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ECON-UA 323
Development Economics

Answers to Extra Practice Problems for Examination 2

(1) True, False, or Uncertain? In each case, back up your answer with a brief but precise explanation.

(a) In the Harris-Todaro model, an exogenous decline in the formal-sector wage increases formal labor demand, and therefore must increase the size of the informal sector via the Todaro paradox.

Answer. False, though it can happen. A lower formal wage has two opposing effects. It raises formal employment $\bar{L}_m$, and more formal jobs attract more migrants (this is the Todaro-paradox channel). But each formal job now pays less, which makes the urban lottery less attractive and discourages migration. To see which wins, take the surplus-labor phase, with w_a fixed, and suppose informal workers earn $w_i = 0$. The Harris-Todaro condition $p\bar{w}_m = w_a$ gives $p = w_a/\bar{w}_m$, so with $p = \bar{L}_m/(\bar{L}_m + L_i)$,

$$L_i = \bar{L}_m(\bar{w}_m) \frac{\bar{w}_m - w_a}{w_a}.$$

A fall in $\bar{w}_m$ raises the first factor and lowers the second. The informal sector grows only if formal labor demand is elastic enough: if its elasticity exceeds $\bar{w}_m/(\bar{w}_m - w_a)$, which is always greater than 1. With inelastic (steep) demand, the informal sector shrinks; and if the formal wage falls all the way to w_a , the informal sector disappears altogether. (Strictly speaking, the Todaro paradox refers to job creation *at a given wage*; here the wage itself changes.)

(b) In the Harris-Todaro model, a wage subsidy paid at the *same* rate per worker to employers in both agriculture and the formal sector cannot affect the allocation of labor, because it leaves the relative cost of labor in the two sectors unchanged.

Answer. False. The formal wage received by workers is fixed at $\bar{w}_m$, so the subsidy lowers formal employers' cost to $\bar{w}_m - \sigma$ and expands formal hiring. In agriculture, the wage is flexible: the subsidy raises labor demand and pushes up the wage that workers receive. As the subsidy rises, w_a climbs towards $\bar{w}_m$; at the right rate the two are equal, the informal sector disappears, and since both sets of employers equate the marginal product to (wage $- \sigma$), marginal products are equalized: the outcome is efficient, without any migration restrictions. The subsidy does change relative costs: the formal sector's cost falls by σ with no offsetting wage increase, while part of the agricultural subsidy is passed on to workers.

(c) If potential migrants are risk-averse, the Harris-Todaro informal sector will be larger than if they are risk-neutral.

Answer. False. Migrating to the city is a gamble: a formal job at $\bar{w}_m$ with probability p , informal work otherwise. A risk-averse person values this gamble at less than its expected value, so at any given chance of a formal job, fewer people want to migrate. In equilibrium the chance of a formal job must therefore be higher, which (with the number of formal jobs given) means a *smaller* informal sector, and more people stay in agriculture.

(d) Traffic on two alternative highways into a city is an example of a complementarity: everyone's choice of route affects everyone else's journey.

Answer. False. Route choices do affect others, but in the opposite direction: each extra car on a highway *lowers* the payoff of using it (congestion), so the more people are expected to take a route, the *fewer* want to. The complementarity map slopes down, there is a single equilibrium in which journey times are equalized, and it is stable: history does not matter. A complementarity requires that more people taking an action makes the action *more* attractive, as with the QWERTY keyboard.

(e) In the “joining the revolution” model, if everyone gets the same benefit from a successful revolution and suffers the same loss if it fails, then the revolution either always happens or never happens, whatever people expect.

Answer. False. With identical people, the payoff to joining depends only on how many others are expected to join, through the probability of success. If few are expected to join, success is unlikely and no one joins; if many are expected to join, success is likely and everyone joins. So (under the usual assumptions that the probability of success is 0 with no participants and 1 with full participation) both the quiescent outcome and the full revolution are equilibria, and which one occurs depends entirely on expectations. Homogeneity makes multiplicity *more* likely, not less: the complementarity map jumps from 0 to 1.

(f) The QWERTY keyboard has survived because, once typists have been trained on it, it is the most efficient layout available.

Answer. False (or at least uncertain). The QWERTY story is about lock-in, not efficiency. The value of a layout to each user depends on how many others use it (typists, schools, employers), so even if another layout were better, no single firm or typist would gain from switching alone, and the established standard survives. Whether Dvorak is actually better is disputed (Liebowitz and Margolis argue the gains are small), but the argument does not depend on it: the point is that persistence by itself is no evidence of efficiency.

(g) Hirschman's strategy of unbalanced growth recommends that the government invest in the most profitable sectors of the economy, since these will have the strongest linkages with other sectors.

Answer. False. Hirschman's idea is to promote a few *leading sectors* chosen for the number and strength of their linkages, and then let the market respond to the imbalances they create. Profitability points the other way: activities that are profitable will tend to be taken up by the private sector anyway, so the government gets the most out of its money by investing in activities with strong linkages that are *not* privately profitable, such as roads and other

infrastructure. A leading sector need not be profitable itself; it must make other sectors profitable.

(h) Forward and backward linkages are equally effective in pulling other sectors into a high-investment equilibrium.

Answer. False (or uncertain). A backward linkage raises the demand for, and so the price of, the benefiting sector's output, a concentrated and clear signal. A forward linkage lowers the price of just one of the many inputs that a sector uses, so its effect on any one sector is diffuse and less likely to tip it into investing; it is also a noisier signal about which sectors have become profitable. That is why backward linkages are often thought to be the more powerful of the two, though in any particular case it depends on the sectors involved.

(i) In the complementarity map for industrialization, firms with high setup costs, which stay dormant in the low-investment equilibrium, may well prefer that equilibrium to the high-investment one.

Answer. False. A dormant firm earns 0 in either equilibrium. In the high-investment equilibrium, any firm that invests does so only because its revenues $R(n)$ cover its setup cost, so it earns at least 0, and firms that would invest in both equilibria earn strictly more in the high one, since $R(n)$ is higher there. So no firm is worse off in the high-investment equilibrium, and many are strictly better off: the equilibria are ranked, precisely because each firm's investment creates *positive* spillovers for the others. That is what makes the low equilibrium a coordination failure rather than a conflict of interest.

(j) Since good institutions (secure property rights, courts that enforce contracts, broad access to schooling and the vote) promote investment and growth, every society will eventually adopt them.

Answer. False. Institutions are shaped by history and by the interests of those who hold power. Sokoloff and Engerman argue that in the New World colonies with plantation economies and great inequality, elites restricted the vote, schooling and access to land, because broad-based institutions would have threatened their position; these arrangements then persisted long after the conditions that created them had gone. Good-for-growth institutions may be resisted by those who would lose from them, and the status quo bias we studied (Fernández-Rodrik) adds to the stickiness.

