0.5$, $A = 0.1$, $A = 0.3$, and $A = 0.5$. Pay particular attention to the value of $x(n)$ at the edges 0 and 1, and to the points where it hits the diagonal of the graph. Also, in the region where $x(n)$ lies strictly between 0 and 1, make sure you explain intuitively why an increase in n is increasing $x(n)$.

The fraction of households x who want to open an account (as a function of the fraction n who are expected to open one) is given by the equation:

$$x = A + (0.8)n^2$$

as long as x , calculated by this equation, lies between zero and 1. Specifically, $x(n) = 0$ for all n such that x as defined above is negative, $x(n)$ is given by the above equation when it generates values between zero and 1, and once the above equation generates larger numbers than 1, we set $x(n) = 1$.



The diagram above captures this for the four cases $A = -0.5$, $A = 0.1$, $A = 0.3$, and $A = 0.5$. Each panel plots the function $A + (0.8)n^2$, but “truncates” it at the bottom by 0 and at the top by 1. So, for instance, when $A = -0.5$, the resulting $x(n)$ curve is flat and then follows the $A + (0.8)n^2$ function once that becomes positive. And when $A = 0.3$, the function starts positive anyway so $x(n)$ faithfully follows it, but once it rises above 1 we truncate $x(n)$ at the top.

In the strictly increasing segment, the idea is that if more households use mobile money, an account becomes more useful: you can pay the shopkeeper or the landlord, or receive money from relatives working in the city, by phone. So $x(n)$, the fraction of households that *want* an account, goes up as well.

(b) Analyze the equilibrium adoption of mobile money in Pesaland as A varies. For which ranges of A does a *unique* equilibrium exist? When multiple equilibria exist, what do they look like? Provide some intuition for your answer.

Intuitively, when A is low, it should be optimal *never* to open an account. And when A is large, it should be optimal to *always* open one, so there should be a unique equilibrium for both low and high A . To go along with this intuition, in the first panel of the diagram, A is low (negative in fact) and there is only one contact between $x(n)$ and the diagonal, and that is at $n = 0$. That is the unique equilibrium, with zero adoption of mobile money. In the second panel, there is again a unique contact with the diagonal, but this time at a strictly positive value as shown. This situation will also have a unique equilibrium, but in that equilibrium there will be some positive adoption. In the third panel, there are three intersections, one with low adoption, one with high, and one with intermediate values of adoption (with $A = 0.3$, the three equilibria are at $n = 0.5$, $n = 0.75$ and $n = 1$). The

intermediate one, at 0.75, you should check is “unstable”. Finally, in the fourth panel, there is again just one contact, and that is at $n = 1$.

In class, we are particularly interested in cases such as those depicted by the third panel, where there may be multiple steady state outcomes to the same underlying system, as in the case of QWERTY and Dvorak.

Which values of A generate these multiple solutions? You can easily see this in your mind’s eye by sliding A up continuously as you move from the first to the fourth panel. At intermediate values of A , the complementarity generates two stable solutions: one with low adoption of mobile money, the other high. And in those cases, there is always an unstable solution in between.

You can do all this diagrammatically. What follows is not needed for this question but if you took this path, you are really understanding the material more deeply.

To find the range of A , we first ask the question: when does the equation

$$A + (0.8)n^2 = n$$

have one solution or more? This will help build our answer. The theory of quadratic equations says that the roots of the equation are given by

$$n = \frac{1 \pm \sqrt{1 - 4 \times (0.8) \times A}}{2 \times 0.8} = \frac{1 \pm \sqrt{1 - (3.2) \times A}}{1.6}$$

Notice that if $A > 1/3.2 \simeq 0.312$, then there is no solution (all roots are imaginary). This is the threshold beyond which you have a picture like Panel 4 of the Figure above. In that case, there is a unique equilibrium $n^* = 1$. Everyone adopts mobile money.

For every value of A smaller than $1/3.2$, there are indeed two real solutions, but we have to see if they both lie between zero and 1. The larger solution is

$$n = \frac{1 + \sqrt{1 - (3.2) \times A}}{1.6}$$

and you can easily check that if this is *larger* than 1, then we have a situation like the first or the second panel with just one unique equilibrium. That is the same as the condition that $\sqrt{1 - (3.2) \times A} > 0.6$, or equivalently,

$$1 - (3.2) \times A > 0.36, \text{ or } A < 0.2.$$

In all such cases, and as long as $A > 0$, there is just one solution to $A + (0.8)n^2 = n$ that lies between 0 and 1, and that is the lower root

$$n = \frac{1 - \sqrt{1 - (3.2) \times A}}{1.6},$$

which corresponds to the situation in Panel 2. And once A goes below 0, this lower root turns negative. We still have a unique equilibrium, but now at $n^* = 0$, as in Panel 1.

Panel 3 — which is the case of multiple equilibria — corresponds to the remaining intermediate range of A , which is $0.2 \leq A \leq 1/3.2$.

(3) Multiplania is a community that wants to recycle its glass and plastic, and encourages its residents to do so. But you have to take the trash pretty far to find recycling bins, so it is inconvenient to do so. Everyone has a benefit $b > 0$ from just throwing all glass and plastic into the garbage.

On the other hand, anybody who doesn't recycle can be *seen* not doing so, and there is an individual stigma or shame attached to it, a cost $s(n)$, where n is the overall fraction of the community who actually recycle. Assume that $s(n) = a + cn$, where $a \geq 0$ and $c > 0$.

The payoff from not recycling is the benefit minus the shame. The payoff from recycling is just taken to be zero.

(i) Interpret the function $s(n)$. In particular, why should c be positive? What about a ?

The function $s(n)$ is increasing in n if $c > 0$. (You could draw a picture.) It means that if more people recycle, the (fewer) people who don't would stand out more easily and in the eyes of the community there would be a bigger sense of shame. If everyone is flouting the norm, then the sense of shame would be pretty small. We have still allowed for people to feel a "baseline" sense of shame a , which may or may not be zero.

(ii) Describe the range of values of (a, b, c) for which there is a unique equilibrium.

