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ECON-UA 323

Answers to Sample Examination 1, Version 1, Fall 2026

NO CALCULATORS, IPADS, LAPTOPS, ETC., ALLOWED. PUT THEM AWAY, PLEASE.

Points 65. Time 75 minutes. The first question carries 30 points; the second 17 points, and the third 14 points. 4 points are reserved for extra credit, presentation and clarity.

Guide for Time Allocation: *The questions in (1) should take no more than 5 minutes each to answer; total 30 minutes. Questions (2) and (3) should take you no more than 15 minutes each. This schedule will allow you to finish the exam in 60 minutes. If you are stuck with a question, move on to the next one and plan to come back later. Keep your answers brief and to the point.*

(1) (30 points, 6 points per part, 5 parts) Are the following statements true or false? It is *not* enough to just guess one or the other. You need to provide an argument for or against, and only then will any credit be awarded.

[a] If a production function defined on two inputs (say capital and labor) has constant marginal returns to *each input*, then it must exhibit increasing returns to *scale*.

False. It is true for the Cobb-Douglas production function, and if you explain that clearly you will get partial credit, but it is not always true. You can check that the production function

$$Y = F(K, L) = AK + BL$$

where $A > 0$ and $B > 0$, has constant marginal returns of A and B when K and L are varied, and at the same time it also has constant returns to scale.

[b] In the Solow model with no technical progress, if the savings rate increases, then there exists *some* change in the population growth that “cancels” the savings rate increase, so that per-capita income in steady state is unchanged.

True. In the Solow model with no technical progress, the steady state k^* is characterized by

$$\Gamma(k^*) = \frac{n + \delta}{s},$$

where $\Gamma(k)$ is the output-capital ratio when per-capita capital is k . Notice that if s goes up to $s' > s$, then we can always find a rate of population growth $n' > n$ such that the ratios

$$\frac{n + \delta}{s} \text{ and } \frac{n' + \delta}{s'}$$

are equal. But then k^* cannot change.

You are not required to derive the formula for steady state k^* to answer this question, but you can if you wish.

[c] In the surplus labor phase of the open-economy Lewis model with laborers equally capable of working in either sector, a small expansion in the modern sector's capital stock raises modern employment, *but not the wage*.

True. Draw the diagram showing that the work subsistence is binding on the wage, and with employment $L_a + L_m < L$. Then explain diagrammatically that a small increase in K_m will raise the demand for labor in the m -sector. Show how the modern demand curve for labor then shifts to the right, but because this expansion is small, there is still unemployment after the shift. Then labor L_m will increase but the wage will still stay at its lower bound z .

Some of you might answer by saying that the rise in K_m could raise labor demand by so much that in the new setting, $L_a + L_m > L$ when the labor demands are calculated at the work subsistence z . Then it is true that the wage will rise, and this will give you partial credit if you answered "False" to this question and gave this example. But you should understand that for a small enough change in the labor demand, this scenario would never come about, and for this reason the answer is "True."

[d] A country that grows at 5% a year for the next 28 years, and then shrinks by 1% a year for the next 140 years is back to approximately where it started (in terms of income).

True. We use the approximate formula for doubling times given by $70/g$. So if a country starts at income y and grows at 5%, income will double to $2y$ in 14 years, and then again to $4y$ in another 14 years, or 28 in all. Now we note that if a country is declining, the same formula applies to get halving times, because it is just like running time backwards. So if our country declines by 1% a year, income will halve in 70 years from $4y$ to $2y$ and then again in another 70 years to y . Done.

[e] This could conceivably be a possible mobility matrix across four relative income categories:

	1	2	3	4
1	77%	15%	12%	0%
2	0%	0%	65%	35%
3	0%	50%	50%	0%
4	0%	0%	0	100%

False. In a valid mobility matrix, each row has to add up to 100%, because the numbers represent the percentages of countries from a starting category which transit to all the categories at the end of the period. But the first row adds up to more than 100%. So, this is not a valid mobility matrix.

(2) (17 points) The country of Kaleckia has 10,000 inhabitants and two potential sectors, a and m . It is a closed economy and needs to produce its own food for work subsistence: $z = 2$ per person for every employed person. The same skills are needed for either sector and labor moves freely across them. The production functions in sectors a and m are given by

$$Y_a = 20\sqrt{K_a L_a} \text{ and } Y_m = 10\sqrt{K_m L_m}$$

In both sectors, labor is chosen to maximize profits.

(a) [4 points] Write down the Kalecki-Rao constraint, and show that the *minimum* value of K_a that just feasibly allows all 10,000 to consume the work subsistence $z = 2$ is given by $K_a = 100$. (This is not a question about markets, but about what's plain feasible.)