(k) History can affect a society's long-run outcomes only if there are multiple equilibria, so that self-fulfilling expectations can pick one of them.

Answer. False. We distinguished multiple *equilibria* (for given conditions, several self-fulfilling outcomes, selected by expectations) from multiple *steady states* (for each initial condition, a unique equilibrium path, but different starting points lead to different long-run outcomes). A Solow model in which the savings rate rises with income, or a growth model with a fixed cost of the advanced technology, can have a unique equilibrium path from every starting point and still lead a poor country to a low steady state and a rich one to a high steady state. History matters there through the initial conditions, not through expectations.

(1) In a Solow model in which the savings rate rises with per-capita income, a country whose capital stock is destroyed in a war will, once peace returns, always grow back to its pre-war level of income.

Answer. Uncertain. In the standard Solow model, the answer would be yes: the economy converges to its unique steady state from any starting point. But if the savings rate rises with income, there can be two stable steady states separated by a threshold. If the war pushes capital below that threshold, then once peace returns, low income means low savings, and the country converges to the *low* steady state instead: the temporary shock has a permanent effect. If the destruction is not that severe (or if, despite the endogenous savings rate, there is only one steady state), the country recovers.

(2) Industrialization and the Complementarity Map. An economy has a continuum of firms, indexed by $x \in [0, 1]$. Each firm can stay *dormant*, in which case it earns a profit of 0, or it can *invest* in a modern technology. A firm that invests must pay a one-time setup cost, which differs across firms:

$$s(x) = 20 + 100x.$$

(So firms are arranged in increasing order of setup cost.) If a fraction n of all firms is expected to invest, each investing firm earns gross revenues of

$$R(n) = 36 + 100n^2.$$

(a) Explain in words why $R(n)$ might increase with n . Give one concrete channel through which investment by *other* firms raises the revenue of an investing firm.

Answer. Investing firms pay higher wages and earn profits, both of which raise national income and therefore the demand for every firm's product. Or: investing firms buy inputs from one another (backward linkages), or make inputs cheaper for one another (forward linkages). Any of these makes the return to investing higher when more firms invest.

(b) Suppose a fraction n of firms is expected to invest. Show that firm x will want to invest if and only if

$$x \leq \frac{R(n) - 20}{100}.$$

Use this to write down the complementarity map $x(n)$: the fraction of firms that *want* to invest when a fraction n is *expected* to invest. Be careful: $x(n)$ must lie between 0 and 1.

Answer. Firm x invests iff $R(n) - s(x) \geq 0$, i.e. $36 + 100n^2 \geq 20 + 100x$, i.e. $x \leq 0.16 + n^2$. Since the firms with x below this cutoff all invest, the fraction investing is the cutoff itself, provided it lies in $[0, 1]$. The cutoff is never negative, but it exceeds 1 once $n^2 > 0.84$, i.e. $n > \sqrt{0.84} \approx 0.92$. So

$$x(n) = \min\{0.16 + n^2, 1\}.$$

(c) Compute $x(n)$ for $n = 0, 0.2, 0.5, 0.8$ and 1, and draw the complementarity map together with the 45° line.

Answer. $x(0) = 0.16$, $x(0.2) = 0.2$, $x(0.5) = 0.41$, $x(0.8) = 0.8$, $x(1) = 1$ (the formula gives 1.16, which is capped at 1). See Panel A of the figure below.

(d) Find *all* the equilibria. (Hint: an equilibrium is a value of n with $x(n) = n$. Look for interior equilibria using the formula, and then check the endpoints separately.) Which equilibria are stable and which are unstable? Explain using your diagram.

Answer. Interior: $0.16 + n^2 = n$, i.e. $n^2 - n + 0.16 = 0$, i.e. $(n - 0.2)(n - 0.8) = 0$. So $n = 0.2$ and $n = 0.8$. Endpoint: $x(1) = 1$, so $n = 1$ is also an equilibrium. ($n = 0$ is not, since $x(0) = 0.16 > 0$.) The map cuts the diagonal from above at 0.2 (stable), from below at 0.8 (unstable, a tipping point), and is stuck at the ceiling at 1 (stable). Starting anywhere below 0.8, the economy drifts to 0.2; starting anywhere above, it drifts to full industrialization.

(e) Compare the low equilibrium $n = 0.2$ with the high equilibrium $n = 1$. Show that *every* firm is at least as well off in the high equilibrium, and that many are strictly better off. In what sense is the low equilibrium a “coordination failure”?

Answer. At $n = 0.2$, $R = 40$: firms with $x \leq 0.2$ earn $40 - 20 - 100x = 20 - 100x \geq 0$, and the rest stay dormant and earn 0. At $n = 1$, $R = 136$: every firm earns $136 - 20 - 100x = 116 - 100x \geq 16 > 0$. So each firm earns strictly more: firm 0, for instance, goes from 20 to 116. Yet no single firm can bring this about, because a lone firm that switches has no effect on R . Everyone would gain if all moved together, but no one gains by moving alone.

(f) *A big push.* The economy is stuck at $n = 0.2$. The government offers a *temporary* subsidy towards setup costs, just large enough to persuade firms to invest *given* that they currently expect revenues of $R(0.2) = 40$. The subsidy is then withdrawn. Which firms must the government subsidize to guarantee that, after the withdrawal, the economy moves on its own to full industrialization? What is the total cost of the subsidy?

Answer. The government must push the economy past the tipping point 0.8: it must get all firms up to (just above) $x = 0.8$ to invest. Firms with $x \leq 0.2$ invest anyway. Firm $x \in (0.2, 0.8]$ needs a subsidy of $s(x) - 40 = 100x - 20$. The total cost is

$$\int_{0.2}^{0.8} (100x - 20) dx = [50x^2 - 20x]_{0.2}^{0.8} = 18.$$

Once the subsidy is withdrawn, n is just above 0.8, where $x(n) > n$, so more firms want to invest and the economy climbs to $n = 1$. A temporary policy has a permanent effect. Compare the cost of 18 with the gain in part (e): firm 0 alone gains 96.

(g) Now suppose instead that revenues are $R(n) = 8 + 200n^2$, with the same setup costs. Redo parts (b) and (d). Show that this time the map has a single interior crossing of the 45° line, but there are still three equilibria. Which are stable? Why does no firm invest when no other firm does?

