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ECON-UA 323

Answers to Sample Examination 1, Version 2, Fall 2026

NO CALCULATORS, IPADS, LAPTOPS, ETC., ALLOWED. PUT THEM AWAY, PLEASE.

Points 65. Time 75 minutes. The first question carries 30 points; the second 17 points, and the third 15 points. 3 points are reserved for extra credit, presentation and clarity.

Guide for Time Allocation: *The questions in (1) should take no more than 5 minutes each to answer; total 30 minutes. Questions (2) and (3) should take you no more than 15 minutes each. This schedule will allow you to finish the exam in 60 minutes. If you are stuck with a question, move on to the next one and plan to come back later. Keep your answers brief and to the point.*

(1) (30 points, 6 points per part, 5 parts) Are the following statements true or false? It is *not* enough to just guess one or the other. You need to provide an argument for or against, and only then will any credit be awarded.

[a] A production function defined on three inputs (say physical capital, human capital and unskilled labor) can exhibit diminishing returns to *each one* of these three inputs, *and* increasing returns to scale.

True. Consider the Cobb-Douglas production function given by

$$Y = AK^a L^b H^c,$$

where each of a , b and c lie strictly between 0 and 1 and $a + b + c \equiv D > 1$. This function has increasing returns to scale, because if we multiply all inputs by $\lambda > 1$ and let Y' denote the new output,

$$Y' = A(\lambda K)^a (\lambda L)^b (\lambda H)^c = \lambda^D AK^a L^b H^c = \lambda^D Y > \lambda Y.$$

And yet, because each of the separate exponents are between 0 and 1, the function has diminishing returns to each input.

[b] The Harrod-Domar model is capable of generating persistent growth in the *per-capita* capital stock (and therefore in per-capita income), even without any technical progress.

True. In the Harrod-Domar model with no technical progress, we know that

$$(1 + n)k(t + 1) = (1 - \delta)k(t) + sAk(t),$$

and dividing through by $k(t)$, we see that

$$\frac{k(t + 1)}{k(t)} = \frac{(1 - \delta) + sA}{1 + n}$$

which could persistently exceed 1 (i.e., exhibit persistent growth).

[c] Under the Kalecki-Rao constraint $Y_a = z(L_a + L_m)$, a change in the modern capital stock K_m has no effect on modern sector output.

False. It is true that under the Kalecki-Rao constraint, L_m fails to change when there is a change in K_m . That is because the magnitude of the constraint does not depend on K_m and can only support the same value of L_m before and after the change. The price p of the manufactured good suitably changes to implement the same L_m . But Y_m still changes, because K_m has changed and that has its own effect on output.

[d] A country which has been growing steadily at 2% per year and now has a per-capita income of \$80,000 would have a per-capita income under \$5000 a century and a half ago.

True. At a growth rate of 2% a year, a country will be doubling its income once every $70/2 = 35$ years. A century and a half contains 4 such spans of 35 years and an extra 10. In the four spans, with income doubling each time, a country should have 16 times its starting income (not even counting the extra 10 years), so that starting income is under \$5000.

[e] This could conceivably be a possible mobility matrix across four relative income categories:

	1	2	3	4
1	80%	20%	0%	0%
2	0%	15%	65%	35%
3	0%	50%	50%	0%
4	0%	0%	0	100%

False. A bonafide mobility matrix must have each row adding up to precisely 100, because a set of countries in any row (=starting category) must be fully divided across all the ending categories, and so the percentages must add up to 100%. This is not satisfied by the depicted mobility matrix on row 2.

(2) (17 points) The country of Kuznetsia has 1500 inhabitants and two potential sectors, a and m . The production functions in sectors a and m are given by

$$Y_a = 600\sqrt{L_a} \text{ and } Y_m = 15\sqrt{K_m L_m}$$

respectively. The price of the a good is 1 and the price of the m good is fixed at $p = 2$, so this is an open economy with no Kalecki-Rao constraint. There is a work subsistence of $z = 10$, defined in units of the agricultural good. In both sectors, labor is chosen to maximize profits.

