

Professor Debraj Ray  
19 West 4th Street, Room 608  
email: [debraj.ray@nyu.edu](mailto:debraj.ray@nyu.edu), homepage: <https://debrajray.com/>  
Webpage for course: <https://pages.nyu.edu/debraj/Courses/26UGDev/index.html>

ECON-UA 323  
*Development Economics*

**Extra Practice Problems for Examination 2**

**(1) True, False, or Uncertain?** In each case, back up your answer with a brief but precise explanation.

(a) In the Harris-Todaro model, an exogenous decline in the formal-sector wage increases formal labor demand, and therefore must increase the size of the informal sector via the Todaro paradox.

(b) In the Harris-Todaro model, a wage subsidy paid at the *same* rate per worker to employers in both agriculture and the formal sector cannot affect the allocation of labor, because it leaves the relative cost of labor in the two sectors unchanged.

(c) If potential migrants are risk-averse, the Harris-Todaro informal sector will be larger than if they are risk-neutral.

(d) Traffic on two alternative highways into a city is an example of a complementarity: everyone's choice of route affects everyone else's journey.

(e) In the "joining the revolution" model, if everyone gets the same benefit from a successful revolution and suffers the same loss if it fails, then the revolution either always happens or never happens, whatever people expect.

(f) The QWERTY keyboard has survived because, once typists have been trained on it, it is the most efficient layout available.

(g) Hirschman's strategy of unbalanced growth recommends that the government invest in the most profitable sectors of the economy, since these will have the strongest linkages with other sectors.

(h) Forward and backward linkages are equally effective in pulling other sectors into a high-investment equilibrium.

(i) In the complementarity map for industrialization, firms with high setup costs, which stay dormant in the low-investment equilibrium, may well prefer that equilibrium to the high-investment one.

(j) Since good institutions (secure property rights, courts that enforce contracts, broad access to schooling and the vote) promote investment and growth, every society will eventually adopt them.

(k) History can affect a society's long-run outcomes only if there are multiple equilibria, so that self-fulfilling expectations can pick one of them.

(l) In a Solow model in which the savings rate rises with per-capita income, a country whose capital stock is destroyed in a war will, once peace returns, always grow back to its pre-war level of income.

**(2) Industrialization and the Complementarity Map.** An economy has a continuum of firms, indexed by  $x \in [0, 1]$ . Each firm can stay *dormant*, in which case it earns a profit of 0, or it can *invest* in a modern technology. A firm that invests must pay a one-time setup cost, which differs across firms:

$$s(x) = 20 + 100x.$$

(So firms are arranged in increasing order of setup cost.) If a fraction  $n$  of all firms is expected to invest, each investing firm earns gross revenues of

$$R(n) = 36 + 100n^2.$$

(a) Explain in words why  $R(n)$  might increase with  $n$ . Give one concrete channel through which investment by *other* firms raises the revenue of an investing firm.

(b) Suppose a fraction  $n$  of firms is expected to invest. Show that firm  $x$  will want to invest if and only if

$$x \leq \frac{R(n) - 20}{100}.$$

Use this to write down the complementarity map  $x(n)$ : the fraction of firms that *want* to invest when a fraction  $n$  is *expected* to invest. Be careful:  $x(n)$  must lie between 0 and 1.

(c) Compute  $x(n)$  for  $n = 0, 0.2, 0.5, 0.8$  and 1, and draw the complementarity map together with the 45° line.

(d) Find *all* the equilibria. (Hint: an equilibrium is a value of  $n$  with  $x(n) = n$ . Look for interior equilibria using the formula, and then check the endpoints separately.) Which equilibria are stable and which are unstable? Explain using your diagram.

(e) Compare the low equilibrium  $n = 0.2$  with the high equilibrium  $n = 1$ . Show that *every* firm is at least as well off in the high equilibrium, and that many are strictly better off. In what sense is the low equilibrium a “coordination failure”?

(f) *A big push.* The economy is stuck at  $n = 0.2$ . The government offers a *temporary* subsidy towards setup costs, just large enough to persuade firms to invest *given* that they currently expect revenues of  $R(0.2) = 40$ . The subsidy is then withdrawn. Which firms must the government subsidize to guarantee that, after the withdrawal, the economy moves on its own to full industrialization? What is the total cost of the subsidy?

(g) Now suppose instead that revenues are  $R(n) = 8 + 200n^2$ , with the same setup costs. Redo parts (b) and (d). Show that this time the map has a single interior crossing of the 45° line, but there are still three equilibria. Which are stable? Why does no firm invest when no other firm does?