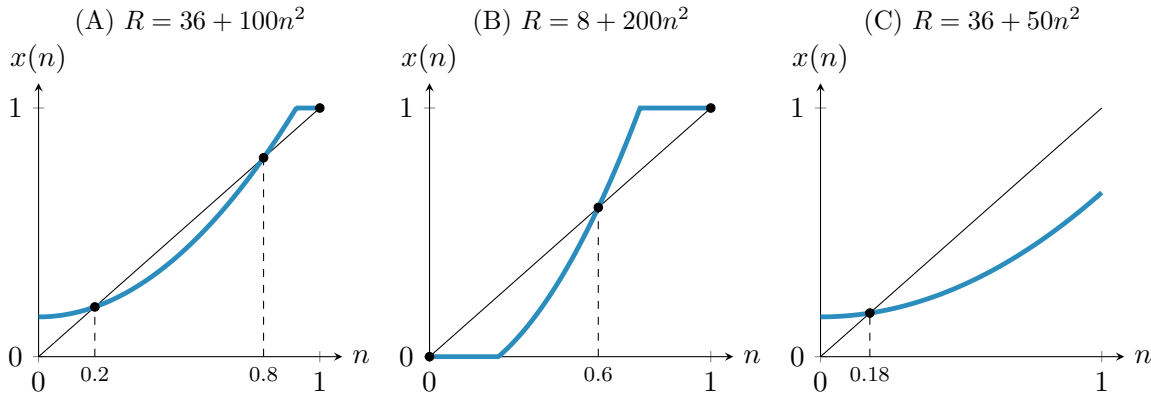
Answer. Firm x invests iff $8 + 200n^2 \geq 20 + 100x$, i.e. $x \leq 2n^2 - 0.12$. This cutoff is negative for $n < \sqrt{0.06} \approx 0.24$ and exceeds 1 for $n > \sqrt{0.56} \approx 0.75$, so

$$x(n) = \min\{\max\{2n^2 - 0.12, 0\}, 1\}.$$

Interior: $2n^2 - 0.12 = n$, i.e. $2n^2 - n - 0.12 = 0$, i.e. $(n - 0.6)(2n + 0.2) = 0$, so $n = 0.6$ (the other root is negative). This single crossing is from *below*, so it is unstable. But $x(0) = 0$ and $x(1) = 1$, so both corners are equilibria, and both are stable. No firm invests when no one else does because $R(0) = 8$ is below even the lowest setup cost, $s(0) = 20$. See Panel B.

(h) Finally, suppose revenues are $R(n) = 36 + 50n^2$. Show that there is now a *unique* equilibrium, and find it. What does this tell you about the relationship between complementarities and multiple equilibria?

Answer. Now $x(n) = 0.16 + 0.5n^2$, which never reaches 1 (its largest value is $x(1) = 0.66$). Solving $0.5n^2 - n + 0.16 = 0$ gives $n = 1 - \sqrt{0.68} \approx 0.18$; the other root (≈ 1.82) lies outside $[0, 1]$. So there is a unique equilibrium, and it is stable. There are still complementarities ($x(n)$ is increasing), but they are too weak to generate multiplicity: complementarities are necessary for multiple equilibria of this kind, but not sufficient. See Panel C.



(3) Income Distribution and the Savings Rate. This problem tests your understanding of the effect of the income distribution on savings rates.

The economy of Sonrisa has people in three income categories: poor, middle class, and rich. The poor earn \$500 per year and have to spend it all to meet their consumption needs. The middle class earn \$3,000 per year, of which \$2,000 is spent and the rest saved. The rich earn \$10,000 per year, and consume 80% of it, saving the rest.

(a) Calculate the overall savings rate in Sonrisa if 30% of the people are poor and 60% are in the middle class.

Answer. The safest way to do this question is to find out total income and total savings in an economy of, say, 100 people in the proportions given, and then divide the latter by the former. There will be 30 poor people, 60 middle-class, and 10 rich people. So total income is \$ $(500 \times 30 + 3000 \times 60 + 10,000 \times 10)$, which is \$ 295,000. Total savings is \$ $(0 \times 30 + 1000 \times 60 + 2000 \times 10)$, which is \$80,000. It follows that the overall rate of savings in the economy is $\frac{80,000}{295,000}$, which is approximately 0.27, or 27%. *Note well that the overall*

rate of savings is affected by the distribution of income, because people in different income categories save different amounts.

(b) Suppose that all growth occurs by moving people from the poor category to the middle-class category. Will the savings rate rise over time or fall? Using the Harrod–Domar model and assuming that population growth is zero and all other variables are exogenous, predict whether the resulting growth rate will rise or fall over time.

Answer. It should be obvious that this kind of growth leads to an increase in the savings rate because the middle-class saves a positive fraction of income, while the poor do not. Using the formula for the Harrod-Domar model, the rate of growth should accelerate.

(c) Outline another growth scenario with positive growth where the rate of growth changes over time in a way opposite to that of (b).

Answer. Consider a scenario where growth occurs by shifting people from the middle-class to the rich. Because the rich save a lower fraction of their income than the middle-class do, this should bring down the rate of savings over time, and therefore the rate of growth.

(d) Understand well that this question asks you about how growth *rates* are changing. In the simple Harrod–Domar model, the growth rate is constant over time because the savings rate is presumed to be unchanging with the level of income. Do you understand why matters are different here?

Answer. Recap this for your own understanding.

(4) Verticalia. Verticalia is a new community in the town of Flatland that is inviting residents to move in. The cool thing is that everyone will live in a mega highrise and hang out with everyone else. This “hanging-out” gives rise to *networking payoffs*, given by

$$P = An,$$

where $A > 0$ is a positive parameter and n is the fraction of people in Flatland that move into Verticalia. But the problem is that more people moving in will also raise per-person rents in Verticalia because of congestion. Rent is given by the function

$$R = 10 + 5n.$$

Assume that the net payoff a person receives in Verticalia is networking payoff minus rent, and that there are no other costs or benefits other than these.

Meanwhile, living in Flatland *outside* Verticalia has its own payoff. We simply assume that payoff to be equal to zero for every resident.

(a) Show that *no one* living in Verticalia; i.e., $n = 0$, is always an equilibrium, no matter how small or large the value of A is.

Answer. If payoff is given by networking payoff minus rent, then it is equal to

$$P - R = An - 5n - 10.$$

If $n = 0$, then this payoff is negative. So if no one expects to live in Verticalia, no one will. This is independent of however large A might be.

(b) Describe precisely the values of A for which $n = 0$ is the *only* equilibrium.