(a) [4 points] If $K_m = 0$, so that the m -sector is inactive, show that Kuznetsia cannot fully employ its entire labor force. Compute the share of the labor force that is indeed hired.

While the m -sector is inactive, the a -sector chooses L_a to maximize

$$Y_a - w_a L_a = 600L_a^{1/2} - w_a L_a,$$

the first order condition for which is given by

$$300L_a^{-1/2} = w_a,$$

so that we can solve out to get $L_a = (300/w_a)^2 = 90000/w_a^2$. This cannot give us full employment, because $w \geq z = 10$, and so

$$L_a = 90000/w_a^2 \leq 900,$$

whereas the total labor force is 1500. But this argument also shows that the wage must indeed drop to work subsistence. (Any strictly higher wage cannot be sustained when there is unemployment, it would be undercut.) So the final answer is that $L_a = 900$: 60% of the workforce is employed, and $w_a = z = 10$.

(b) [5 points] Next, suppose K_m jumps up to 400, so the modern sector springs to life. However, suppose that only 10% of the labor force have the skills to work in the m -sector. What will be the wages in the two sectors now? And is there full employment? Why or why not?

If $K_m = 400$, then the m -sector will choose L_m to maximize

$$pY_m - w_m L_m = p15\sqrt{K_m L_m} - w_m L_m = 600\sqrt{L_m} - w_m L_m$$

(remembering $p = 2$ and $K_m = 400$), the first order condition for which is given by

$$300L_m^{-1/2} = w_m,$$

which we can solve out to get $L_m = 90000/w_m^2$. But only 10%, or 150, can work in the m -sector, and so in equilibrium, the wage rate w_m must solve the equation

$$(1) \quad L_m = 90000/w_m^2 = 150,$$

which means that $w_m = \sqrt{600} \approx 25$, which is way above $w_a = z = 10$. The m -sector employs 150 at this wage, the a -sector 900 at wage $w_a = 10$ (as already shown), and 450 people are left unemployed, because they cannot be employed in a nor do they have the skills to work in m . There is a large wage differential between $w_a = 10$ and $w_m = \sqrt{600}$ but it cannot be closed, because everyone who *can* work in sector m is already working there — there is no one else left to undercut the wage.

(c) [5 points] A few years later, the capital stock K_m is still fixed at 400, but now everyone can work in the modern sector. Is there still some unemployment? What will be the new wages in the two sectors of Kuznetsia?

Now everyone can work in either sector, so the wages across sectors will have to be the same. Our first question is: is there still unemployment? Well, for that to be the case, the m -sector must hire less than 600 people (recall that the rest are in a), *and* the wage w_m has to be down to $z = 10$ (otherwise it will be undercut). But we know from equation (1) that labor demand is given by

$$L_m = 90000/w_m^2,$$

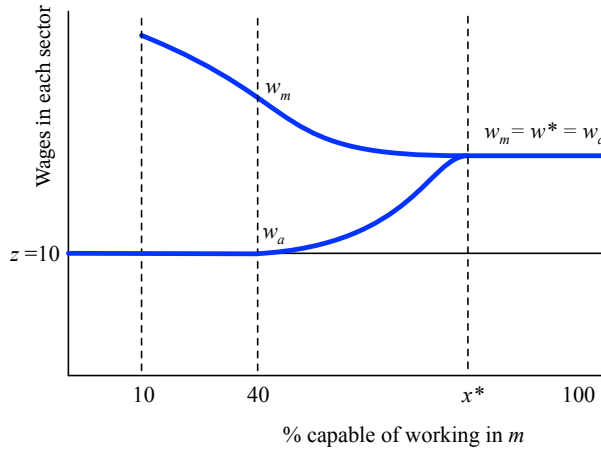
which is larger than 600 when $w_m = z = 10$. So there must be full employment, and the two wages w_a and w_m *must be equal*. Calling this common wage w^* , the full employment condition can be written as

$$L_a + L_m = 90000/w^{*2} + 90000/w^{*2} = L = 1500,$$

and canceling terms and transposing, we obtain $w^* = \sqrt{120}$. For *full* credit, you should note that the wage w^* rises above the work subsistence $z = 10$.

