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ECON-UA 323
Development Economics

Problem Set 3

(1) (a) Review how the demand for labor is determined by a profit-maximizing firm with a given capital stock. To do, start with the modern-sector production function

$$Y = MK_m^\mu L_m^{1-\mu},$$

along with output price p and wage w_m . Carefully work out every step leading to equation (6.14).

Profits are given by

$$pY_m - w_m L_m = pMK_m^\mu L_m^{1-\mu} - w_m L_m.$$

Entrepreneurs choose labor L_m to maximize profits, which means that labor is hired up to the point where the value of its marginal product equals the wage. Writing down these first-order conditions by taking derivatives with respect to L_m , we see that:

$$pM(1 - \mu)K_m^\mu L_m^{-\mu} = w_m.$$

Moving terms around, we obtain a formula for the employment of modern labor, *given* the capital stock in the modern sector:

$$L_m = \left[\frac{M(1 - \mu)}{w_m/p} \right]^{1/\mu} K_m.$$

In class we also took a stab at converting this information into an equation for m -profits. It looked complicated but it really is not. All we do is substitute the optimal value of L_m into the expression for profits. Doing so, we see that:

$$\begin{aligned} m\text{-Profits} &= pMK_m^\mu L_m^{1-\mu} - w_m L_m \\ &= pMK_m^\mu \left[\frac{Mp(1 - \mu)}{w_m} \right]^{(1-\mu)/\mu} K_m^{1-\mu} - w_m \left[\frac{Mp(1 - \mu)}{w_m} \right]^{1/\mu} K_m \\ &= \underbrace{\left[\frac{\mu M^{1/\mu} p^{1/\mu} (1 - \mu)^{(1-\mu)/\mu}}{w_m^{(1-\mu)/\mu}} \right]}_B K_m. \end{aligned}$$

For this problem, show both algebraically and intuitively how the TFP parameter M and the wage rate w_m affect labor demand, all other things equal.

The terms within the brackets contain parameters that affect labor demand in obvious ways. For instance, higher TFP M will raise labor demand, and a higher value of the wage w_m or a smaller output price p , will lower labor demand. When these terms remain constant, *modern*

employment expands in proportion to modern capital. All these are basic properties that rely only on constant returns to scale, though our Cobb-Douglas formulation lets us write down specific formulae in closed form.

(b) Now let's change notation to the archaic sector, where the production function is:

$$Y = AK_a^\alpha L_a^{1-\alpha}.$$

Of course, if this sector is capitalistic, then the answer is the same as in part (a), only with different notation. Otherwise, find a formula for the wage, or per-person payment in the family farm, if all output is divided equally.

If all families in the archaic sector simply share their output, then there is no demand for labor as such. Rather, L_a is what it is, and a wage w_a (or average income) is then defined by the *average* product of the agrarian production function — “average product,” because all output is being shared equally and so each person gets the average. The following formula illustrates this for the case of the Cobb-Douglas production technology:

$$w_a = \frac{Y_a}{L_a} = AK_a^\alpha L_a^{-\alpha} = A[K_a/L_a]^\alpha.$$

This takes the place of the labor demand curve.

(c) If we provisionally think of L_a as a fixed labor supply to the a -sector and reduce it exogenously (for a given K_a), show that under either method of determining wages ((a) or (b)), wages must go up in the a -sector.

If the labor force in Sector a is L_a , then under the profit-maximization setup, wages are given by marginal product, so that

$$w_a = A(1 - \alpha)K_a^\alpha L_a^{-\alpha} = A(1 - \alpha)[K_a/L_a]^\alpha,$$

and under the family farm setup, it is given by the average product as shown in the answer to the previous question. In both cases, because of diminishing returns to the labor input, the wage rate rises with the capital-labor ratio K_a/L_a , as we can see from simply inspecting these two equations. In particular, if the labor input L_a falls for some *fixed* value of K_a , both the marginal and the average products must rise.

(2) Let's study the Lewis open-economy model in which capital is accumulated by saving a fraction of industrial profits, generating the accumulation equation:

$$K_m(t + 1) = (1 - \delta)K_m(t) + s \times \text{Profits}.$$

Now suppose a tax of τ is placed on profits, which is transferred lump sum to all workers in the economy irrespective of where they are located. (a) Write down the new accumulation equation between time t and time $t + 1$.

The new equation is:

$$K_m(t + 1) = (1 - \delta)K_m(t) + s(1 - \tau) \times \text{Profits},$$

because retained profits are now just $1 - \tau$ of what they were in the absence of the tax. So accumulation slows down.

Suppose that workers get a chance to vote on whether to place the tax of τ or not.