(h) Finally, suppose revenues are  $R(n) = 36 + 50n^2$ . Show that there is now a *unique* equilibrium, and find it. What does this tell you about the relationship between complementarities and multiple equilibria?

**(3) Income Distribution and the Savings Rate.** This problem tests your understanding of the effect of the income distribution on savings rates.

The economy of Sonrisa has people in three income categories: poor, middle class, and rich. The poor earn \$500 per year and have to spend it all to meet their consumption needs. The middle class earn \$3,000 per year, of which \$2,000 is spent and the rest saved. The rich earn \$10,000 per year, and consume 80% of it, saving the rest.

(a) Calculate the overall savings rate in Sonrisa if 30% of the people are poor and 60% are in the middle class.

(b) Suppose that all growth occurs by moving people from the poor category to the middle-class category. Will the savings rate rise over time or fall? Using the Harrod–Domar model and assuming that population growth is zero and all other variables are exogenous, predict whether the resulting growth rate will rise or fall over time.

(c) Outline another growth scenario with positive growth where the rate of growth changes over time in a way opposite to that of (b).

(d) Understand well that this question asks you about how growth *rates* are changing. In the simple Harrod–Domar model, the growth rate is constant over time because the savings rate is presumed to be unchanging with the level of income. Do you understand why matters are different here?

**(4) Verticalia.** Verticalia is a new community in the town of Flatland that is inviting residents to move in. The cool thing is that everyone will live in a mega highrise and hang out with everyone else. This “hanging-out” gives rise to *networking payoffs*, given by

$$P = An,$$

where  $A > 0$  is a positive parameter and  $n$  is the fraction of people in Flatland that move into Verticalia. But the problem is that more people moving in will also raise per-person rents in Verticalia because of congestion. Rent is given by the function

$$R = 10 + 5n.$$

Assume that the net payoff a person receives in Verticalia is networking payoff minus rent, and that there are no other costs or benefits other than these.

Meanwhile, living in Flatland *outside* Verticalia has its own payoff. We simply assume that payoff to be equal to zero for every resident.

(a) Show that *no one* living in Verticalia; i.e.,  $n = 0$ , is always an equilibrium, no matter how small or large the value of  $A$  is.

(b) Describe precisely the values of  $A$  for which  $n = 0$  is the *only* equilibrium.

(c) If the conditions in part (ii) are not met, show that there are multiple equilibria and describe all of them, including the unstable one.

(d) Now change the rent function to  $R = 1 + 36n^2$ , and assume that  $A = 20$ . Show that there are now just two stable equilibria, one in which no-one lives in Verticalia, and another in which just half of them do.

**(5) Harris-Todaro with Open Unemployment.** Pueblo has a labor force of 1000. The formal urban sector pays a fixed wage  $\bar{w}_m = 5$  and employs exactly  $\bar{L}_m = 250$  workers. A migrant who fails to get a formal job does not automatically find informal work: with probability  $q = 1/2$  she finds an informal job paying  $w_i = 2$ , and with probability  $1 - q$  she is openly unemployed and earns nothing. Formal jobs are allocated by lottery among all urban residents, so the probability of a formal job is  $p = \bar{L}_m / (\bar{L}_m + L_u)$ , where  $L_u$  is the number of urban residents without a formal job (informal workers plus the openly unemployed). Agriculture has a downward-sloping demand for labor,  $w_a = 8 - L_a/100$ . People are risk-neutral and migration is costless.

(a) Write down the expected income of a migrant as a function of  $p$ , and the Harris-Todaro equilibrium condition for Pueblo.

(b) Show that the equilibrium has  $L_a = 500$ , and find  $p$ , the agricultural wage, and the numbers of informal workers and openly unemployed.

(c) The government creates 200 more formal jobs, so  $\bar{L}_m = 450$ . Show that the new equilibrium has  $L_a = 400$  and find the new numbers of informal and openly unemployed workers. Did the policy reduce open unemployment? Would it have done so if agriculture had been in the Lewis surplus-labor phase with a fixed wage of 3?

(d) Return to part (b). Suppose a public-works program guarantees an informal job to anyone who wants one, so that  $q$  rises from  $1/2$  to 1. Without solving for the new equilibrium, show that at the old allocation the expected urban income now exceeds the agricultural wage, and describe the direction in which migration, the agricultural wage, and the size of the urban pool without formal jobs all move.

**(6) Status Quo Bias.** A country of 100 citizens votes on a trade reform. If adopted, each of 60 citizens (the “winners”) gains \$5 and each of the other 40 (the “losers”) loses \$6. Everyone is risk-neutral and votes in her own interest; the reform passes if more than 50 vote for it.