(d) [3 points] Using the answers to (b) and (c) as anchors, draw a graph with a written explanation that shows both the evolution of employment and the path of wages in Kuznetsia as the proportion of workers capable of working in the new m -sector climbs up from 10% to 100%.

This question can be answered for different levels of credit. The full answer is given in this diagram:



Explanation. Initially all employment is in a and the wage is $w_a = z = 10$. Then the modern sector comes in and 10% are initially capable of working there, so we start off there at a relatively high wage ($w_m = \sqrt{600} \approx 25$) while the wage w_a stays at 10. This state of affairs continues until 40% or more can work in the m -sector (remember a is absorbing 60%) and after this there is competition across the two sectors for workers, there is full employment and the wage starts to climb. Everyone who *can* work in m is working in m but there will still be a wage differential initially. That goes away at some percentage x^* at which there is enough overlap of potential employment between the two sectors. At this stage, the wage everywhere has climbed to its final level $w^* = \sqrt{120}$ that you described in the previous question.

You can answer this question at different levels which will all get you some amount of credit but only the completeness of the above answer will get you full credit. If someone computes x^* , they will get *extra* credit!

(3) (15 points) The country of Lento has a production function which depends on capital and labor, and at any date t , it is given by

$$Y = 8K^{1/4}[e(t)L]^{3/4}.$$

where $e(t) = (1 + \pi)^t$ is the exogenous productivity of labor at time t , which grows at a constant rate π .

(a) [3 points] Find a formula for per-effective-capita production $\hat{y}$ as a function of the per-effective-capita capital stock $\hat{k}$.

Divide both sides of the production function by effective labor $e(t)L$; then:

$$\frac{Y}{e(t)L} = \frac{8K^{1/4}[e(t)L]^{3/4}}{e(t)L} = 8 \left[\frac{K}{e(t)L} \right]^{1/4},$$

so that, defining per-effective-capita magnitudes by $\hat{y} = Y/e(t)L$ and $\hat{k} = K/e(t)L$, we have:

$$(2) \quad \hat{y} = 8\hat{k}^{1/4}.$$

(b) [4 points] In Lento, capital has depreciation rate δ , but there is no population growth of labor (L is constant). The savings rate is given by s . Show that steady state path of output per-capita is given by

$$(3) \quad y^*(t) = 16 \left(\frac{s}{\delta + \pi} \right)^{1/3} (1 + \pi)^t,$$

describing precisely all the steps that lead to this conclusion.

Let us recall the Solow equation for capital accumulation, which is:

$$K(t+1) = (1 - \delta)K(t) + sY(t).$$

Dividing through by $e(t)L$ on both sides:

$$\frac{K(t+1)}{e(t)L} = (1 - \delta) \frac{K(t)}{e(t)L} + s \frac{Y(t)}{e(t)L} = (1 - \delta)\hat{k}(t) + s\hat{y}(t),$$

and because $K(t+1)/e(t)L = [e(t+1)/e(t)][K(t+1)/e(t+1)L] = (1 + \pi)\hat{k}(t+1)$, we get

$$(4) \quad (1 + \pi)\hat{k}(t+1) = (1 - \delta)\hat{k}(t) + s\hat{y}(t) = (1 - \delta)\hat{k}(t) + 8s\hat{k}(t)^{1/4},$$

where the last equality uses equation (2) in part (a). In steady state, $\hat{k}(t) = \hat{k}(t+1) = \hat{k}^*$, and using this in equation (4), we see that

$$(1 + \pi)\hat{k}^* = (1 - \delta)\hat{k}^* + 8s\hat{k}^{*1/4},$$

or

$$\hat{k}^* = 8^{4/3} \left(\frac{s}{\delta + \pi} \right)^{4/3} = 16 \left(\frac{s}{\delta + \pi} \right)^{4/3}.$$