For there to be a unique equilibrium, the function that maps expected number of recyclers n into the *actual* number of recyclers x must lie everywhere above or everywhere below the 45° line. For the first, we need

$$b < s(n) = a + cn$$

for all n . In that case, $x = 1$ no matter what the value of n , and everyone recycles. But for the above to hold for *every* n , it is necessary and sufficient that it holds at $n = 0$. That implies $a > b$, which is the required condition.

For the second case, we need

$$b > s(n) = a + cn$$

for all n . In that case, $x = 0$ no matter what the value of n , and no one recycles. But for the above to hold for *every* n , it is necessary and sufficient that it holds at $n = 1$. That means $b > a + c$, which is the required condition.

(iii) Describe the range of values of (a, b, c) for which there is are multiple equilibria, and describe all the equilibria, including the unstable one.

In the remaining region; that is, whenever

$$a \leq b \leq a + c,$$

there are multiple equilibria. Now there is one equilibrium where no one recycles — after all, if $n = 0$, then x is also equal to zero because $b \geq s(0) = a$. There is another where everyone recycles — after all, if $n = 1$, then x is also equal to 1 because $b \leq s(1) = a + c$. Assuming that strict inequality holds everywhere above (it will get you a bit of extra credit if you make this point clear), there is a third equilibrium located at the point n^* where

$$b = s(n^*) = a + cn^*,$$

or equivalently, at

$$n^* = \frac{b-a}{c}.$$

This equilibrium is always unstable. Draw the picture.

(iv) In the world of part (iii), consider a policy that imposes a fine $F > 0$ on every garbage thrower. Show that the threshold value for the population share of recyclers that tips the society over into the good equilibrium (i.e., the unstable equilibrium value) must fall with the fine.

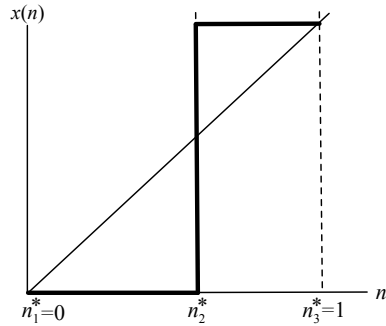
With a fine F , the net payoff from *not* recycling is given by

$$b - s(n) - F = b - a - cn - F.$$

A bad equilibrium in which $n = 0$ can still be supported, therefore, if when $n = 0$, everyone derives a net benefit from not recycling; that is, if

$$b - a - F > 0.$$

So the situation looks as in this diagram:



and the unstable threshold is given by the point n_2^* where the net payoff from recycling is exactly zero; that is, $b - a - cn - F = 0$, or

$$n_2^* = \frac{b-a-F}{c}.$$

It is easy to see from the above equation that as we increase the fine, the threshold population needed for the unstable tip-over to recycling becomes lower.

(v) Using part (iv), can you discuss a situation where we begin with a bad equilibrium (i.e., one in which no one recycles), impose a fine, then remove the fine after a temporary imposition, with the society now in a good equilibrium with recycling?

Suppose that we are in the situation of part (iii), where we have multiple equilibria but are currently stuck in the bad equilibrium with $n^* = 0$. Choose a fine F such that $F > b$. Now there is no reason to rely on shame, because the fine is higher than the benefit from throwing garbage, so now everyone will recycle. After this is in place, remove the fine. The equilibrium with $n^* = 1$ will stay in place without the fine. Only if many people simultaneously start throwing garbage might society move back to the equilibrium where everyone throws garbage on the street. In this sense, temporary policy interventions can have a permanent effect.

(4) Suppose that each of N citizens in the country of Taxland needs to pay a tax of T every year to the government. Each citizen may decide to pay or to evade the tax. Assume that each citizen is risk-neutral and simply seeks to maximize expected income net of any taxes or fines.

If an evader is caught, Taxland imposes a fine of amount F , where $F > T$. However, the Government of Taxland has limited resources to detect evaders. Assume that out of all the people who evade taxes, *the government has the capacity to audit a maximum of m of them, where $1 \leq m < N$.*

Distinguish between two cases: In Case 1, m persons are randomly chosen from the population to be audited. If any of these is an evader, then conditional on being audited, he will be fined for sure. In Case 2, each evader — and no one else — has an anomaly on his tax return which alerts the government. Up to m of them will be randomly audited — and fined.

(a) Write down the expected payoffs in each case from evasion, and show that there are no complementarities in Case 1, while there are complementarities in Case 2.

If I am an evader, then in Case 1, I will be caught with probability $p = m/N$, where N is the total population. In Case 2, the probability of being audited is m/n , where n is now the number of actual evaders (counting me). Well, it is slightly more complicated than that, because if $m \geq n$, then I will be audited with probability 1, and otherwise with probability m/n , so in this case the exact description is that $p = \min\{m/n, 1\}$. E.g., if $n = 3$ and $m = 1$, then there are three evaders and the chance of my getting caught is one out of three or $1/3$. If $n = 2$ and $m = 3$, then I will be audited for sure.

Anyway, denote the audit probability in either Case by p . If I am not caught, then I pay nothing. But if I am caught, then I pay a fine of F . Thus my expected payout is p times F . As a potential evader, I will compare this loss with the sure payment of T (if I do not evade), and take the course of action that creates smaller losses. It's a matter of comparing T to pF , and choosing the smaller expected payment. Notice that p could be endogenous and that is what this question is all about.

Notice that in Case 1, this value is independent of the number of evaders n , so that there are no complementarities (the net payoff from evasion is constant). But in Case 2, *p falls with the number of evaders n , at least when $n \geq m$.* This is a case of complementarities: if one person becomes an evader, she makes it easier for other people to evade. This is because the probability of getting caught comes down, so that the expected losses from evasion come down as well.

(b) Contrast Case 1 and Case 2. In Case 1 show that there is typically a unique equilibrium. In Case 2, show that it is always an equilibrium for nobody in society to evade taxes. Are there another equilibria as well? Describe parametric configurations of (m, n, T, F) for which such equilibria will exist, and also describe the equilibria.