(a) Suppose nobody knows whether she will be a winner or a loser. Does the reform pass?

(b) Suppose everyone knows her type. Does the reform pass?

(c) Now suppose that exactly 30 of the winners know that they are winners, while the other 70 citizens know only that 30 winners have been identified and that 60 winners exist in total. Show that the reform fails.

(d) Let  $k$  be the number of winners who know their identity, everyone else being uncertain as in (c). For which values of  $k$  does the reform fail?

(e) Suppose the reform was adopted at some earlier date, so that by now everyone knows her type. A proposal to repeal it comes to a vote. Does it pass? Combine your answers to explain what “status quo bias” means here.

**(7) Endogenous Savings and Two Steady States.** Consider the Solow model with no technical progress. Per-capita output is  $y = 10k^{1/2}$ , and population growth plus depreciation sum to  $n + \delta = 0.1$ , so that the per-capita accumulation equation is  $k(t+1) = k(t) + sy(t) - 0.1k(t)$  (ignore the  $1+n$  denominator for simplicity). The savings rate depends on income: people save 5% of income when per-capita income is below 300, and 40% of it when per-capita income is 300 or more.

(a) Treating  $s$  as a fixed number for the moment, show that the steady state per-capita income is  $y^* = 1000s$ .

(b) Find all the steady states of the economy with the income-dependent savings rate, and check that each one is consistent with the savings rate that generates it.

(c) Describe where the economy ends up as a function of its initial per-capita income  $y(0)$ . Explain why two economies with identical technology and identical savings *behavior* can have very different long-run incomes.

(d) The economy sits at the low steady state. A donor offers a *one-time* grant of capital. What is the smallest grant per person that has a permanent effect, and what is the effect? Contrast this with the effect of the same grant in a Solow model where  $s$  is a fixed parameter.

**(8) Joining the Revolution.** A population of size 1 is deciding whether to join a revolution. If the revolution succeeds, each participant gets  $B > 0$ ; if it fails, each participant suffers a loss  $L > 0$ . Bystanders get 0 either way. The probability of success is  $p(n) = n^2$ , where  $n$  is the fraction of the population that participates.

(a) Show that an individual who expects a fraction  $n$  to participate wants to join if and only if  $n > \sqrt{L/(B+L)}$ .

(b) Let  $B = 3$  and  $L = 1$ . Draw the complementarity map, find all the equilibria, and classify them as stable or unstable.

(c) Now suppose participants differ in what they stand to gain:  $B$  is uniformly distributed on  $[0, 4]$  across the population, with  $L = 1$  for everyone. Show that the fraction wanting to join is  $x(n) = 1 - (1 - n^2)/(4n^2)$  when this is positive (and 0 otherwise), and that the interior equilibrium solves  $4n^3 - 5n^2 + 1 = 0$ . Find all the equilibria and classify them. (The cubic has  $n = 1$  as a root; factor it out.)

**(9) Complementarities or Omitted Variables?** Munshi and Myaux study contraceptive use in villages in Matlab, Bangladesh, where use rose from 40% in 1983 to 63% in 1993 after an intensive family-planning program began in 1978. Their hypothesis is that individual use responds to use by others in the village, a complementarity working through social norms.

- (a) Write down a regression that would test this hypothesis, with a couple's contraceptive use in year  $t$  on the left-hand side and village-level use in year  $t - 1$  on the right, and explain why a positive coefficient on village use is *not* by itself evidence of a complementarity.
- (b) The authors can include village fixed effects and year fixed effects. Which parts of the omitted variable do these remove, and which part survives?
- (c) Explain how splitting village-level use into use among the couple's own religious group and use among the other group helps, and what assumption the argument rests on.

**(10) Harris-Todaro: Efficiency, Migration Bans and Subsidies.** Labora has a labor force of 1000. The formal sector pays a fixed wage of  $\bar{w}_m = 8$ . Formal employers hire up to the point where the marginal product of labor equals what a worker costs them, and the marginal product in the formal sector is  $10 - L_m/100$  when  $L_m$  people are employed there. Agriculture has a flexible wage equal to its marginal product,  $w_a = 10 - L_a/100$ . Anyone in the city without a formal job is unemployed and earns nothing. Total output in agriculture is  $10L_a - L_a^2/200$ , and in the formal sector it is  $10L_m - L_m^2/200$ .