Answer. For n to be the only equilibrium, it must be the case that the net payoff to living in Verticalia is negative for all fractions n ; i.e.,

$$An - 5n - 10 < 0$$

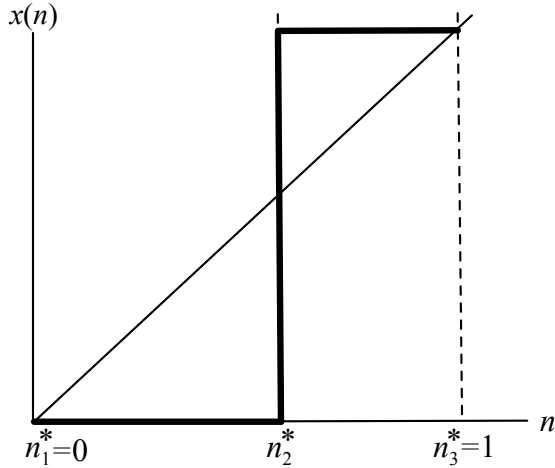
for all n between 0 and 1. Solving this inequality, we see this will happen when $A < 15$.

(c) If the conditions in part (ii) are not met, show that there are multiple equilibria and describe all of them, including the unstable one.

Answer. First, if A is *strictly* larger than 15, then at $n = 1$, the net payoff is strictly positive, so that everyone expects everyone else to live in Verticalia, then everyone will. Moreover, this is stable, because even if $n < 1$ but close enough to it, the net payoff to living in Verticalia

$$An - 5n - 10$$

will be strictly positive. So the system will push towards $n = 1$. The usual complementarity map is useful here:



Where is the unstable point n_2^* located? It is at the point where $An - 5n - 10 = 0$, or

$$n_2^* = \frac{10}{A - 5}.$$

Because $A > 15$, it is easy to check that $0 < n_2^* < 1$. Note how the complementarity map cuts the 45° line at n_2^* “from below,” so you will need to argue that this intermediate equilibrium is unstable.

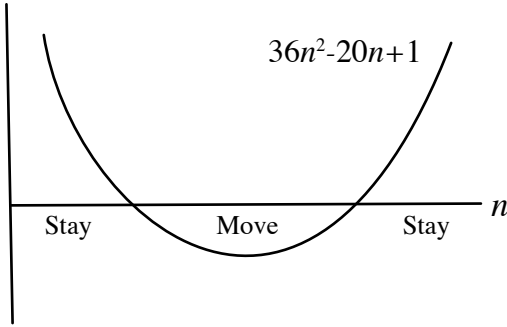
There is one “knife-edge” case that we left out, and if you include it you will get a small amount of extra credit. When A *exactly* equals 15, then there is one stable equilibrium at $n = 0$. For every $0 < n < 1$, the net payoff is strictly negative, and only *at* $n = 1$ does it become zero. So now the unstable equilibrium coincides with $n = 1$.

(d) Now change the rent function to $R = 1 + 36n^2$, and assume that $A = 20$. Show that there are now just two stable equilibria, one in which no-one lives in Verticalia, and another in which just half of them do.

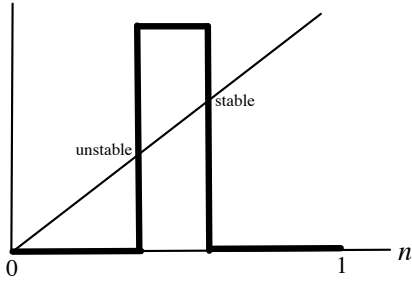
Answer. Under these conditions, the conditions for wanting to move to Verticalia are given by $20n - 36n^2 - 1 \geq 0$ or equivalently

$$36n^2 - 20n + 1 \leq 0.$$

See diagram, which plots this quadratic equation:



So the relevant “complementarity map” looks like this:



Indeed, we don’t have a complementarity throughout any more! For small values of n there is a complementarity, as the networking effect dominates the rent congestion effect. For large values of n it is the other way around as the congestion effect dominates the networking effect and the map jumps *down*. These jumps happen at the roots of the quadratic, shown in the previous diagram. So all we have to do is solve for the roots, which are given by

$$\frac{20 \pm \sqrt{400 - 4 \cdot 36 \cdot 1}}{2 \cdot 36} = \frac{20 \pm \sqrt{256}}{72},$$

which yields the solutions $1/18$ for the lower unstable root and the solution $1/2$ for the upper stable root.

(5) Harris-Todaro with Open Unemployment. Pueblo has a labor force of 1000. The formal urban sector pays a fixed wage $\bar{w}_m = 5$ and employs exactly $\bar{L}_m = 250$ workers. A migrant who fails to get a formal job does not automatically find informal work: with probability $q = 1/2$ she finds an informal job paying $w_i = 2$, and with probability $1 - q$

she is openly unemployed and earns nothing. Formal jobs are allocated by lottery among all urban residents, so the probability of a formal job is $p = \bar{L}_m/(\bar{L}_m + L_u)$, where L_u is the number of urban residents without a formal job (informal workers plus the openly unemployed). Agriculture has a downward-sloping demand for labor, $w_a = 8 - L_a/100$. People are risk-neutral and migration is costless.

(a) Write down the expected income of a migrant as a function of p , and the Harris-Todaro equilibrium condition for Pueblo.

Answer. A migrant gets $\bar{w}_m$ with probability p ; otherwise, with probability $1 - p$, she faces a second lottery that pays w_i with probability q and 0 with probability $1 - q$, worth $qw_i = 1$ in expectation. So the expected urban income is $5p + (1 - p) \cdot 1 = 1 + 4p$. Equilibrium requires this to equal the sure agricultural wage: $8 - L_a/100 = 1 + 4p$, with $p = 250/(1000 - L_a)$, since everyone not in agriculture is in the city.

(b) Show that the equilibrium has $L_a = 500$, and find p , the agricultural wage, and the numbers of informal workers and openly unemployed.

Answer. Substituting p , the condition is $7 - L_a/100 = 1000/(1000 - L_a)$. Multiply by $100(1000 - L_a)$: $(700 - L_a)(1000 - L_a) = 100,000$, or $L_a^2 - 1700L_a + 600,000 = 0$, which factors as $(L_a - 500)(L_a - 1200) = 0$. The feasible root is $L_a = 500$. Then $w_a = 3$, the urban population is 500, $p = 250/500 = 1/2$ (check: $1 + 4(1/2) = 3$), and the 250 urban residents without formal jobs split into 125 informal workers and 125 openly unemployed.

(c) The government creates 200 more formal jobs, so $\bar{L}_m = 450$. Show that the new equilibrium has $L_a = 400$ and find the new numbers of informal and openly unemployed workers. Did the policy reduce open unemployment? Would it have done so if agriculture had been in the Lewis surplus-labor phase with a fixed wage of 3?