In Case 1 there is a unique equilibrium, yielding full evasion or no evasion depending on whether $T - (mF/N)$ is positive or negative. (There are only multiple equilibrium outcomes in the knife-edge case in which $T = mF/N$.) In Case 2, to see that “no evasion” is an

equilibrium, suppose that nobody in the economy is evading. You are a potential evader. If you pay your taxes you will pay T . If you evade, then $n = 1$, while $m \geq n$, so p has to be 1, which is just another way of saying that you will be caught for sure. So that your expected loss is simply F . But $F > T$ by assumption. It follows that if nobody else is evading, you won't evade either. The same mental calculation holds for everybody, so that "no evasion" all around is an equilibrium.

What about everybody evading? Suppose that this is indeed happening around you, and you are considering evasion. If you do evade, then $n = N$, so that your expected losses are mF/N . It follows that if $mF/N < T$, you will jump on the bandwagon and evade as well. Thus "widespread evasion" is *also* an equilibrium provided that the condition $T > mF/N$ holds. (There is also an intermediate equilibrium with $n^* = mF/T$ evaders, at which $mF/n^* = T$ and each evader is exactly indifferent. Like the other interior equilibria in this problem set, it is unstable: one more evader makes evasion strictly attractive, one fewer makes it strictly unattractive.)

(5) Suppose that people's attitudes can take three possible positions: L , M , and R , where you can think of L as leftist, R as rightist, and M as middle-of-the-road. Consider a society in which *it is known that* a fraction α are M types, and the remaining fraction $1 - \alpha$ are divided equally between L and R , but no individual is known to be L , M , or R at first sight. Suppose that each individual gets satisfaction S from expressing his or her own true views, but feels a loss ("social disapproval") in not conforming to a middle-of-the-road position. The amount of the loss depends on the fraction α of M types: suppose that it equals the amount $\alpha/(1 - \alpha)$.

(a) Show that there is a threshold value of α such that everybody in society will express their own view if α is less than the threshold, but will all express M -views if α exceeds the threshold.

The gain from being your own self is S . If you are an L -type or an R -type, you will also feel a loss equal to $\frac{\alpha}{1-\alpha}$. Therefore the net gain from being your own type L or R is $S - \frac{\alpha}{1-\alpha}$. This is negative if $\alpha > \frac{S}{1+S}$. Above this threshold, everybody will say that they are type M , and below that everyone will freely report their own view.

(b) What happens if we change the specification somewhat to say that the "social disapproval loss" equals $\beta/(1 - \beta)$, where β is the expected fraction of people who *choose* to express M views (and not necessarily the true fraction of M types)?

In this case, there are two possibilities. First, assume that $\alpha > \frac{S}{1+S}$, the threshold derived in part (a). Note that a fraction α (the true M -types) will always say that they are type M , because they have nothing to gain by stating any other position. But by part (a), the other types will hide their identity, which raises the value of β (the announced M -types) above the value of α . This process can only stop when everybody announces that they are type M . On the other hand, if $\alpha \leq \frac{S}{1+S}$, there is an equilibrium in which everybody announces their true type, and so $\beta = \alpha$. You can check that nobody will want to deviate from their announcements. But at the same time, there is another conformity equilibrium in which everybody announces that they are type M (and in which β takes on the value of one).

(c) Indicate how you would extend the analysis to a case in which there are potential conformist urges attached to each of the views L , M , and R , and not just M .

If there are potential conformist urges attached to each of the views L , M and R (and not just M), then other equilibria appear. There may be conformist equilibria in which everybody announces L , or in which everybody announces R (try and provide a simple algebraic example of this).

(6) The country of Linkland has no steel industry and no car industry. Two firms are each considering a large investment: Steelco would build a steel mill, and Autoco would build a car plant. The two investments are *linked*. Autoco would be the steel mill's main customer: without a local car plant, Steelco would have to export its steel at a low price. And without local steel, Autoco would have to import steel at a high price. The two firms decide simultaneously whether to Invest or Not Invest, with the following payoffs (in millions of dollars, Steelco's payoff first):

	Autoco Invests	Autoco Does Not
Steelco Invests	40, 50	-30, 0
Steelco Does Not	0, -20	0, 0

(a) Using the idea of a complementarity map, find all the equilibria of this interaction. Which one do both firms prefer? Explain why the other one can nevertheless persist.

Each firm's decision depends on what it expects the other to do, so we can describe it with a complementarity map: for each firm, how it responds to the *expected* behavior of the other. If Steelco expects Autoco to invest, it invests ($40 > 0$); if it expects Autoco to stay out, it stays out ($0 > -30$). Likewise, Autoco invests if it expects Steelco to invest ($50 > 0$), and stays out if it expects Steelco to stay out ($0 > -20$). So each firm simply does what it expects the other to do. The complementarity is visible in the payoffs: the other firm's investment raises Steelco's return from investing from -30 to 40, and Autoco's from -20 to 50.

An equilibrium is a situation in which these expectations are borne out, the analogue of a point where the complementarity map meets the 45-degree line: what each firm is expected to do is what it actually does. There are exactly two such situations. In one, both firms expect the other to invest, and both invest, with payoffs (40, 50). In the other, both expect the other to stay out, and both stay out, with payoffs (0, 0). A situation in which only one firm invests is not an equilibrium: the investing firm would regret it, because its expectation that the other would invest has turned out to be wrong.

Both firms prefer the first. But the second persists because neither firm wants to invest *alone*: if each expects the other to stay out, staying out is the right response, and the expectation is self-fulfilling. Nothing about technology or demand rules out industrialization; what is missing is coordination. This is the linkages argument from class: each investment raises the return to the other, and that is exactly a complementarity.

(b) The government wants both firms to invest. It offers Steelco a *guarantee*: if Steelco invests and Autoco does not, the government will pay Steelco an amount G . What is the

smallest G that makes “both invest” the only equilibrium? How much does the guarantee actually cost the government?