(b) Would workers unambiguously support the tax? Why or why not? Does your answer vary depending whether or not the economy is currently in the Lewis surplus labor phase?

This question asks you to think about the costs and benefits accruing to workers. The tax revenue is distributed to all workers equally, so they are definitely better off on that score. However, accumulation slows down. How might this affect a worker?

Someone currently working would have benefited from wage increases as accumulation occurs. Remember how $w_a = w_m = w$ climbs in the Lewis model as the economy continues to grow. *But that happens only after the Lewis turning point is reached.* In the surplus labor phase, there is no wage increase at all, and so the workers do not internalize the negative effect of a tax rate — and therefore vote in its favor.

This has the interesting implication that if we are “deep inside” the surplus labor phase, employed workers might actually slow the growth process down, because they could vote in favor of taxes, seeing not much benefit in wage increases from accumulation.

Unemployed workers are in a more ambiguous situation. Even in the Lewis surplus labor phase, they might be willing to vote against taxes because accumulation could cause them to become employed. Doing so at work subsistence is better than not joining at all.

In all of this there is an interesting twist in the tale. An economy just starting its structural transformation — far away from the Lewis turning point — might support high taxation — at least the workers, both employed and unemployed, would. That way they at least get the transfers from the tax revenue. So there could be stickiness for such economies — they stay poor. In contrast, an economy that’s already close to the Lewis turning point will receive more support for lower taxes, because unemployed workers will see the benefits of rapid growth in terms of prospective employment, and in addition, even after the turning point has been reached, employed workers will see their wages rise. So an economy closer to the turning point can grow faster than an economy that is far away from it. Contrast with the Solow convergence prediction.

(3) TwoGood is a simple economy with sectors a and b , which produces just two commodities. The a -good is food, and each person needs to consume 5 units of it if they are to be able to work. After 5, the a -good is not needed any more in consumption, and every person then switches to the m -good, which is a manufactured consumption good, which they don’t mind consuming in ever-increasing quantities up to whatever their budget is.

Here are the production functions for TwoGood:

$$Y_a = 200\sqrt{L_a} \text{ and } Y_m = 100\sqrt{L_m}.$$

These are operated by profit-maximizing entrepreneurs who *only* consume the m -good, not the a -good.

The a -good is numeraire, and the price of the m -good is denoted by p , while there is a common wage w — no extra education is needed to move from one sector to the other.

(a) Prove that if the wage is w and price is p , then $L_a = 10,000/w^2$ and $L_m = 2500(p/w)^2$.

If the wage is w , then in the a -sector, employers choose L_a to maximize:

$$Y_a - wL_a = 200\sqrt{L_a} - wL_a.$$

The first order condition to this problem sets the marginal product of labor equal to the wage, so that

$$100L_a^{-1/2} = w$$

Squaring both sides and transposing terms gives us the answer. Use completely parallel steps to solve for L_m , noting this time that L_m is chosen to maximize:

$$100p\sqrt{L_m} - wL_m.$$

Assume first that p is fixed on the international market, and that TwoGood can trade either good all it likes as long as it maintains a balance of payments. Assume that there are 2,000 laborers in the economy, so that $L_a + L_m \leq 2,000$ (if “ $<$ ” holds, the rest are unemployed).

However, the wage w is *endogenous*, though it must be at least 5.

(b) Prove that if $p < 4$, then the wage must settle at 5, and there is unemployment. On the other hand, if $p > 4$, prove that

$$w = \sqrt{5 + \frac{5p^2}{4}} > 5,$$

and there is full employment.

Recall the conditions for equilibrium studied in class. We must have

$$L_a + L_m = \frac{10,000 + 2500p^2}{w^2} \leq L = 2000,$$

but if strict inequality holds above, then w must settle at work subsistence, which is 5. The trick is to examine what labor demand is at the wage 5. Putting $w = 5$ in the expression above, we see that

$$L_a + L_m = \frac{10,000 + 2500p^2}{w^2} = \frac{10,000 + 2500p^2}{25} = 400 + 100p^2,$$

which is smaller than 2000 whenever $p^2 < 16$. It follows that if $p < 4$, the wage settles at 5, and there is some unemployment.

However, if $p > 4$, it follows from the same logic that

$$L_a + L_m = 400 + 100p^2 > 2000,$$

and now the wage must rise above 5 to bring total demand into line with supply; that is

$$\frac{10,000 + 2500p^2}{w^2} = 2000,$$

and you can solve this to see that

$$w = \sqrt{5 + \frac{5p^2}{4}},$$

which is easily seen to exceed 5 when $p > 4$. There is no unemployment in this case.

Now suppose that there is *no international trade*; then, the domestic price p becomes endogenous. To make our calculations easy in what follows, we will assume that the labor force is very large, so that the wage rate w is always at work subsistence, which is 5.

