Development Economics

Slides 4

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Combines production function with consumption-savings choices.

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Combines production function with consumption-savings choices.

- A constant fraction of income is saved, and the rest consumed:

$$S(t) = sY(t)$$

- Savings equals investment:

$$S(t) = I(t)$$

- Investment adds to capital stock:

$$K(t+1) = (1-\delta)K(t) + I(t) = (1-\delta)K(t) + sY(t)$$

where δ is the rate of depreciation.

- This is the **accumulation equation**.

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- Combine with per-capita production function $y = f(k)$:

$$(1+n)k(t+1) = (1-\delta)k(t) + sf(k(t))$$

-

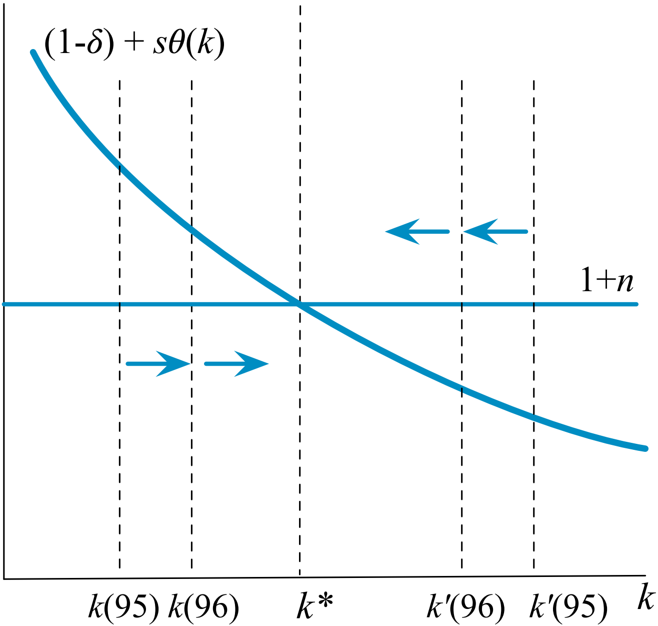
$$\Rightarrow \quad \frac{k(t+1)}{k(t)} = \frac{(1-\delta) + s\theta(k(t))}{1+n}$$

- where $\theta(t) \equiv y(t)/k(t)$ is the output-capital ratio.

Theory of Economic Growth: The Solow Model

$$\frac{k(t+1)}{k(t)} = \frac{(1-\delta) + s\theta(k(t))}{1+n}$$

- This is the **fundamental equation for the growth model**.
- A lot now hangs on what we view as exogenous or endogenous.
- In the Solow model, the output-capital ratio θ is endogenous.



$(1-\delta) + s\theta(k)$
$1+n$
$k(95)$
$k(96)$
k^*
$k'(96)$
$k'(95)$
k

The Steady State

- **Steady state capital-labor ratio** of the economy is k^*, which solves:

$$\underbrace{\theta^* = \frac{y^*}{k^*} = \frac{f(k^*)}{k^*}}_{\text{all the same}} = \frac{n+\delta}{s}$$

- $k^* = K(t)/L(t)$, but $K(t)$ and $L(t)$ keep rising at rate n.
- Argument crucially depends on diminishing returns to inputs.