With the guarantee, Steelco’s payoff from investing when Autoco does not becomes $-30 + G$. If $G > 30$, investing is better for Steelco *whatever* Autoco does ($40 > 0$ and $-30 + G > 0$), so Steelco invests. Autoco, knowing this, invests too ($50 > 0$). So any G just above 30 million makes “both invest” the unique equilibrium. (At exactly $G = 30$ Steelco is indifferent, and the bad equilibrium survives as a knife-edge case.)

The remarkable thing is the cost: *zero*. The guarantee pays out only if Steelco invests and Autoco does not, and in equilibrium that never happens. A credible promise is enough to change expectations, and once expectations change, the promise never has to be honored.

(c) Compare this with a *subsidy* S that is paid to Steelco whenever it invests. How large must S be to achieve the same result, and what does it cost? Would it be better for the government to offer its guarantee to Autoco instead of Steelco?

A subsidy paid whenever Steelco invests changes Steelco’s payoffs from investing to $40 + S$ and $-30 + S$. Investing becomes better for Steelco *whatever* Autoco does if $S > 30$, so the subsidy must also be just over 30 million. It achieves the same outcome, but now the government actually pays about 30 million, because Steelco does invest. The guarantee is far cheaper, because it is conditioned on the bad event, while the subsidy is paid in the good one.

The guarantee could equally go to Autoco: Autoco then invests whatever Steelco does if $-20 + G_A > 0$, that is, $G_A > 20$, and Steelco follows. This also costs nothing in equilibrium, but the government’s *exposure* is smaller (20 million rather than 30). If the government’s promise is more believable when the amount at stake is smaller, as it usually is for a cash-strapped government, it should back the firm that would lose less from investing alone.

(d) Now suppose that Steelco can build its mill first, and Autoco decides after seeing what Steelco has done. What happens? What does this suggest about the value of being able to commit to an investment early?

Start with Autoco, which moves second. If Steelco has invested, Autoco’s best choice is to invest ($50 > 0$); if Steelco has not, Autoco stays out. Steelco can foresee this, so it compares 40 (invest, and Autoco follows) with 0 (stay out), and invests. Both firms invest, and the bad equilibrium disappears.

The coordination problem in (a) came from the firms having to move at the same time, each unsure of the other. A visible, irreversible early commitment by one firm removes that uncertainty: the second firm no longer has to guess. In practice this works only if the first investment is easy to observe and hard to reverse, which is true of a steel mill but not of many other investments. A related solution is for one business group to own both firms: a combined firm earns $40 + 50 = 90$ from investing in both and so does so, internalizing the linkage. This is one reason why large diversified business groups have been prominent in many developing economies.

(e) A real economy has many linked sectors, not two. Use this example to explain why a “big push” of the kind proposed by Rosenstein-Rodan is hard to carry out in practice.

With two firms and known payoffs, the government knows exactly whom to back and by how much. A real economy has hundreds of sectors linked in complicated ways. To coordinate a big push, the government would need to know which sectors are linked to which, how large each investment should be, and what each firm would lose if it invested alone, so as to size the guarantees. If it gets this wrong, say by backing too much steel and too little transport, some guarantees will be called and become very expensive. It also needs the fiscal capacity and the credibility to make its promises believable. These informational and fiscal demands, more than the logic of complementarities itself, are what make a big push so difficult in practice.

Optional Questions: These questions do not need to be handed in.

(7) This is a question about an abstract situation in which people have the option to “be honest” or to “cheat.” We define payoffs for these two actions.

Let us suppose that people are arrayed in order of decreasing honesty, and that the “honesty payoff” to person i , where i is an index between 0 and 1, is given by $h(i) = 1 - ai$, where i runs from 0 to 1, and a is a positive parameter. This is the direct payoff person i gets from being honest in a particular situation. But she also gets a cheating payoff equal to bn , where n is the fraction of individuals being dishonest in that situation, and $b > 0$ is a positive parameter. Each person has just two choices, to be honest or dishonest, receiving the honesty payoff if she takes the honest action and the cheating payoff if she takes the dishonest action.

Describe the equilibria of this model for different values of the parameters a and b .

Note that in any equilibrium, by the way we have ordered people in order of decreasing honesty, it must be that if i is honest, so is j , where $j < i$ in the index. Now suppose that the fraction of dishonest people is n ; that means from the above argument that $1 - n$ is the index of the “last” honest person. The honesty payoff of that threshold person is $1 - a(1 - n)$, and her cheating payoff is bn .

Can we get an equilibrium in which everyone is honest? That is possible if $n = 0$, which requires $1 - a(1 - n) \geq bn$ evaluated at $n = 0$, or $a \leq 1$. (Interpret this.)

Can we get an equilibrium in which everyone is dishonest? That is possible if $n = 1$, which requires $1 - a(1 - n) \leq bn$ evaluated at $n = 1$, or $b \geq 1$. (Interpret this.)

Now you can write down conditions for unique and multiple equilibrium. Remember that there could also be an interior equilibrium, in which $0 < n^* < 1$. That requires the condition $1 - a(1 - n^*) = bn^*$, or $n^* = (1 - a)/(b - a)$. This lies strictly between 0 and 1 in two cases: when $a < 1 < b$, and when $a > 1 > b$. In the first case both corner equilibria also exist, and the interior one is unstable for the usual reasons: a slightly larger fraction of cheats makes cheating more attractive at the margin, and the society tips to everyone cheating; a

slightly smaller fraction tips it to everyone being honest. In the second case neither corner is an equilibrium, so the interior one is the *unique* equilibrium, and it is stable: the honesty payoffs fall steeply across people (large a) while the gain from cheating rises only slowly with n (small b), so the society settles on a mix of honest and dishonest people.

(8) In the question that follows, n refers to the *number* of people rather than a fraction of the population. In the land of Pampa, living in the countryside gives you a fixed payoff of 100 (Pampa has lots of land), while living in a city gives you a payoff that first increases with the number of people living in the city (agglomeration), and then declines after the number of people goes above a certain threshold (congestion). Let us write this payoff as

$$r = 20n - n^2/2,$$

where n is the number of city dwellers in that particular city.

