# Development Economics

Slides 11

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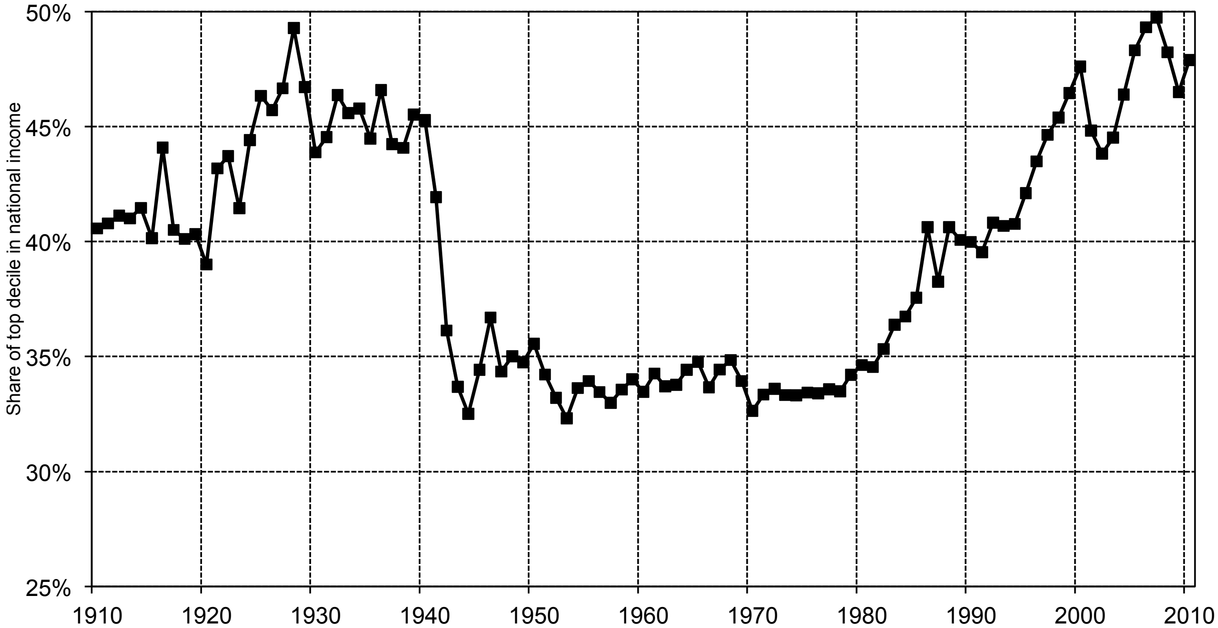
## The Return of Inequality

- **The financial crisis sparked a new interest in inequality.**
  - But inequality has been historically high
  - Growing steadily through late 20th century

  Wolff, Piketty, Saez, Atkinson, many others

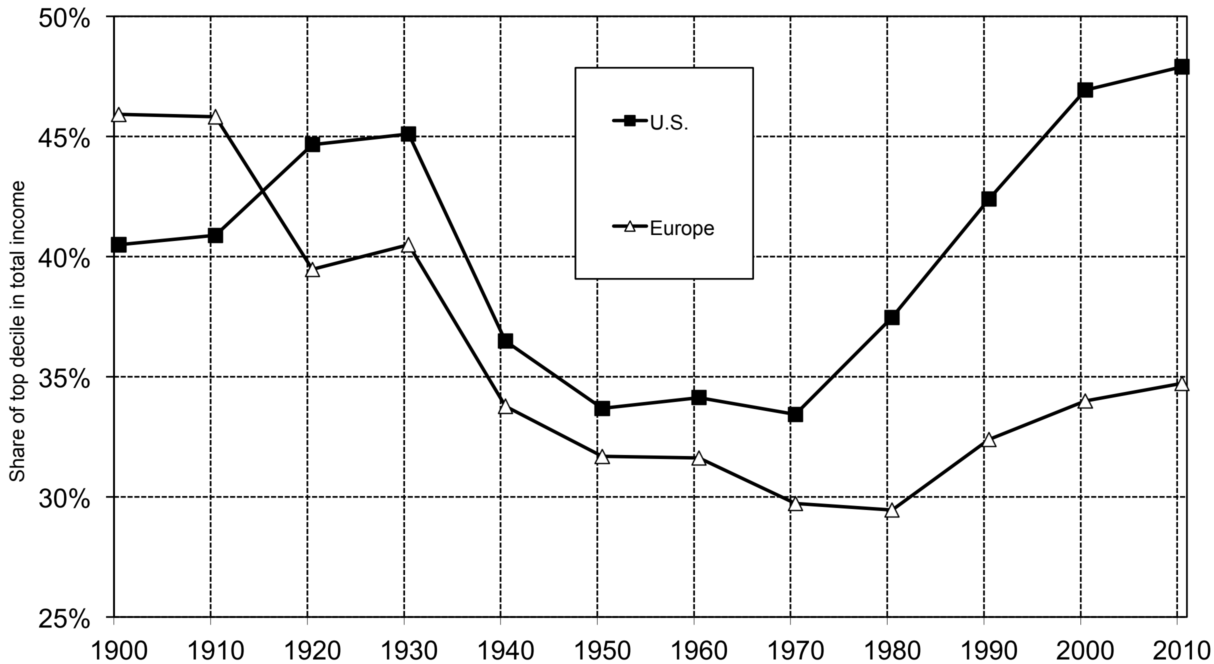
- A recent book by Piketty 2014
  - brought rising inequality to the public consciousness
  - summarized the evidence (compelling and useful)
  - was a runaway hit in the United States, touching a raw nerve