The Kalecki-Rao constraint states that domestic production must be enough to “feed” every worker at the rate of $z = 2$ per worker, so it can be written as

$$Y_a = 20\sqrt{K_a L_a} = 2(L_a + L_m).$$

Now we want to find the minimum value of K_a that will just about permit $L_a + L_m$ to hit 10,000. To do this exercise (which is not necessarily one that the market would do on its own — see below), we want to have all hands in agriculture, so that the value of Y_a above is as large as possible. That means that if $L_m > 0$, reduce it and increase L_a . In short, to solve our problem we set $L_m = 0$ and find the minimum value of K_a such that the above equation, now written as

$$20\sqrt{K_a L_a} = 2L_a, \text{ or } L_a = 100K_a$$

has to hit $L_a = 10,000$. Solving the above equation, we see that the minimum needed value of $K_a = 100$. Anything less than this and we must have unemployment.

(b) [5 points] Now show that with profit maximizing production in both sectors and with $K_a = 100$, total employment in *market equilibrium* is just 5000 — very different from (a).

Let's find the demand curves for labor under profit-maximizing production. In the a -sector, producers maximize:

$$(1) \quad Y_a - wL_a = 20\sqrt{100L_a} - wL_a = 200\sqrt{L_a} - wL_a$$

by choosing L_a . Carrying out this exercise, we see that the first-order condition is given by:

$$(2) \quad 100L_a^{-1/2} = w, \text{ so that } L_a = 10,000/w^2.$$

It follows that at any wage $w \geq z$, output is given by

$$(3) \quad Y_a = 20\sqrt{K_a L_a} = 200\sqrt{L_a} = 20,000/w,$$

so by the Kalecki-Rao constraint and using equation (3), the total employment is given by

$$(4) \quad L_a + L_m = \frac{1}{z}Y_a = \frac{1}{2} \frac{20000}{w}.$$

Because $w \geq z = 2$, this means that total *market* employment is no more than 5000. It follows that in equilibrium, w has to come down to the subsistence level $z = 2$, and total market employment is just 5000 (substitute $w = 2$ in equation (4)).

(c) [3 points] Explain clearly why this employment figure is *completely independent* of the value of K_m , but why it may be expected to change when K_a changes.

Notice that in the calculation for part (b), we used only the size of K_a but K_m played no role anywhere. So mathematically, it is clear that $L_a + L_m = 5000$ when $K_a = 100$, no matter what the size of K_m is. This independence comes from the Kalecki-Rao constraint: once agrarian employment and output is pinned down, so is the total amount of surplus left

over to support a modern work force at the subsistence level. It is true that in the m -sector, producers maximize:

$$pY_m - wL_m = 10p\sqrt{K_m L_m} - wL_m = 10p\sqrt{K_m}\sqrt{L_m} - wL_m$$

by choosing L_m . Carrying out this exercise, we see that the first-order condition is given by:

$$5p\sqrt{K_m}L_m^{-1/2} - w, \text{ so that } L_m = 25K_m p^2 / w^2.$$

But when K_m changes, the market price p changes just so as to keep $K_m p^2$ the same, and L_m becomes independent of the size of K_m .

In contrast, when K_a changes, the surplus left over to “support” a new level of L_m does indeed change, and in general both total employment and individual employment in each sector will climb with K_a . We will do more of this in the next part below.

(d) [5 points] Find the minimum value of K_a that permits full employment *in market equilibrium*. Explain why the answers to (a) and (d) are different.

Let’s re-do the demand curves for labor for any K_a . In the a -sector, producers maximize:

$$(5) \quad Y_a - wL_a = 20\sqrt{K_a L_a} - wL_a$$

by choosing L_a . The first-order condition is given by:

$$(6) \quad 10\sqrt{K_a}L_a^{-1/2} = w, \text{ so that } L_a = 100K_a/w^2,$$

which of course reduces to equation (2) when $K_a = 100$. So for any $w \geq z$, output is given by

$$(7) \quad Y_a = 20\sqrt{K_a L_a} = 200K_a/w$$

(again, check out the match with equation (3) when $K_a = 100$). By the Kalecki-Rao constraint applied to equation (7), the total employment in market equilibrium is given by

$$(8) \quad L_a + L_m = \frac{1}{z}Y_a = \frac{1}{2} \frac{200K_a}{w} = \frac{100K_a}{w}$$

As long as there is unemployment, w is at work subsistence, and so $w = 2$. Therefore $L_a + L_m = 50K_a$, and so the minimum value of K_a that’s needed to take market equilibrium to full employment is $K_a = 200$.