(a) Let N be the total population in Pampa. If only one city can exist in the entire country, trace out the set of equilibria (i.e., population allocations between countryside and city) as N varies from 0 to infinity.

Suppose there is one city. Notice that r is maximized by setting $n = 20$ (write down the maximization problem for r and set the derivative with respect to n to equal zero). It follows that the maximum value of r is given by $20^2 - 20^2/2 = 200$. But of course, the *actual* value of r will depend on how many people choose to live in Pampa. To find this, look at the intersections of the line $20n - n^2/2$ with the value 100. That means you solve the quadratic equation

$$20n - n^2/2 = 100, \text{ or } n^2/2 - 20n + 100 = 0,$$

which has the pair of solutions

$$n_1 \text{ and } n_2 = 20 \pm \sqrt{400 - 4 * (1/2) * 100}, \text{ or } n_1 = 20 - \sqrt{200} \text{ and } n_2 = 20 + \sqrt{200}.$$

Note that if the population N is less than n_1 , *there is just one equilibrium*, everyone in the country. If $N > n_1$, but no bigger than n_2 , then there are three equilibria.² The two stable ones involve everyone being in the city, or everyone being in the country. There is just one more equilibrium in which the two returns have to be equalized, which means that $n = n_1$ and the rest, which is $N - n_1$, live in the country. Draw a diagram to show that this is unstable.

If $N > n_2$ then again there are three equilibria. One is everyone in the country and one is unstable (as usual). The third involves n_2 people in the city and $N - n_2$ in the country.

(b) Now suppose that new cities can come up, each yielding exactly the same payoff function as above, and that people can move freely between the countryside and any city, and from one city to another. Describe a particular equilibrium, call it the *maximal city equilibrium*, one in which each city is as large as it can possibly be in equilibrium. Explain how this equilibrium changes as the overall population N varies.

²We left out the knife-edge case in which N just happens to be equal to n_1 , in which case there are only two equilibria, either $n = 0$ (stable) and $n = n_1$, which is unstable.

No city can ever have more than $n_2 = 20 + \sqrt{200} \simeq 34.1$ people in equilibrium: beyond n_2 a city pays less than the 100 available in the countryside, and its residents would leave. So the largest a city can be is n_2 , and the maximal city equilibrium packs as many cities as possible to exactly this size:

- If $N < n_1 \simeq 5.9$, no city can pay 100, so everyone lives in the countryside.
- If $n_1 \leq N \leq n_2$, everyone lives in a single city of size N . That is the largest city the population allows.
- If $N > n_2$, let M be the largest whole number with $Mn_2 \leq N$. There are M cities of size n_2 , and the remaining $N - Mn_2$ people live in the countryside.

In the last case everyone, in the cities and in the countryside, earns exactly 100. This is an equilibrium because no one gains by moving on her own: joining an existing city lowers its payoff below 100, and founding a new city alone yields only $20 - 1/2 = 19.5$.

As N grows beyond n_2 , the countryside population rises one-for-one with N until it reaches n_2 ; at that point a new city of size n_2 forms and the countryside empties out again. So the number of cities rises in steps, one step every $n_2 \simeq 34$ people, and the countryside population follows a sawtooth pattern, always lying between 0 and n_2 .

(c) In contrast, a *maximal city dweller* equilibrium is one in which *as many people as possible* live in cities. Again describe this equilibrium, and explain how it changes as the overall population N varies.

Two observations pin down what any equilibrium with cities must look like.

First, if some people live in the countryside, every city must pay exactly 100: if a city paid more, people from the countryside would move in, and if it paid less, people would leave it. So every city has either n_1 or n_2 people. A city of size n_1 is unstable (one more arrival raises the payoff above 100 and draws in more people; one departure lowers it below 100 and the city empties out), so in a stable equilibrium every city has n_2 people.

Second, if there are two or more cities, they must all have the same size, and that size must be at least 20, the size at which the city payoff peaks. To see why, suppose some city had fewer than 20 people. It is then on the rising part of the payoff curve, so a person who moves into it from another city raises its payoff further, while the city she left (if it is at or above 20 people) does at least as well without her. She gains, so the allocation is not an equilibrium. With all cities at or above 20, payoffs fall with size, and equal payoffs then require equal sizes.

So everyone can live in cities exactly when the population can be split into K cities of equal size N/K that satisfy these conditions:

- If $N < n_1$, no city can pay 100, so everyone lives in the countryside.
- If $n_1 \leq N \leq n_2$, everyone lives in a single city.
- If $n_2 < N < 40$, one city cannot hold everyone (its payoff would fall below 100), and two cities would each have fewer than 20 people. So not everyone can live in cities.

The maximal city dweller equilibrium has one city of size n_2 , with the remaining $N - n_2$ (fewer than 6 people) in the countryside.

- If $N \geq 40$, everyone lives in cities: there are K cities of size N/K for any whole number K with $20 \leq N/K \leq n_2$. Such a K always exists, because the ranges of populations that K and $K + 1$ cities can absorb, $[20K, n_2K]$ and $[20(K + 1), n_2(K + 1)]$, overlap once $K \geq 2$. For instance, two cities cover populations from 40 to about 68, and three cover 60 to about 102.

For $N \geq 40$, then, the countryside is empty, and as N grows the cities first grow and then split into more, smaller cities. Notice too that there may be more than one such equilibrium: with $N = 60$, for example, both two cities of 30 and three cities of 20 house everyone.

(d) Which of these two equilibria would the citizens of Pampa prefer to be in?

They prefer the maximal city dweller equilibrium. In the maximal city equilibrium of part (b), each city is pushed all the way to the point n_2 where congestion has driven its payoff down to that of the countryside, so everyone earns exactly 100. In the maximal city dweller equilibrium of part (c), the population is spread over more cities, each held closer to the size of 20 at which the city payoff peaks, so city dwellers typically earn well above 100 and nobody earns less. With $N = 50$, for instance, part (b) has one city of about 34 and 16 people on the land, all earning 100, while part (c) has two cities of 25 in which everyone earns $20(25) - 25^2/2 = 187.5$. (When several maximal city dweller equilibria exist, the best one has the most cities, because its cities are closest to the peak size of 20: with $N = 60$, three cities of 20 pay 200 each, against 150 each for two cities of 30.)