**Figure I.1. Income inequality in the United States, 1910-2010**



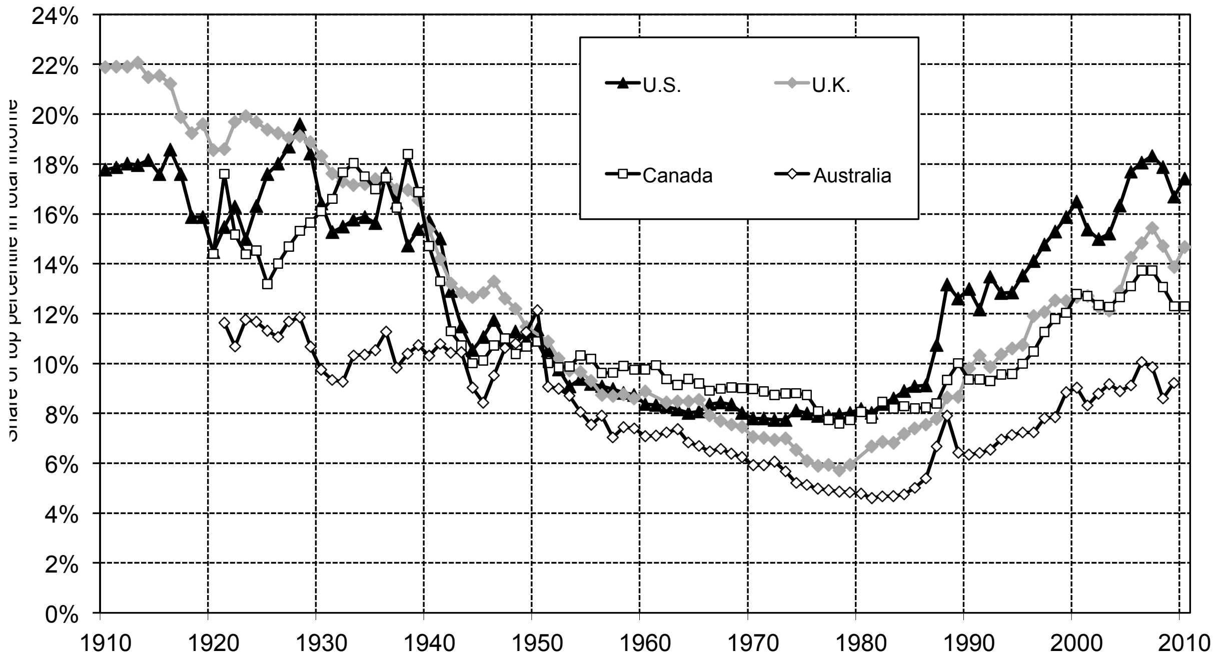

Source: Piketty (2014)