Why is this threshold bigger than the K_a threshold in part (a)? The reason is that in part (a), we are not studying market equilibrium. We are doing all we can feasibly do to get total work subsistence payouts as high as possible, so that involves putting everyone in the a -sector. The market equilibrium does not do this: the m -sector is always active as long as $K_m > 0$. So the threshold value of K_a needed for full employment is higher.

(3) (14 points) The country of Rápido has a production function which depends on capital and labor, and at any date t , it is given by

$$Y = 9K^{1/3}[e(t)L]^{2/3}.$$

where $e(t) = (1 + \pi)^t$ is the exogenous productivity of labor at time t , growing at a constant rate π .

(a) [4 points] Find a formula for per-effective-capita production $\hat{y}$ as a function of the per-effective-capita capital stock $\hat{k}$, and explain the steps to get there.

Divide both sides of the production function by effective labor $e(t)L$; then:

$$\frac{Y}{e(t)L} = \frac{9K^{1/3}[e(t)L]^{2/3}}{e(t)L} = 9 \left[\frac{K}{e(t)L} \right]^{1/3},$$

so that, defining per-effective-capita magnitudes by $\hat{y} = Y/e(t)L$ and $\hat{k} = K/e(t)L$, we have:

$$\hat{y} = 9\hat{k}^{1/3}.$$

(b) [5 points] In Rápido, capital has depreciation rate δ , population growth n , and savings rate s . Show that steady state path of output per-capita is given by

$$y^*(t) = 27 \left(\frac{s}{\delta + \pi + n} \right)^{1/2} (1 + \pi)^t,$$

describing precisely all the steps that lead to this conclusion.

Let us recall the Solow equation for capital accumulation, which is:

$$K(t+1) = (1 - \delta)K(t) + sY(t),$$

where we use the assumption that there is no depreciation. Dividing through by $e(t)L$ on both sides:

$$\frac{K(t+1)}{e(t)L} = (1 - \delta) \frac{K(t)}{e(t)L} + s \frac{Y(t)}{e(t)L} = (1 - \delta)\hat{k}(t) + s\hat{y}(t),$$

and because $K(t+1)/e(t)L = [e(t+1)/e(t)][K(t+1)/e(t+1)] = (1 + \pi)\hat{k}(t+1)$, we get

$$(1 + \pi)(1 + n)\hat{k}(t+1) = (1 - \delta)\hat{k}(t) + s\hat{y}(t) = (1 - \delta)\hat{k}(t) + 9s\hat{k}(t)^{1/3},$$

where the last equality uses part (a). In steady state $\hat{k}^*$, we therefore have

$$(1 + \pi)(1 + n)\hat{k}^* = (1 - \delta)\hat{k}^* + 9s\hat{k}^{*1/3},$$

and solving this system for its positive solution, we have:

$$\hat{k}^* \simeq 9^{3/2} \left(\frac{s}{\delta + \pi + n} \right)^{3/2},$$

where at this stage we use the approximation that $(1 + \pi)(1 + n) \simeq 1 + \pi + n$. It follows that

$$\hat{y}^* = 9\hat{k}^{*1/3} = 9 \left[9^{3/2} \left(\frac{s}{\delta + \pi + n} \right)^{3/2} \right]^{1/3} = 9^{3/2} \left(\frac{s}{\delta + \pi + n} \right)^{1/2} = 27 \left(\frac{s}{\delta + \pi + n} \right)^{1/2},$$

and opening up $\hat{y}$ to trace out per-capita income y^* , we see that $y^*(t) = \hat{y}(t)(1 + \pi)^t$, so that

$$y^*(t) = 27 \left(\frac{s}{\delta + \pi + n} \right)^{1/2} (1 + \pi)^t,$$

which establishes the equation we want.

(c) [5 points] Now set technical change and depreciation both equal to zero in Rápido. Show that the coefficient on log savings rate is equal in magnitude to, but has the opposite sign of,

the coefficient on log population growth, in any regression of steady state per-capita output on these parameters. Discuss this intuitively.

With technical change and depreciation both set to zero, steady state output is given by

$$y^* = 27 \left(\frac{s}{n} \right)^{1/2}.$$

Take logs to see that

$$\ln y^* = \ln 27 + \frac{1}{2} \ln s - \frac{1}{2} \ln n.$$

So the predicted regression coefficients on $\ln s$ and $\ln n$ are equal in magnitude, but opposite in sign.

There is good reason for this. An increase in the fraction of savings out of national income will raise capital accumulation, thereby moving up the Solow steady state. An increase in the population growth rate *lowers* capital accumulation *per capita*. The two effects tend to cancel out when there is no technical progress or depreciation. An increase in the population growth rate by 1% can be exactly nullified by the same percentage increase in the savings rate, so that the resulting growth in capital makes up exactly for the resulting higher population numbers.