Answer. Now $(700 - L_a)(1000 - L_a) = 100 \times 4 \times 450 = 180,000$, or $L_a^2 - 1700L_a + 520,000 = 0$, which factors as $(L_a - 400)(L_a - 1300) = 0$, so $L_a = 400$ and $w_a = 4$. The urban population is 600, $p = 450/600 = 3/4$ (check: $1 + 4(3/4) = 4$), and 150 urban residents lack formal jobs: 75 informal and 75 openly unemployed. Open unemployment fell from 125 to 75, because the agricultural wage rose from 3 to 4 as 100 workers left, raising the hurdle for migration. With a fixed agricultural wage of 3, p would have to stay at $1/2$, so $L_u = \bar{L}_m = 450$: the urban pool without formal jobs would *rise* from 250 to 450, and open unemployment from 125 to 225, the Todaro paradox.

(d) Return to part (b). Suppose a public-works program guarantees an informal job to anyone who wants one, so that q rises from $1/2$ to 1. Without solving for the new equilibrium, show that at the old allocation the expected urban income now exceeds the agricultural wage, and describe the direction in which migration, the agricultural wage, and the size of the urban pool without formal jobs all move.

Answer. With $q = 1$ the expected urban income is $5p + 2(1 - p) = 2 + 3p$. At the old allocation $p = 1/2$, so it is $3.5 > 3 = w_a$: the program makes the city more attractive without creating a single formal job. Workers migrate until equality is restored, so L_a falls, $w_a = 8 - L_a/100$ rises, and the urban pool L_u rises (which lowers p and the expected urban

income back towards w_a). The program removes open unemployment by definition, but it enlarges the informal sector by more than the number of formerly unemployed, because it also pulls in new migrants: the Todaro logic again. (If you do solve it, the condition is $(600 - L_a)(1000 - L_a) = 75,000$, giving $L_a \approx 461$.)

(6) Status Quo Bias. A country of 100 citizens votes on a trade reform. If adopted, each of 60 citizens (the “winners”) gains \$5 and each of the other 40 (the “losers”) loses \$6. Everyone is risk-neutral and votes in her own interest; the reform passes if more than 50 vote for it.

(a) Suppose nobody knows whether she will be a winner or a loser. Does the reform pass?

Answer. Each citizen expects $(0.6)(5) - (0.4)(6) = 3 - 2.4 = 0.6 > 0$, so everyone votes yes and the reform passes 100 to 0. Total gains, $300 - 240 = 60$, are positive, so this is also the efficient outcome.

(b) Suppose everyone knows her type. Does the reform pass?

Answer. The 60 winners vote yes and the 40 losers vote no: it passes 60 to 40.

(c) Now suppose that exactly 30 of the winners know that they are winners, while the other 70 citizens know only that 30 winners have been identified and that 60 winners exist in total. Show that the reform fails.

Answer. Among the 70 uncertain citizens there are $60 - 30 = 30$ winners and 40 losers, so each of them is a winner with probability $30/70 = 3/7$. Her expected gain is $(3/7)(5) - (4/7)(6) = (15 - 24)/7 = -9/7 < 0$, so all 70 vote no. The reform fails 30 to 70, although 60 of the 100 citizens would gain from it and total gains are positive.

(d) Let k be the number of winners who know their identity, everyone else being uncertain as in (c). For which values of k does the reform fail?

Answer. The k identified winners vote yes. Each of the other $100 - k$ citizens is a winner with probability $(60 - k)/(100 - k)$ and votes yes if and only if $5(60 - k) - 6(40) \geq 0$, that is, $300 - 5k \geq 240$, or $k \leq 12$. So for $k \leq 12$ everyone votes yes and the reform passes. For $k > 12$ only the identified winners vote yes, which is a majority only if $k > 50$. The reform therefore fails for $13 \leq k \leq 50$ and passes for $k \leq 12$ or $k > 50$: more information about who gains can sink a reform that a majority would benefit from.

(e) Suppose the reform was adopted at some earlier date, so that by now everyone knows her type. A proposal to repeal it comes to a vote. Does it pass? Combine your answers to explain what “status quo bias” means here.

Answer. Once in place, the reform has revealed everyone’s type, so the 60 winners vote to keep it and repeal fails 40 to 60. Put together with (c): the reform cannot get *in* by majority vote when identities are partially known, but once in it cannot be voted *out*. Whichever state the country happens to be in persists: that is status quo bias. Two identical countries, one that happened to adopt the reform under a dictator or in a moment of uncertainty and

one that did not, will stay different. The bias comes from uncertainty about the identity of gainers, not from any aggregate loss.

(7) Endogenous Savings and Two Steady States. Consider the Solow model with no technical progress. Per-capita output is $y = 10k^{1/2}$, and population growth plus depreciation sum to $n + \delta = 0.1$, so that the per-capita accumulation equation is $k(t+1) = k(t) + sy(t) - 0.1k(t)$ (ignore the $1+n$ denominator for simplicity). The savings rate depends on income: people save 5% of income when per-capita income is below 300, and 40% of it when per-capita income is 300 or more.

(a) Treating s as a fixed number for the moment, show that the steady state per-capita income is $y^* = 1000s$.

Answer. In steady state $sy^* = 0.1k^*$. With $y^* = 10k^{*1/2}$ we have $k^* = (y^*/10)^2$, so $sy^* = 0.1(y^*)^2/100$, which gives $y^* = 1000s$ (and $k^* = 10,000s^2$).

(b) Find all the steady states of the economy with the income-dependent savings rate, and check that each one is consistent with the savings rate that generates it.

Answer. With $s = 0.05$, $y^* = 50$, which is below 300, so a 5% savings rate is indeed what prevails there: this is a steady state, with $k^* = 25$. With $s = 0.4$, $y^* = 400$, which is at or above 300, so the 40% savings rate is self-consistent: a second steady state, with $k^* = 1600$. (A hypothetical steady state with $y^* = 250$ at $s = 0.25$ would not be self-consistent, since at $y = 250$ the savings rate is 5%.) So there are exactly two steady states, at per-capita incomes 50 and 400.

(c) Describe where the economy ends up as a function of its initial per-capita income $y(0)$. Explain why two economies with identical technology and identical savings *behavior* can have very different long-run incomes.