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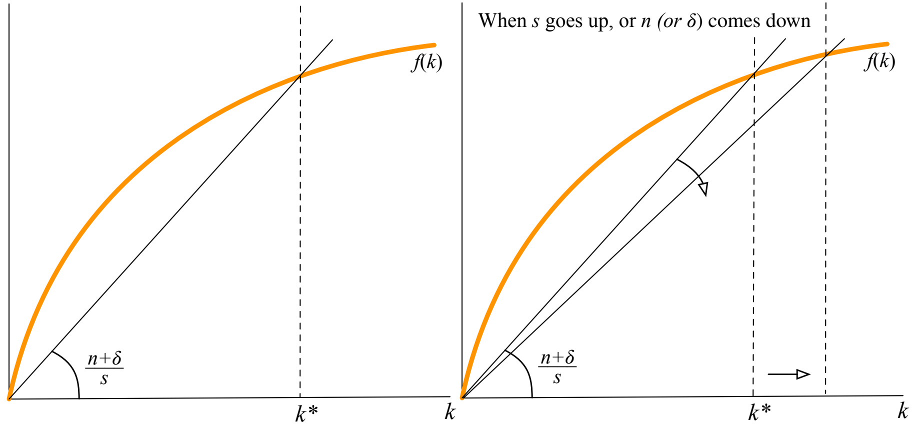
  - $k^* = K(t)/L(t)$, but $K(t)$ and $L(t)$ keep rising at rate n.
  - Argument crucially depends on diminishing returns to inputs.
- No long-run growth over and above population growth:
  - Any extra growth only comes from technical progress, as we shall see.

The Steady State

- **How s, n and δ affect k^*:**



The Steady State for Cobb-Douglas Production

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- With Cobb-Douglas production, $f(k) = Ak^a$, so:

$$\frac{f(k^*)}{k^*} = Ak^{*a-1} = \frac{n+\delta}{s}$$

$\Rightarrow$

$$k^* = \left(\frac{sA}{n+\delta}\right)^{1/(1-a)} \text{ and } y^* = A^{1/(1-a)}\left(\frac{s}{n+\delta}\right)^{a/(1-a)}$$

- Can directly verify the properties we established geometrically earlier.

Two Sources of Per-Capita Growth

Exogenous versus endogenous growth

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Exogenous versus endogenous growth

- **Exogenous growth:** comes from “outside” the model.

  ongoing technical progress

- **Endogenous growth:** comes from “inside” the model.

  induced technical progress

  absence of diminishing returns

Exogenous Growth: Technical Progress

- $Y(t) = F(K(t), e(t)L(t))$
- where $e(t)$ is **labor efficiency**, so $e(t)L(t)$ is **effective labor**;
- **exogenous technical progress:** $e(t+1) = e(t)(1+\pi)$, $\pi > 0$.

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- Divide through in the accumulation equation by $e(t)L(t)$ instead of $L(t)$:

$$(1+\pi)(1+n)\hat{k}(t+1) = (1-\delta)\hat{k}(t) + sf(\hat{k}(t)),$$

  - where $\hat{k} = K/eL$ is in "effective per-capita" units.

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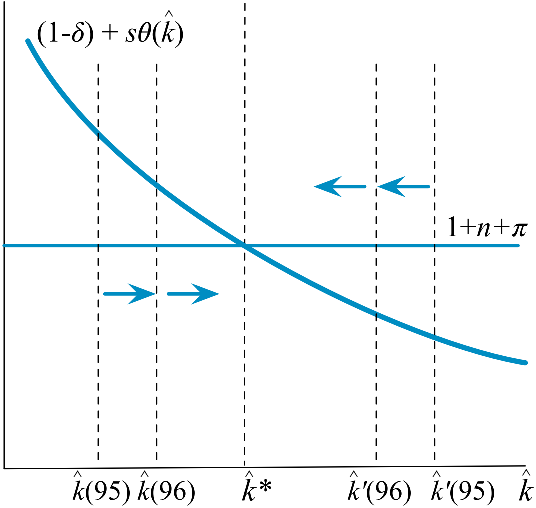
  - where $\hat{k} = K/eL$ is in “effective per-capita” units.
- **Approximate version**:

$$(1+n+\pi)\hat{k}(t+1) = (1-\delta)\hat{k}(t) + sf(\hat{k}(t)),$$

  - And now do exactly what you did before ...