(c) *Assume* that the wage rate settles at 5. Use the first formula in part (a) and the Kalecki-Rao constraint to argue that $L_a = 400$ and $L_m = 400$.

If we assume that $w = 5$, then we know that $L_a = 10000/w^2 = 10000/25 = 400$. Moreover, output Y_a solves

$$Y_a = 200\sqrt{L_a} = 4000.$$

The Kalecki-Rao constraint states that

$$Y_a = 5(L_a + L_m), \text{ or in other words, } L_m = 400.$$

(d) With $w = 5$, what must the price p be to coax m -employers into voluntarily hiring $L_m = 400$?

Well, we know that the demand function for labor in sector m is given by

$$L_m = 2500(p/w)^2 = 100p^2,$$

so to coax employers in m into hiring $L_m = 400$, it must be that $100p^2 = 400$, or $p = 2$.

(e) Continue to assume $w = 5$. Suppose we make the production function in the m -sector twice as productive as before, so that $Y_m = 200\sqrt{L_m}$. What happens to L_m and L_a ? Contrast your finding with the open economy in which p is exogenously given and either good can be freely exported or imported.

Look at the calculations in part (c). Nothing has changed in those calculations before and after the change in the m -production function, so it must still be true that $L_a = 400$ and $L_m = 400$. But now, the labor demand curve in m has changed. By going through the steps in part (a), it is easy to check that

$$L_m = 10,000(p/w)^2 \text{ instead of } 2500(p/w)^2 \text{ as before.}$$

So to coax $L_m = 400$ at wage 5, it must now be the case that

$$L_m = 10,000(p/w)^2 = 10,000(p^2/25) = 400p^2$$

must be made to equal 400, from which it follows that $p = 1$. The price drops to nullify the higher labor demand in m that's generated by the m -sector becoming more productive, and no new employment is created.

In the open economy version, we are in the Lewis model and goods can be freely exported and imported. At any fixed price p and wage $w = 5$, labor demand in sector m will climb without constraint, from a value of $100p^2$ to $400p^2$.

(f) Continuing our assumption that $w = 5$, suppose we make the production function in the a -sector twice as productive as before, so that $Y_a = 400\sqrt{L_a}$, while the production function for L_m remains as it originally was. What happens to L_m and L_a ?

Originally, labor in agriculture was $L_a = 400$ and output was $Y_a = 200\sqrt{L_a} = 4000$. Now that changes. First, we compute the new demand for labor. This is found by maximizing

$$Y_a - wL_a = 400\sqrt{L_a} - wL_a,$$

and using the first-order condition just as in part (a), we see that

$$200L_a^{-1/2} = w = 5, \text{ or } L_a = 1600.$$

So L_a goes up (by a lot). What about L_m ? Well, output in sector a is now

$$Y_a = 400\sqrt{L_a} = 16,000,$$

so the Kalecki-Rao constraint tells us that

$$Y_a = 16,000 = 5(L_a + L_m) = 5(1600 + L_m),$$

or $L_m = 1600$. All of this expansion comes from unemployment — remember, we assumed that the labor force was very large and that w always equals 5. If instead, we impose a labor force, say 2000 as in the first part of this question, then what will happen is that unemployment will completely vanish and the wage rate will climb above 5 to guarantee that total labor use is exactly at 2000, and not 3200 as in this answer.

Additional Questions for Practice: I won't hand out answers to these (except for Question 6) as you can check them easily by looking at the slides and the chapters.

(4) Review the sources of change that drive the structural transformation. For each of the following transformations, discuss the sources that might be most relevant:

- (a) the agriculture-industry transformation during the Industrial Revolution;
- (b) the agriculture-industry transformation in China;
- (c) the IT revolution and the transformation from manual to digital work;
- (d) the ongoing replacement of labor by AI.

(5) Describe the two fundamental resource flows — labor and basic needs — that link the agricultural sector with the industrial sector. Discuss the market forces that are relevant to the magnitudes of these flows. (This is an open-ended question that makes you think about just how economic forces operate in an open vs a closed economy.)

(6) A farm household in rural Mexico consists of five adult sisters and no other dependents. Total annual income depends on the number of sisters working on the farm through the year and is given by the following schedule:

<i>Number of sisters</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>
Total farm output (in \$)	1,000	1,800	2,400	2,800	3,000

Each sister can decide to migrate to Mexico City, where a typical job, commensurate with her skills, pays \$650. A person who migrates to the city cannot come back and work on the farm that year. Furthermore, all city jobs are temporary one-year appointments. Also assume, for all questions that follow, that there is no difference in the cost of living between the city and the countryside. Now consider three different scenarios.