Answer. If $y(0) < 300$ the savings rate is 5%, the economy behaves like a Solow economy with $s = 0.05$, and it converges to $y^* = 50$: from above 50 capital shrinks, from below it grows, but it never reaches 300 (its path is monotone and bounded by 50 from the relevant side). If $y(0) \geq 300$ the savings rate is 40% and the economy converges to 400, never falling below 300 along the way (from above 400 it falls towards 400, from 300 it rises). So the basin of attraction of the low steady state is $[0, 300)$ and that of the high one is $[300, \infty)$. Two countries with the same technology and the same savings *rule* end up at 50 or 400 depending only on where they started: history, not parameters, determines the outcome. This is the macro-feedback version of a development trap: income determines savings, and savings determine income.

(d) The economy sits at the low steady state. A donor offers a *one-time* grant of capital. What is the smallest grant per person that has a permanent effect, and what is the effect? Contrast this with the effect of the same grant in a Solow model where s is a fixed parameter.

Answer. The grant must lift the economy into the high basin, that is, to $y \geq 300$, which requires $k \geq (300/10)^2 = 900$. Starting from $k^* = 25$, the smallest grant is 875 units of capital per person. After that the economy saves 40% and climbs on its own to $y^* = 400$: a

temporary intervention with a permanent effect, and one that raises income eightfold. Any smaller grant is washed out: the economy returns to $y = 50$. With a fixed savings rate there is only one steady state, so a capital grant of any size raises income only temporarily and the economy returns to the same steady state; permanent effects would require a permanent change in s .

(8) Joining the Revolution. A population of size 1 is deciding whether to join a revolution. If the revolution succeeds, each participant gets $B > 0$; if it fails, each participant suffers a loss $L > 0$. Bystanders get 0 either way. The probability of success is $p(n) = n^2$, where n is the fraction of the population that participates.

(a) Show that an individual who expects a fraction n to participate wants to join if and only if $n > \sqrt{L/(B+L)}$.

Answer. Joining pays $p(n)B - [1 - p(n)]L = n^2B - (1 - n^2)L = n^2(B + L) - L$, which is positive if and only if $n^2 > L/(B + L)$, that is, $n > \sqrt{L/(B + L)}$. The threshold rises with L and falls with B .

(b) Let $B = 3$ and $L = 1$. Draw the complementarity map, find all the equilibria, and classify them as stable or unstable.

Answer. The threshold is $\sqrt{1/4} = 1/2$. If $n < 1/2$ nobody wants to join, so $x(n) = 0$; if $n > 1/2$ everybody does, so $x(n) = 1$; at $n = 1/2$ everyone is indifferent. The map is a step function that jumps from 0 to 1 at $n = 1/2$. Equilibria: $n = 0$ (nobody expects anyone to join, so nobody joins) and $n = 1$ (everyone expects everyone, so everyone joins), both stable, plus the knife-edge $n = 1/2$, where the map crosses the 45° line from below: unstable, since a slightly larger expected turnout brings everyone out and a slightly smaller one brings nobody. A regime can be perfectly safe and perfectly doomed with the same fundamentals; what differs is what people expect of each other.

(c) Now suppose participants differ in what they stand to gain: B is uniformly distributed on $[0, 4]$ across the population, with $L = 1$ for everyone. Show that the fraction wanting to join is $x(n) = 1 - (1 - n^2)/(4n^2)$ when this is positive (and 0 otherwise), and that the interior equilibrium solves $4n^3 - 5n^2 + 1 = 0$. Find all the equilibria and classify them. (The cubic has $n = 1$ as a root; factor it out.)

Answer. Person i joins if and only if $B_i > (1 - n^2)/n^2$. Since B is uniform on $[0, 4]$, the fraction with B_i above a number $c \in [0, 4]$ is $1 - c/4$, so $x(n) = 1 - (1 - n^2)/(4n^2)$ whenever $(1 - n^2)/n^2 \leq 4$, that is, $n \geq 1/\sqrt{5} \approx 0.45$, and $x(n) = 0$ for smaller n . The map now rises gradually instead of jumping. An interior equilibrium solves $1 - (1 - n^2)/(4n^2) = n$; multiplying by $4n^2$ gives $4n^2 - 1 + n^2 = 4n^3$, or $4n^3 - 5n^2 + 1 = 0$, and $(n - 1)(4n^2 - n - 1) = 0$. The quadratic gives $n = (1 + \sqrt{17})/8 \approx 0.64$ (the other root is negative). So the equilibria are $n = 0$ (since $x(0) = 0$), $n \approx 0.64$, and $n = 1$ (since $x(1) = 1$). Just above 0.64 the map lies above the diagonal (for instance $x(0.7) \approx 0.74$), so the crossing at 0.64 is from below and that equilibrium is unstable; 0 and 1 are stable. Heterogeneity smooths the map but leaves the structure intact: two stable outcomes separated by a tipping point.

(9) Complementarities or Omitted Variables? Munshi and Myaux study contraceptive use in villages in Matlab, Bangladesh, where use rose from 40% in 1983 to 63% in 1993 after an intensive family-planning program began in 1978. Their hypothesis is that individual use responds to use by others in the village, a complementarity working through social norms.

(a) Write down a regression that would test this hypothesis, with a couple's contraceptive use in year t on the left-hand side and village-level use in year $t - 1$ on the right, and explain why a positive coefficient on village use is *not* by itself evidence of a complementarity.

Answer. Something like $y_{it} = A + \gamma y_{i,t-1} + \beta x_{t-1}^{v(i)} + \eta Z_{it} + C_t^{v(i)} + \epsilon_{it}$, where y_{it} is couple i 's use (0 or 1), $x_{t-1}^{v(i)}$ is last year's use in i 's village v , Z collects individual characteristics, and C_t^v is everything unobserved that affects use in village v at date t . The problem is C_t^v : if the village received a more energetic health worker, or a new clinic, or a good harvest, then every couple's use rises together, individual use and village use are correlated, and β comes out positive with no complementarity at all. This is the identification problem Manski raised: a common unobserved cause looks exactly like a social interaction.

(b) The authors can include village fixed effects and year fixed effects. Which parts of the omitted variable do these remove, and which part survives?

Answer. Decompose C_t^v into a component that depends only on the village (C_1^v , say the village's remoteness or the attitude of its religious leaders), a component that depends only on time (C_{t2} , say a nationwide campaign or a price change), and a component that varies in a village-specific way over time. Village fixed effects absorb the first, year fixed effects the second. The third survives: anything that changes within a particular village in a particular year, such as that village's health worker becoming more effective. Fixed effects do not solve the problem.

(c) Explain how splitting village-level use into use among the couple's own religious group and use among the other group helps, and what assumption the argument rests on.