- (a) Show that the formal sector employs 200 people, and that in the Harris-Todaro equilibrium there are 200 unemployed and 600 people in agriculture. What is the agricultural wage?
- (b) Find the allocation of the 1000 workers between agriculture and the formal sector that maximizes total output. Compare total output there with total output in the Harris-Todaro equilibrium.
- (c) The government bans anyone without a formal job from living in the city. Describe the new allocation and the agricultural wage, and compute total output. Is the outcome efficient? Explain.
- (d) Instead of a ban, the government pays a subsidy of  $\sigma$  per worker to employers in *both* sectors. (Formal workers still receive 8; agricultural workers receive the market wage  $w_a$ , and agricultural employers pay  $w_a - \sigma$ .) Show that a subsidy of  $\sigma = 3$  eliminates unemployment and achieves the efficient allocation, with no restriction on migration. How much does the subsidy cost?

**(11) History or Expectations? Moving to Nuevo.** Two regions, Viejo and Nuevo, are home to a population of size 1. Living and working in Viejo pays 12 per period, however many people live there. Nuevo has complementarities: it pays

$$r(n) = 2 + 20n$$

per period, where  $n$  is the fraction of the population in Nuevo. (Think of shared infrastructure, a thicker job market, or a richer network of suppliers, all of which improve as Nuevo grows.) The fraction living in Nuevo at the start is  $n_0$ , which is given by history.

- (a) Draw the complementarity map: the fraction  $x(n)$  of people who want to live in Nuevo when a fraction  $n$  is expected to live there. Find all the equilibria, and say which are stable.

(b) *History*. Suppose that every period a few people move from the region that currently pays less to the region that currently pays more. Describe what happens over time if  $n_0 = 0.4$ , and if  $n_0 = 0.6$ .

(c) *Expectations*. Now suppose instead that people can move instantly, and that Nuevo's return adjusts instantly to the number of people living there. Start with  $n_0 = 0.1$ . Show that if everyone expects everyone else to move to Nuevo, they will all move, and that if everyone expects no one to move, no one will. What role does history play now?

(d) *Sluggish returns*. Suppose now that Nuevo's return adjusts with a lag: a person in Nuevo this period earns  $2 + 20n$ , where  $n$  is the fraction who lived there *last* period. People move once and for all, and they care about the total of all their future payoffs (no discounting, to keep things simple). Start with  $n_0 = 0.4$ , and suppose everyone else moves to Nuevo in period 1. Show that each person would rather wait and move in period 2. What happens to the expectations-driven equilibrium of part (c)?

(e) *An advantage to moving early*. Keep the lag of part (d), but now suppose that moving gets more expensive over time: moving in period 1 costs nothing, but moving in period 2 or later costs  $m > 0$  (housing in Nuevo fills up, the best plots are taken, and the early arrivals get the better jobs). Show that if everyone else moves in period 1, each person also wants to move in period 1 if and only if  $m > 12 - r(n_0)$ . Compute this threshold for  $n_0 = 0$ ,  $n_0 = 0.2$  and  $n_0 = 0.4$ .

(f) Use part (e) to explain the following statement: "History matters most for economies that start far from the tipping point, while expectations matter most for economies that start close to it." Why might an announcement by the government (for instance, of a new port to be built in Nuevo) succeed in some regions and fail in others?

**(12) Status Quo Bias: Who Knows What?** Reformia has 100 citizens, who vote by simple majority on a reform. If it is adopted, 55 citizens will be winners, each gaining 1, and 45 will be losers, each losing 1. Everyone knows these numbers. Citizens are risk-neutral and vote in their own interest. A citizen who does not know whether she will win or lose takes her chance of winning to be the fraction of winners among all those who, like her, do not know.

(a) Show that if no one knows her own identity, the reform passes.

(b) Suppose  $k$  of the winners know in advance that they will win, and no one else knows anything. Show that the reform fails for every  $k$  from 11 to 50.

(c) Now suppose instead that  $j$  of the *losers* know in advance that they will lose, and no one else knows anything. Show that the reform passes, whatever the value of  $j$ . What happens if 20 winners and 20 losers know their identities? Why does it matter so much *whose* identity is revealed?

(d) Before the vote, the government runs a pilot program that reveals to 20 citizens whether they will win or lose. Suppose first that the 20 are chosen at random, so that 11 of them turn out to be winners and 9 losers. Does the reform pass? Now suppose instead that the

pilot is run in a region where the reform's benefits are concentrated, so that all 20 turn out to be winners. Does it pass now?

(e) Suppose the reform had been adopted without a vote, and a year later everyone knows whether she won or lost. Would a vote to repeal it succeed? Fernández and Rodrik used this model to explain why trade liberalizations are often adopted only in times of crisis, and are then rarely reversed. Explain, using your answers.