Why can't Pampa simply move from the first equilibrium to the second? Because it requires a group of people to found a new city *together*. A new city pays nothing worth having until it has at least $n_1 \simeq 6$ people, and a lone founder earns only 19.5, so no one wants to go first. This is the complementarity at the heart of this part of the course: an equilibrium that everyone would like to escape persists because no individual can escape it alone. (The two equilibria coincide when $N \leq n_2$, and when $n_2 < N < 40$, where there is room for only one city.)

(e) Are there still other equilibria in Pampa?

Yes, many.

- *Everyone in the countryside* is always an equilibrium, whatever the population. If there are no cities, a person who moves to the city alone earns 19.5, far less than 100. This is the extreme coordination failure.
- *Fewer full cities than possible*: any number of cities of size n_2 from 1 up to M , with everyone else in the countryside, is an equilibrium, by the same argument as in part (b).
- *Other ways of housing everyone*: for $N \geq 40$ there can be several numbers of equal-sized cities that house the whole population, as the example with $N = 60$ shows.

- *Unstable equilibria*: cities of size n_1 , with the rest of the population in the countryside, give everyone a payoff of 100 and so are equilibria too, but a single arrival or departure unravels them.

The point of the question is that a model with agglomeration and congestion generates a whole range of equilibria, from no urbanization at all to full urbanization in well-sized cities, and history and expectations determine which one a country ends up in.

(9) The country of Skillover has the good fortune of generating spillovers in skills! Each citizen in Skillover simultaneously decides whether to acquire “high skill” at cost $c > 0$, or remain low-skilled. Let y_h and y_ℓ denote the incomes earned by high- and low-skilled workers. Now here comes the spillover:

$$y_h = y_h^0 + nH \text{ and } y_\ell = y_\ell^0 + nL,$$

where $y_h^0 > y_\ell^0$ are baseline values for the two incomes, H and L are positive constants, and n is the fraction of the population that chooses to become high skilled. Thus in Skillover, a person’s productivity in both kinds of jobs is positively linked not only to her own skills, but also to the skills of her fellow workers.

(a) There are always positive spillovers ($H > 0$ and $L > 0$) but when exactly is there a *complementarity*?

Our formulation captures the following idea: a person’s productivity is positively linked not only to his own skills, but also to that of his fellow workers. But more than that is true: note that $y_h - y_\ell = (y_h^0 - y_\ell^0) + n(H - L)$, which means that the difference between the incomes from low and high skills *widens* with more people acquiring high skills, as long as $H > L$. In this case, it follows that whenever a person chooses to acquire skills, she increases the return to skill acquisition by everybody else. This is precisely the complementarity that underlies any coordination problem. So the answer is: there is a complementarity if $H > L$.

If $H \leq L$, there is still the spillover created by education but it does not generate a complementarity. If H is smaller than L , then the gap between high-skill and low-skill income actually *narrows* as n goes up, leading to people wanting to take less education as the education around them gets larger.

For the next question, define $\Delta \equiv y_h^0 - y_\ell^0$ to be the “baseline difference” between skilled and unskilled incomes.

(b) Show that if $\Delta < c < \Delta + (H - L)$, there are three possible equilibria: one in which everybody acquires skills, one in which nobody does, and a third in which only a fraction of the population becomes high-skilled. Give an algebraic expression for this fraction in the last case. Explain why this equilibrium is “unstable” and is likely to give way to one of the two extreme cases.

Assume that $\Delta < c < \Delta + (H - L)$. First let us see if “no skill acquisition” can be an equilibrium. To this end, suppose that no one in society is acquiring skills: then $n = 0$. If you are thinking of becoming high-skilled, then the gain in your income is $y_h - y_\ell$, which is just Δ (because $n = 0$). If $\Delta < c$ (which is assumed, see above), then it is not worthwhile for

you to acquire skills. We have thus shown that if everybody believes that nobody else will acquire skills, then no one will indeed acquire skills. Such beliefs therefore form a self-fulfilling prophecy.

Now let us see if universal skill acquisition can be an equilibrium. Suppose that you believe that everybody else will acquire skills: then $n = 1$. Therefore, if you are thinking of becoming high-skilled, then the gain in your income is $y_h - y_\ell$, which is $\Delta + (H - L)$ (because $n = 1$). If $\Delta + (H - L) > c$ (which is assumed — see above) then it is worthwhile for you to acquire skills. We have thus shown that if everybody believes that everybody else will acquire skills, then everyone will acquire skills. These beliefs also form a self-fulfilling prophecy.

Finally, there is a third equilibrium in which just the right amount of people invest in skill acquisition so that everybody is indifferent between acquiring or not acquiring skills. This is given by a fraction of skilled people n^* such that $\Delta + n^*(H - L) = c$, that is,

$$n^* = \frac{c - \Delta}{H - L},$$

which lies strictly between 0 and 1 precisely because $\Delta < c < \Delta + (H - L)$. This is an equilibrium because no one is doing anything suboptimal given his or her beliefs. But you can intuitively see why this equilibrium must be unstable. If for some reason the fraction of skilled people exceeds n^* even by a tiny amount, then it becomes strictly preferable for everyone else to acquire skills, so that we rapidly move to the universal skills equilibrium. If on the other hand, n falls below n^* (if only by a tiny amount), everyone will desist from acquiring skills, so that we move towards the no skills equilibrium.

(c) Describe the set of equilibria when $H < L$.