(a) Suppose the family is completely individualistic: those sisters who work on the farm share the farm income equally among themselves. There are no remittances to or from any family member who goes to the city. Find the number of sisters who will migrate.

Compute average product on the farm; it's given by the following table:

<i>Number of sisters</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>
Total farm output (in \$)	1,000	1,800	2,400	2,800	3,000
Average farm output (in \$)	1,000	900	800	700	600

So it is obvious that one sister will migrate, because she gets 650 instead of 600, but after that no one else will: they each get 700 on the farm instead of 650.

(b) Now suppose the family is completely altruistic: the total family income, whether from a city job or from the farm, is pooled and shared equally, and everyone seeks to maximize this total. How many sisters will this family send off to Mexico City to look for jobs?

If the family is completely altruistic and seeks to maximize family income, then the appropriate variable to look at is *marginal* product: because the family as a whole is seeing if the sister should contribute here on the farm — which is marginal product — compared to earning the outside wage of 650. So now we have a table that records marginal product:

<i>Number of sisters</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>
Total farm output (in \$)	1,000	1,800	2,400	2,800	3,000
Marginal farm output (in \$)	1,000	800	600	400	50

Using this table, it is easy to see that *three* sisters will migrate, because 650 is higher than each of these three layers of marginal product.

(c) Here is a third possibility. Those sisters who migrate to the city do so selfishly and never send home any remittances, even if they are employed (maybe those in the countryside have no way to verify whether their sisters in the city have a job). However, the family sits down and makes the following arrangement: those sisters who try out their luck in the city will each be sent \$50 a year from the farm income. Find how many sisters will decide to migrate. (Assume that if a sister is indifferent between migrating and not migrating, she does migrate.)

We can work with the average product table again because the migrating sisters only worry about their own income. But the non-migrating sisters do make good their promise of sending money from the farm to the migrants. So we take that into account. If m sisters are on the

farm and $5 - m$ migrate, then each migrant gets $650 + 50 = 700$, while each sister who stays behind retains

$$\text{Average Product}_m = \left[50 \times \frac{5 - m}{m} \right],$$

because the total payout from the farm is $50 \times (5 - m)$, divided among the m sisters who are left. Using this formula, we can extend the average product table:

<i>Number of sisters</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>
Output	1,000	1,800	2,400	2,800	3,000
Average	1,000	900	800	700	600
Average minus remittances	$1,000 - \frac{50 \times 4}{1}$	$900 - \frac{50 \times 3}{2}$	$800 - \frac{50 \times 2}{3}$	$700 - \frac{50}{4}$	600

The numbers in the bottom row represent what each non-migrating sister would get if she stays behind. For instance, if two sisters have already left, the average product of the remainder is 800, but 50 each per migrating sister will have to be subtracted, but spread across three sisters, so the net return is $800 - [50 \times 2/3] \approx 767$. These returns will have to be compared with the 700 ($650 + 50$) from migration. It is easy to see now that two sisters will migrate. (d) Compare the numbers in parts (a)–(c). In light of the comparison, discuss the following assertion. “The extent of rural urban migration depends, *ceteris paribus*, on the nature and degree of altruistic links within families.”

If migration is a selfish act, with no remittances or compensations running in either direction, an individual will simply have to compare his *own* income under the two scenarios. In this question, that’s like comparing average product to the wage elsewhere. That is part (a). But if migration is a *family-based* act, then I would migrate if the wage I earn is higher than my *marginal* product on the farm. That is part (b). Because marginal product is typically lower than average product, this raises the tendency to migrate, relative to the selfish benchmark. Finally, part (c) considers a case where the migrants are selfish but their family still cares about them and sends them remittances. That raises their willingness to migrate, but possibly not as much as in part (b), because they are still comparing average (and not marginal) products on the farm with wages elsewhere.

(7) Are the following statements true or false? It is *not* enough to just guess one or the other. You need to provide an argument for or against, and only then will any credit be awarded.

(a) If some particular wage equates the overall supply and demand for labor in a two-sector model, *no higher wage* can do that as well for the same capital endowments.

(b) Wages are equal across sectors in the Lewis model but not so in the Kuznets variant of the model.

(c) If employment is restricted by the Kalecki-Rao constraint, producers can’t be maximizing profits at any prices and wages.

(d) In the open economy version of the structural transformation model, the international price of food adjusts to equate supply and demand in a single country.

(e) The services sector declines relative to manufacturing over the course of economic development, because the informal sector mainly produces services — and we know that informality declines with development.

(f) In the closed economy version of the structural transformation model, it is investment in sector m that raises L_m , while an investment in sector a reduces L_m .