Answer. Muslims and Hindus in these villages interact little across the religious line, so if use spreads through social interaction, a couple should respond to use among its *own* group and not to use among the other group. Run the regression with the two group-level variables entered separately. If the own-group coefficient is large and the cross-group coefficient is small, that is what a norm-based complementarity predicts. For the omitted-variable story to produce the same pattern, the unobserved factor would have to vary not only by village and year but also by *religious group within the village and year*, which is much harder to believe of a health worker or a clinic. The argument rests on the assumption that any common shock hits both groups in a village alike; a shock specific to one group in one village in one year would still confound it.

(10) Harris-Todaro: Efficiency, Migration Bans and Subsidies. Labora has a labor force of 1000. The formal sector pays a fixed wage of $\bar{w}_m = 8$. Formal employers hire up to the point where the marginal product of labor equals what a worker costs them, and the marginal product in the formal sector is $10 - L_m/100$ when L_m people are employed there. Agriculture has a flexible wage equal to its marginal product, $w_a = 10 - L_a/100$. Anyone in

the city without a formal job is unemployed and earns nothing. Total output in agriculture is $10L_a - L_a^2/200$, and in the formal sector it is $10L_m - L_m^2/200$.

(a) Show that the formal sector employs 200 people, and that in the Harris-Todaro equilibrium there are 200 unemployed and 600 people in agriculture. What is the agricultural wage?

Answer. Formal employers hire until $10 - L_m/100 = 8$, so $L_m = 200$. With L_u unemployed, $L_a = 800 - L_u$, and the migrant's chance of a formal job is $200/(200 + L_u)$. The Harris-Todaro condition is

$$\frac{200}{200 + L_u} \times 8 = 10 - \frac{800 - L_u}{100} = \frac{200 + L_u}{100},$$

so $(200 + L_u)^2 = 160,000$, giving $200 + L_u = 400$: $L_u = 200$ and $L_a = 600$. The agricultural wage is $10 - 6 = 4$, which equals the expected urban wage $\frac{1}{2} \times 8$.

(b) Find the allocation of the 1000 workers between agriculture and the formal sector that maximizes total output. Compare total output there with total output in the Harris-Todaro equilibrium.

Answer. Output is maximized when the marginal products are equal and no one is idle: $10 - L_m/100 = 10 - L_a/100$ with $L_m + L_a = 1000$, so $L_m = L_a = 500$, with marginal product 5 in both. Total output is $2 \times (5000 - 1250) = 7500$. In the Harris-Todaro equilibrium, output is $(2000 - 200) + (6000 - 1800) = 6000$. The loss comes from two sources: 200 people sit idle, and the formal sector is too small because its wage is too high.

(c) The government bans anyone without a formal job from living in the city. Describe the new allocation and the agricultural wage, and compute total output. Is the outcome efficient? Explain.

Answer. The formal sector still employs 200, and the remaining 800 must work in agriculture, where the wage falls to $10 - 8 = 2$. Output is $1800 + (8000 - 3200) = 6600$: higher than under Harris-Todaro, because no one is idle. But it is not efficient: the marginal product is 8 in the formal sector and only 2 in agriculture, so moving workers into the formal sector would raise output (all the way to 7500). The restriction cures the unemployment but leaves the formal sector too small.

(d) Instead of a ban, the government pays a subsidy of σ per worker to employers in *both* sectors. (Formal workers still receive 8; agricultural workers receive the market wage w_a , and agricultural employers pay $w_a - \sigma$.) Show that a subsidy of $\sigma = 3$ eliminates unemployment and achieves the efficient allocation, with no restriction on migration. How much does the subsidy cost?

Answer. Formal employers now pay $8 - 3 = 5$, so they hire until $10 - L_m/100 = 5$: $L_m = 500$. If no one is unemployed, $L_a = 500$, and agricultural employers hire until $10 - L_a/100 = w_a - 3$, so $w_a = 5 + 3 = 8$. The agricultural wage equals the formal wage, so no one has a reason to gamble on the city: unemployment disappears without any ban. Both sectors employ 500, the efficient allocation of part (b). The subsidy goes to all 1000 workers, so it costs $3 \times 1000 = 3000$, which must be raised somehow (and taxes have their own distortions).

(11) History or Expectations? Moving to Nuevo. Two regions, Viejo and Nuevo, are home to a population of size 1. Living and working in Viejo pays 12 per period, however many people live there. Nuevo has complementarities: it pays

$$r(n) = 2 + 20n$$

per period, where n is the fraction of the population in Nuevo. (Think of shared infrastructure, a thicker job market, or a richer network of suppliers, all of which improve as Nuevo grows.) The fraction living in Nuevo at the start is n_0 , which is given by history.

(a) Draw the complementarity map: the fraction $x(n)$ of people who want to live in Nuevo when a fraction n is expected to live there. Find all the equilibria, and say which are stable.

Answer. A person prefers Nuevo if and only if $2 + 20n > 12$, that is, $n > 0.5$. So $x(n) = 0$ for $n < 0.5$ and $x(n) = 1$ for $n > 0.5$; at $n = 0.5$ everyone is indifferent. The equilibria are $n = 0$ (everyone in Viejo), $n = 1$ (everyone in Nuevo), both stable, and $n = 0.5$, which is unstable: it is the tipping point.

(b) *History.* Suppose that every period a few people move from the region that currently pays less to the region that currently pays more. Describe what happens over time if $n_0 = 0.4$, and if $n_0 = 0.6$.

Answer. At $n_0 = 0.4$, Nuevo pays $2 + 8 = 10 < 12$, so people trickle out of Nuevo; as they do, n falls, Nuevo pays even less, and the outflow continues until everyone is in Viejo. At $n_0 = 0.6$, Nuevo pays $14 > 12$, people trickle in, Nuevo's return keeps rising, and eventually everyone is in Nuevo. In this story the initial allocation decides everything: history tells us which side of the tipping point we start on, and that is the end of it.

(c) *Expectations.* Now suppose instead that people can move instantly, and that Nuevo's return adjusts instantly to the number of people living there. Start with $n_0 = 0.1$. Show that if everyone expects everyone else to move to Nuevo, they will all move, and that if everyone expects no one to move, no one will. What role does history play now?

Answer. If everyone else will be in Nuevo, it pays $22 > 12$, so each person wants to be there too: all move, and the expectation is confirmed. If no one is expected to move, Nuevo pays $2 + 2 = 4 < 12$ to anyone who does, so no one moves (and if anything, the few people in Nuevo leave): again the expectation is confirmed. Both outcomes are possible starting from $n_0 = 0.1$, and indeed from any starting point. History plays no role at all: expectations alone pick the outcome.