When $L > H$, this means that low-skill income is more responsive to the fraction of high-skilled people, than high-skill income. The condition $y_h - y_\ell = [y_h^0 - y_\ell^0] + n(H - L) = \Delta + n(H - L)$ is now seen to yield the opposite conclusion: that there will be a unique equilibrium. To see this, first suppose that even when everyone is skilled, the income gap exceeds the cost; that is:

Case (i). $\Delta + (H - L) \geq c$. Then, remembering that $H - L < 0$, it must be that for every $0 \leq n < 1$,

$$y_h - y_\ell = \Delta + n(H - L) > c,$$

and so the unique equilibrium is that everyone acquires skills.

Now consider the opposite situation, that even when no one is skilled, the income gap between high and low skilled people is too low:

Case (ii). $\Delta \leq c$. Then, again remembering that $H - L < 0$, it must be that for every $0 < n \leq 1$,

$$y_h - y_\ell = \Delta + n(H - L) < c,$$

and so the unique equilibrium is that no one acquires skills.

Finally, look at the intermediate case:

Case (iii). $\Delta > c > \Delta + (H - L)$. Then it is easy to see that there is once again a unique equilibrium at n^* skilled people, where n^* is such that

$$c = \Delta + n^*(H - L).$$

In this case there cannot be any multiple equilibrium, for exactly the same reason as the traffic congestion example in the text cannot exhibit multiple equilibria.

(d) Consider another variation, this time with no skill spillover onto incomes. Specifically, suppose that $y_h = y_h^0$ and $y_\ell = y_\ell^0$ are both fixed and independent of n (that is, $H = L = 0$). However, suppose that the *cost of individual education* is indeed affected by the presence of skilled people $c(n) = (1 - n)/n$. (The idea here is that it is easier to learn if there are more educated people around). Describe the set of equilibria.

In this case, note that the cost of acquiring skills becomes infinitely high as n becomes close to zero, while the cost declines to near zero as n approaches one. Thus we see again that there are three equilibria. In the first, there is no skill acquisition because everyone, expecting that there is no skill acquisition, feels that the cost of acquiring high skills will be very high, and so desists from doing so. At the same time, the expectation that everyone acquires skills is also a self-fulfilling prophecy, because in this case the cost of education is very low. And there is a third equilibrium where people are indifferent between the two options. Just as in part (b), this equilibrium must be described by the condition that $y_h - y_\ell = y_h^0 - y_\ell^0 = \frac{1-n^*}{n^*}$, that is, $\Delta = (1 - n^*)/n^*$, or

$$n^* = \frac{1}{1 + \Delta}.$$

As before, this middle equilibrium is unstable: a few more skilled people lower the cost of education below Δ , and everyone acquires skills; a few fewer raise it above Δ , and no one does.

(10) In the 1960s, facing massive unemployment in the cities (much of it disguised in the informal sector), the Kenyan government embarked on a “Keynesian” policy of creating new urban jobs through public investment. By many accounts, the size of the informal sector in Kenya went up instead of dropping in the months that followed. Give an economic explanation of this phenomenon, using the Harris-Todaro model.

This is the “paradox of job creation.” In the Harris-Todaro model, migrants compare the agricultural wage w_a with the *expected* urban income. Ignoring informal earnings for simplicity, the migration equilibrium condition is

$$\frac{L_m}{L_m + L_I} \bar{w}_m = w_a,$$

where L_m is formal employment at the fixed formal wage $\bar{w}_m$, and L_I is the informal sector (or the urban unemployed). Rearranging, the total urban labor force is $L_m + L_I = L_m \bar{w}_m / w_a$, so that

$$L_I = L_m \left(\frac{\bar{w}_m}{w_a} - 1 \right).$$

Since $\bar{w}_m > w_a$, every new formal job attracts *more* than one migrant: it raises the chance of landing a formal job, which draws $\bar{w}_m/w_a$ new people to the city, only one of whom gets

the job. So creating formal urban jobs, with the agricultural wage roughly unchanged, *raises* the size of the informal sector. This is what happened in Kenya, where the government's job-creation program of the 1960s was followed by an increase in urban unemployment and informality. The cure for urban informality cannot be urban job creation alone; it has to involve the rural sector as well.

(11) Carefully review the different migration policies studied in the chapter on Structural Transformation. Explain under what circumstances the flexible equilibrium allocation is the efficient allocation and how different policies situate themselves relative to the efficient allocation.

When wages are fully flexible, the migration equilibrium equates the marginal product of labor across the two sectors, and so (in the absence of other distortions) it maximizes total output: it is the efficient allocation. The problem in the Harris-Todaro model is the floor on the formal wage. With that floor, too few people are hired in the formal sector, and too many migrants wait in the informal sector, where they produce little, so total output falls short of the efficient level. Policies can be ranked against that benchmark:

- *Urban job creation* (with the wage floor unchanged) can worsen matters, because of the paradox of job creation in the previous question: it draws more people into the informal sector.
- *Migration restrictions* can eliminate the informal sector, but they keep too many people in agriculture and depress the agricultural wage, as in question 1(d). They improve on the Harris-Todaro outcome but do not reach the efficient allocation, because the formal sector is still too small.
- *A wage subsidy* to formal employers can reach the efficient allocation: if the subsidy brings the employers' cost of labor down to the flexible-equilibrium wage, they hire the efficient number of workers, while workers still receive the floor wage. Question 1(e) is an example. Its drawback is that it has to be financed, and the taxes that pay for it may create distortions of their own.
- *Combinations* of a subsidy with migration restrictions can also achieve efficiency, and may need a smaller subsidy.

The general lesson is that the inefficiency comes from the wage floor, so policies that work directly on the cost of formal labor do better than policies that work on migration or on the number of urban jobs.

(12) Here is another two-sector problem. Sector A has the production function

$$Y_a = AL_a^\alpha$$

and there is no profit-maximizing production: output is shared equally.

Sector M has the production function

$$Y_m = ML_m^\mu$$

and all production is carried by profit-maximizing firms. There is no unemployment; all wages are fully flexible.

(a) Find formulae for average incomes or wages in each sector under any *given allocation* of labor L_a and L_m that add up to L .

(b) Prove that people could want to migrate from agriculture to industry even when average incomes in agriculture exceed the wage rate in industry.