It follows that

$$\hat{y}^* = 8\hat{k}^{*1/4} = 8 \left[16 \left(\frac{s}{\delta + \pi} \right)^{4/3} \right]^{1/4} = 16 \left(\frac{s}{\delta + \pi} \right)^{1/3},$$

and opening up $\hat{y}$ to trace out per-capita income y^* , we see that $y^*(t) = \hat{y}(t)(1 + \pi)^t$, so that

$$y^*(t) = 16 \left(\frac{s}{\delta + \pi} \right)^{1/3} (1 + \pi)^t,$$

which establishes (4).

(c) [4 points] If Lento initially has per-capita income *below* its steady state path, explain why its growth rate of per-capita income will *exceed* π .

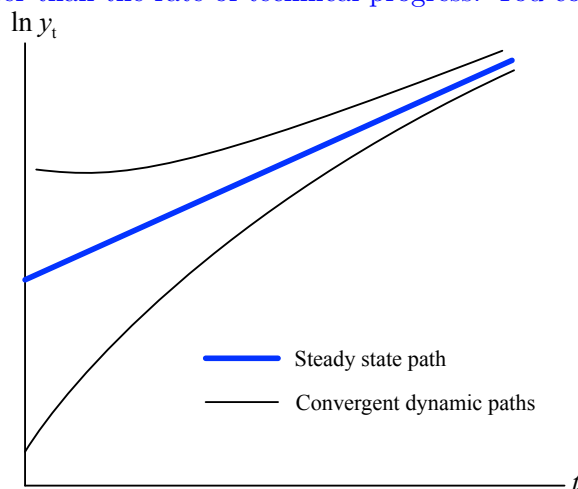
Look at the growth accumulation equation (4). Divide by $\hat{k}(t)$ on both sides; then

$$(5) \quad (1 + \pi) \frac{\hat{k}(t+1)}{\hat{k}(t)} = (1 - \delta) + 8\hat{k}(t)^{-3/4},$$

The right hand side declines steadily as a function of $\hat{k}(t)$, going from a high of near-infinity (when $\hat{k}(t)$ is small) and converging to $1 - \delta$ as $\hat{k}(t)$ becomes large. At the steady state $\hat{k}^*$, of course, $\hat{k}(t+1)/\hat{k}(t) = 1$. In summary,

$$\frac{\hat{k}(t+1)}{\hat{k}(t)} > 1 \text{ when } \hat{k}(t) < \hat{k}^* \text{ and } \frac{\hat{k}(t+1)}{\hat{k}(t)} < 1 \text{ when } \hat{k}(t) > \hat{k}^*$$

where $\hat{k}^*$ is the steady state. In particular, capital in effective labor units is growing when the economy is below its steady state, which means in turn that capital per capita and income per capita are growing faster than the rate of technical progress. You could also use a diagram:



In this diagram, the steady state for Lento is shown in blue. The diagram is in logs of per-capita income so exponential growth appears as linear in time. The curved line below the blue line shows what happens if Lento starts from an initial per-capita income that is below its steady state path. Then income will catch up to the steady state. For that to happen, income per-capita in Lento must be growing — during this transition — at a rate that exceeds the rate of technical progress, which is the steady state growth rate.

(d) [4 points] If Lento initially has per-capita income *above* its steady state path, then will the growth rate of its per-capita income be positive, negative, or ambiguous? Discuss.

The opposite would be true were Lento to start above the steady state line, where the subsequent trajectory is represented by the curve above the steady state path. See diagram again. Now, capital in effective labor units is *shrinking* when Lento is above its steady state, so capital per capita and income per capita are growing *slower* than the rate of technical progress.

What does this mean for the overall rate of growth — is it positive, negative or ambiguous. The answer is that it is ambiguous: the fact that capital in effective units is coming down tends to make the growth negative, but at the same time, there is technical progress, which tends to make the growth positive. The net effect is ambiguous.