**Figure 9.8. Income inequality: Europe vs. the United States, 1900-2010**



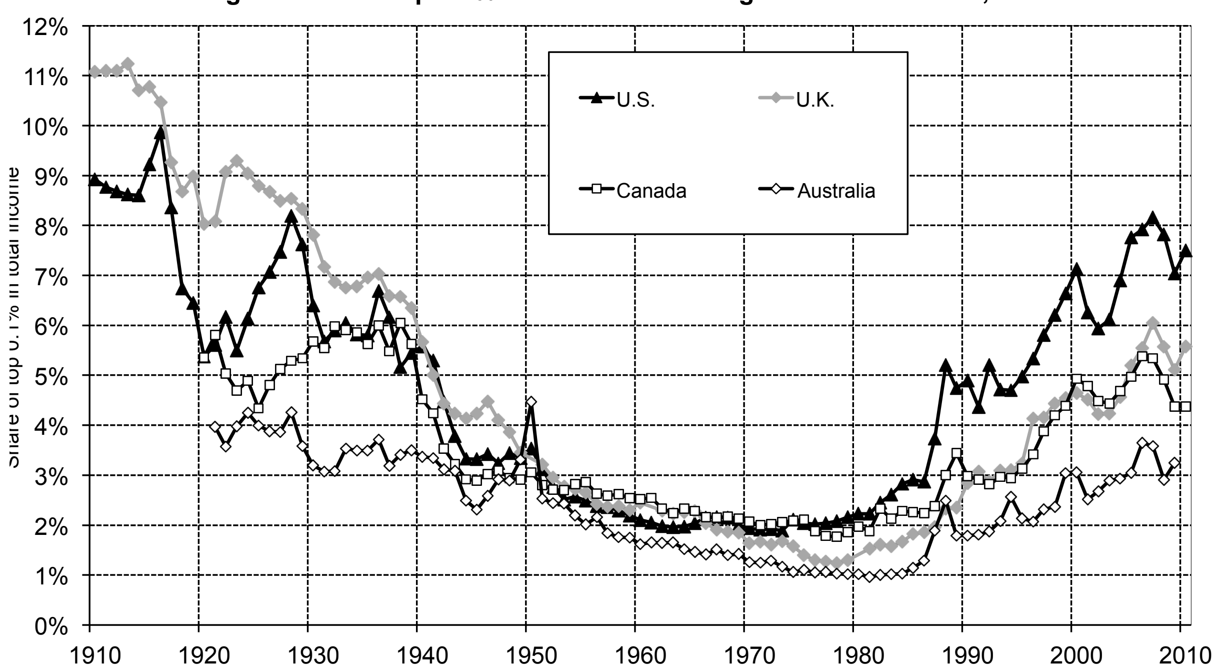

Source: Piketty (2014)

**Figure 9.2. Income inequality in Anglo-saxon countries, 1910-2010**




Source: Piketty (2014)

**Figure 9.5. The top 0.1% income share in Anglo-saxon countries, 1910-2010**




Source: Piketty (2014)

## Intrinsic Versus Instrumental

- Inequality is of intrinsic as well as instrumental interest
- **Intrinsic**:
- inequality measurement: evaluate and compare distributions
- evolution of inequality in societies (e.g., Piketty)
- **Instrumental**:
- inequality and various outcomes: growth, nutrition, employment
- inequality and history-dependence
- Inequality and incentives …

## Conceptualizing and Measuring Inequality

- **Income distribution** $(y_1, \ldots, y_N)$.
- inequality measure maps each vector $(y_1, \ldots, y_N)$ to a number.
- Rankings matter, not the exact numbers.
- Axiomatic approach.
- **Anonymity principle**: Names do not matter.
- So rewrite as $(y_1, N_1; y_2, N_2; \ldots; y_m, N_m)$,
- where $m$ is number of distinct incomes,
- $N_i$ is population with income $y_i$,
- $y_1 < y_2 < \cdots < y_m$.

## Conceptualizing and Measuring Inequality

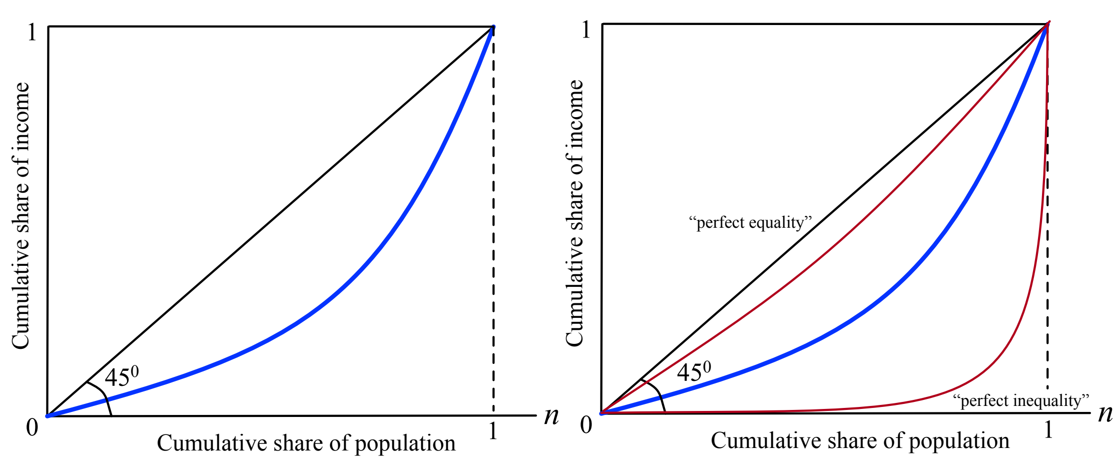
- **Population principle**:
  - Cloning entire population (and incomes) doesn't alter inequality.
  - So only the population *share* $n_i = \frac{N_i}{N}$ is relevant: share: $\sum_{i=1}^{m} n_i = 1$.
- A critical look:
  - 2-person society where one person has income? Perfect inequality?
  - Read the debate here [hyperlink embedded]

- **Relative Income principle**:
  - Scaling incomes up or down does not affect inequality.
- A critical look:
  - presumes linear link between income and "utility".
  - For instance, think of subsistence constraint.