(d) *Sluggish returns.* Suppose now that Nuevo's return adjusts with a lag: a person in Nuevo this period earns $2 + 20n$, where n is the fraction who lived there *last* period. People move once and for all, and they care about the total of all their future payoffs (no discounting, to keep things simple). Start with $n_0 = 0.4$, and suppose everyone else moves to Nuevo in period 1. Show that each person would rather wait and move in period 2. What happens to the expectations-driven equilibrium of part (c)?

Answer. If she moves in period 1 along with everyone else, she earns $r(0.4) = 10$ in period 1 (based on last period's population) and 22 in every period after. If she waits, she earns 12

in Viejo in period 1 and then moves, earning 22 from period 2 on. Waiting is better by 2. Since everyone reasons the same way, everyone waits; but then no one is in Nuevo in period 2 either, and the expectation that everyone will move unravels. With sluggish returns, each person wants others to go first, and the economy is stuck with history: from $n_0 = 0.4$ it drifts to Viejo, exactly as in part (b).

(e) *An advantage to moving early.* Keep the lag of part (d), but now suppose that moving gets more expensive over time: moving in period 1 costs nothing, but moving in period 2 or later costs $m > 0$ (housing in Nuevo fills up, the best plots are taken, and the early arrivals get the better jobs). Show that if everyone else moves in period 1, each person also wants to move in period 1 if and only if $m > 12 - r(n_0)$. Compute this threshold for $n_0 = 0$, $n_0 = 0.2$ and $n_0 = 0.4$.

Answer. Moving in period 1 yields $r(n_0)$ in period 1 and 22 thereafter. Waiting yields 12 in period 1, then 22 thereafter, minus the moving cost m . So moving now is better if and only if $r(n_0) > 12 - m$, that is, $m > 12 - r(n_0) = 10 - 20n_0$. The threshold is 10 for $n_0 = 0$, 6 for $n_0 = 0.2$, and 2 for $n_0 = 0.4$. When m exceeds the threshold, “everyone moves in period 1” is an equilibrium; and “no one moves” remains an equilibrium as well, since a lone mover would earn only $r(n_0) < 12$ and Nuevo would never grow. So with an advantage to going early, expectations can once again overcome history.

(f) Use part (e) to explain the following statement: “History matters most for economies that start far from the tipping point, while expectations matter most for economies that start close to it.” Why might an announcement by the government (for instance, of a new port to be built in Nuevo) succeed in some regions and fail in others?

Answer. The threshold $10 - 20n_0$ falls as n_0 rises towards the tipping point of 0.5. An economy that starts close to the tipping point needs only a small advantage to moving early before a self-fulfilling rush becomes possible: there, expectations can decide the outcome. An economy that starts far below it (say $n_0 = 0$) needs a very large advantage, so in practice the expectation of a rush cannot sustain itself, and history decides. An announcement works by coordinating expectations: if the region is close enough to the tipping point (or the early-mover advantage is large), the announcement can trigger the move to Nuevo by itself, and the port may hardly need to be built. In a region that starts far from the tipping point, the same announcement will not be believed, and only a real change in returns, such as actually building the port, can move it.

(12) Status Quo Bias: Who Knows What? Reformia has 100 citizens, who vote by simple majority on a reform. If it is adopted, 55 citizens will be winners, each gaining 1, and 45 will be losers, each losing 1. Everyone knows these numbers. Citizens are risk-neutral and vote in their own interest. A citizen who does not know whether she will win or lose takes her chance of winning to be the fraction of winners among all those who, like her, do not know.

(a) Show that if no one knows her own identity, the reform passes.

Answer. Each citizen wins with probability 0.55, so her expected gain is $0.55 - 0.45 = 0.1 > 0$: everyone votes yes, and the reform passes unanimously.

(b) Suppose k of the winners know in advance that they will win, and no one else knows anything. Show that the reform fails for every k from 11 to 50.

Answer. The $100 - k$ uninformed citizens include $55 - k$ winners and 45 losers, so each expects $(55 - k - 45)/(100 - k) = (10 - k)/(100 - k)$, which is negative when $k > 10$. Then the uninformed all vote no, and the reform gets only the k votes of the informed winners, which is not a majority as long as $k \leq 50$.

(c) Now suppose instead that j of the *losers* know in advance that they will lose, and no one else knows anything. Show that the reform passes, whatever the value of j . What happens if 20 winners and 20 losers know their identities? Why does it matter so much *whose* identity is revealed?

Answer. The uninformed now include all 55 winners and $45 - j$ losers, so each expects $(55 - 45 + j)/(100 - j) = (10 + j)/(100 - j) > 0$, and votes yes. The reform gets $100 - j \geq 55$ votes, a majority, for every j from 0 to 45. With 20 winners and 20 losers informed, the 60 uninformed include 35 winners and 25 losers, so each expects $(35 - 25)/60 > 0$ and votes yes; with the 20 informed winners, that is 80 votes, and the reform passes. Revealing losers takes losers out of the uninformed pool, which makes the uninformed more optimistic; revealing winners does the opposite. Information that reaches the likely winners first is what creates the bias against reform.

(d) Before the vote, the government runs a pilot program that reveals to 20 citizens whether they will win or lose. Suppose first that the 20 are chosen at random, so that 11 of them turn out to be winners and 9 losers. Does the reform pass? Now suppose instead that the pilot is run in a region where the reform's benefits are concentrated, so that all 20 turn out to be winners. Does it pass now?

Answer. Random pilot: the 80 uninformed include 44 winners and 36 losers, so each expects $(44 - 36)/80 = 0.1 > 0$ and votes yes; with the 11 informed winners that is 91 votes, and the reform passes. A random pilot leaves the uninformed with the same mix as before. Targeted pilot: the 80 uninformed include 35 winners and 45 losers, so each expects $(35 - 45)/80 < 0$ and votes no; the reform gets only 20 votes and fails. A pilot that informs the likely winners can kill a reform that a majority would benefit from.

(e) Suppose the reform had been adopted without a vote, and a year later everyone knows whether she won or lost. Would a vote to repeal it succeed? Fernández and Rodrik used this model to explain why trade liberalizations are often adopted only in times of crisis, and are then rarely reversed. Explain, using your answers.

Answer. No: the 55 winners vote to keep it, against 45 losers. Before the reform, uncertainty about who will gain, combined with knowledge among some of the likely winners, can block it (part (b)); once it is in place, the uncertainty is gone, and the majority who gained keep it. So a reform that would not be chosen in normal times can be pushed through in a crisis (when governments have more room to act, or when outside lenders impose conditions), and it then survives, because after the fact the winners are a majority. The status quo is sticky in both directions: it takes something unusual to change it, and once changed, the new status quo is just as sticky.