Steady State in Effective Units With Technical Progress



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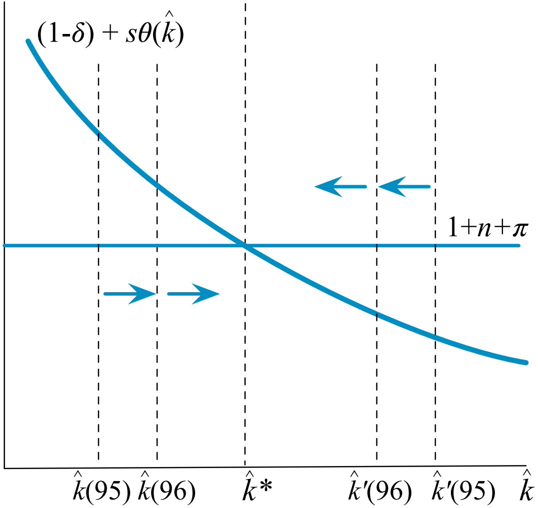

- **Steady state:** $\frac{f(\hat{k}^*)}{\hat{k}^*} \simeq \frac{n+\delta+\pi}{s}$, **Per-capita long run growth rate** = π.

Steady State in Effective Units With Technical Progress

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- In the Cobb-Douglas case, $\hat{y} = f(k^*) = A\hat{k}^a$, so

$$\hat{k}^* \simeq \left(\frac{sA}{n+\delta+\pi}\right)^{1/(1-a)},$$

- …and steady state output in effective labor units is:

$$\hat{y}^* \simeq A^{1/(1-a)} \left(\frac{s}{n+\delta+\pi}\right)^{a/(1-a)}.$$

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- Actual per-capita output and capital grow at the rate of π

Steady State Growth With Technical Progress

- **Path of steady state output per capita:**

$$y^*(t) = \hat{y}^*(1+\pi)^t = A^{1/(1-a)} \left(\frac{s}{n+\delta+\pi}\right)^{a/(1-a)} (1+\pi)^t.$$

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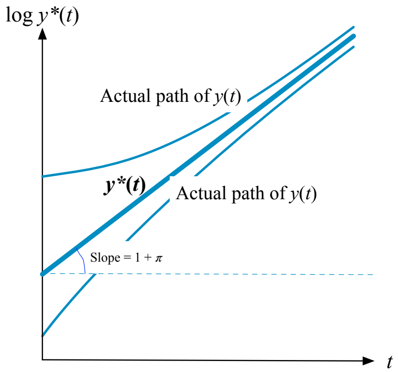

Steady State Growth: Parametric Changes

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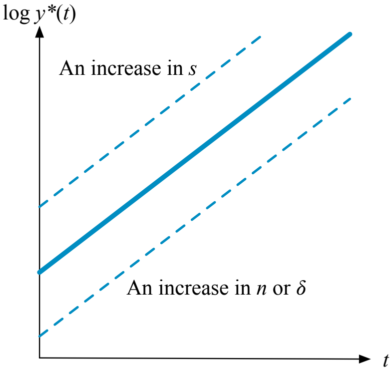

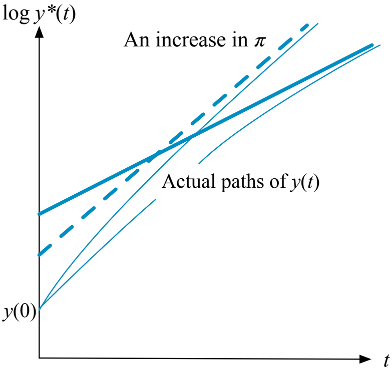

Endogenous Growth: The Harrod-Domar or AK Model

- Notice how steady state moves with a.
- When a hits 1, the model behaves *very* differently:

$$(1+n)k(t+1) = (1-\delta)k(t) + sAk(t).$$

- Moving terms around:

$$\text{Rate of growth } = \frac{k(t+1)-k(t)}{k(t)} = \frac{sA-(n+\delta)}{1+n}.$$

- Or an approximate but easier-to-use version

$$g \simeq sA - n - \delta.$$

- **Now parameters have growth effects, unlike Solow model.**