(a) In sector a , output is shared equally, so each person earns the average product, $w_a = Y_a/L_a = AL_a^{\alpha-1}$. In sector m , profit-maximizing firms hire until the wage equals the marginal product, $w_m = \mu ML_m^{\mu-1}$.

(b) This problem hinges on the fact that the average product exceeds the marginal product on a production function with diminishing returns to labor. I sketch the answer very quickly. The typical family farm with labor pool L wants to maximize

$$\text{Total Income} = Y_a + Y_m = AL_a^\alpha + (L - L_a)w_m,$$

by its choice of L_a (how many to keep on the farm, versus how many to send to the city). That leads to the first order condition:

$$(5) \quad A\alpha L_a^{\alpha-1} = w_m.$$

In turn, w_m is given by the marginal product of labor in sector m :

$$(6) \quad w_m = M\mu L_m^{\mu-1}.$$

Comparing (5) and (6), we see that marginal products are equalized across the two sectors. But here is the crucial difference: while the wage in m is indeed given by the marginal product of labor in sector m , the wage in sector a is given by *average* product of labor in sector a . So, as a consequence, notice that

$$w_a = \frac{AL_a^\alpha}{L_a} = AL_a^{\alpha-1} > A\alpha L_a^{\alpha-1} = w_m,$$

which means that a migration equilibrium occurs in this setting at a wage w_a that *strictly exceeds* the wage w_m . Think hard about why this is the case. Why is that people migrate from a to m , even though $w_a > w_m$?

(13) This question is about the “hot money” discussed in the box “Monkeying With the Rupee.” Foreign investors have put money into the country of Pecunia. Think of a large number of investors (a continuum of total size 1), each with one dollar invested. Each investor decides whether to keep her money in Pecunia or pull it out. An investor who pulls out gets her dollar back. An investor who stays earns a return $r > 0$ and ends up with $1 + r$ *unless* Pecunia’s currency collapses. Pecunia’s central bank holds dollar reserves θ , where $0 < \theta < 1$: it can pay out up to a fraction θ of the foreign investment at the current exchange rate. If the fraction w of investors who pull out exceeds θ , the reserves run out, the currency is devalued by a fraction d (where $0 < d < 1$), and those who stayed end up with only $(1 + r)(1 - d)$. For simplicity, assume that those who pull out always get their dollar.

(a) Show that if $(1 + r)(1 - d) < 1$, there are two equilibria: one in which everyone stays and one in which everyone pulls out. Why can a crisis happen even though nothing about Pecunia’s economy has changed?

Suppose everyone else stays, so $w = 0 \leq \theta$. The currency holds, and staying earns $1 + r > 1$, so no investor wants to leave: “everyone stays” is an equilibrium. Now suppose everyone else pulls out, so $w = 1 > \theta$. The reserves run out and the currency is devalued, so staying earns $(1 + r)(1 - d) < 1$, less than the dollar an investor gets by leaving. So she leaves too: “everyone pulls out” is also an equilibrium. (Each investor is tiny, so her own decision does not affect whether the currency collapses.)

The two equilibria share exactly the same fundamentals: the same return r , the same reserves θ , the same devaluation d . What differs is only what investors expect other investors to do. A crisis can therefore be triggered by anything that shifts expectations, a rumor, a crisis elsewhere, a change of mood, even though nothing about Pecunia’s economy has changed. This is the herding behavior described in the box: the money that flowed in can flow out just as fast, and the flight is self-fulfilling.

(b) Suppose that $r = 10\%$ and $d = 20\%$. Is a run possible? How large would Pecunia’s reserves have to be to rule out a run altogether?

$(1 + r)(1 - d) = (1.1)(0.8) = 0.88 < 1$, so a run is possible. To rule it out with reserves alone, Pecunia must be able to pay out everyone: $\theta \geq 1$, reserves as large as the entire stock of foreign investment. Then even if everyone pulls out, the currency holds, staying always earns $1 + r$, and no one has a reason to leave. But holding that many reserves is very costly, because reserves typically earn far less than the country could earn by investing the money at home. Borrowing the reserves, for instance through a credit line from the IMF, is a way of achieving the same thing more cheaply.

(c) Pecunia considers a capital control: a tax τ on every dollar taken out of the country. With the numbers in part (b), how large must τ be to rule out the run? What are the costs of such a policy?

With the tax, an investor who pulls out gets $1 - \tau$. A run is an equilibrium only if staying during a run is worse than leaving: $0.88 < 1 - \tau$, that is, $\tau < 0.12$. So a tax of at least 12% on capital outflows eliminates the run equilibrium. (Malaysia imposed controls of this kind during the Asian crisis of 1998.)

The costs: investors anticipate that their money may be trapped or taxed when they want to leave, for good reasons as well as bad, so less foreign money comes in to begin with. Controls also invite evasion, and they can damage a country’s reputation with investors for a long time after they are lifted.

(d) Now suppose that the size of the devaluation depends on how large the run is: $d = \delta(w - \theta)$ whenever $w > \theta$, and $d = 0$ otherwise, where $\delta > 0$. Find the condition under which a run is an equilibrium, and show that larger reserves make a run less likely.

“Everyone stays” is still an equilibrium, because with $w = 0$ there is no devaluation. In a run, $w = 1$, so the devaluation is $d = \delta(1 - \theta)$, and a run is an equilibrium if

$$(1 + r)[1 - \delta(1 - \theta)] < 1, \quad \text{that is,} \quad \delta(1 - \theta) > \frac{r}{1 + r}, \quad \text{or} \quad \theta < 1 - \frac{r}{\delta(1 + r)}.$$

Larger reserves shrink the devaluation that a full run would cause, and once θ exceeds $1 - r/[\delta(1 + r)]$, even a full run does not hurt the stayers enough to make leaving worthwhile, so the run equilibrium disappears. Reserves do not have to cover all the foreign money; they only have to make a collapse mild enough that staying remains attractive. (For example, with $r = 10\%$ and $\delta = 0.5$, reserves above $1 - 0.1/0.55 \simeq 0.82$ rule out a run.)