## Conceptualizing and Measuring Inequality

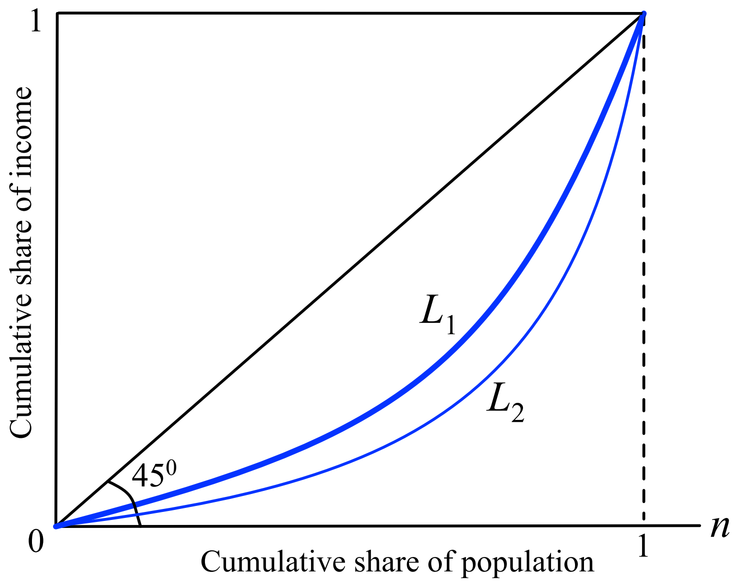
- **Pigou-Dalton Transfers principle**:
  - An income transfer from relatively poor to relatively rich must increase inequality.
  - Postpone critical look to later.
- **Summary:** $I(\mathbf{y})$, where $\mathbf{y} = (y_1, \ldots, y_N)$.
  - Anonymity: $I(\mathbf{y}) = I(\sigma\mathbf{y})$, where $\sigma$ is a permutation.
  - Population: $I(\mathbf{y}) = I(\mathbf{y}, \mathbf{y})$.
  - Relative Income: $I(\mathbf{y}) = I(\lambda\mathbf{y})$ for all $\lambda > 0$.
  - Transfers: $I(\mathbf{y}) < I(y_1, \ldots, y_i - \delta, \ldots, y_j + \delta, \ldots, y_N)$.

## The Lorenz Curve



## The Lorenz Criterion

- If Lorenz curve $L(2)$ lies **everywhere below** (or "**to the right of**") $L(1)$, then inequality under $L(2)$ is higher than inequality under $L(1)$:




## The Lorenz Criterion

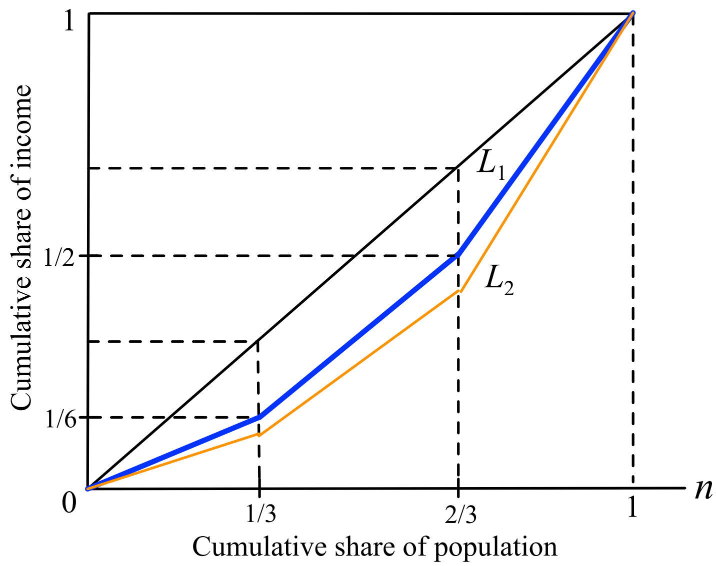
- An inequality measure $I$ is **Lorenz-consistent** if it agrees with the Lorenz criterion whenever one Lorenz curve lies completely below the other (it could touch in parts).

**Theorem 1**

*Inequality measure $I$ is Lorenz consistent if and only if it satisfies Anonymity, the Population principle, the Relative Income principle and the Pigou-Dalton Transfers principle.*

- **Illustration.** 3 groups of the same size:
  - Situation 1 incomes: $(10, 20, 30)$. Situation 2 incomes: $(14, 44, 62)$.
  - Rescale and **note that the transfers principle can be applied here**.

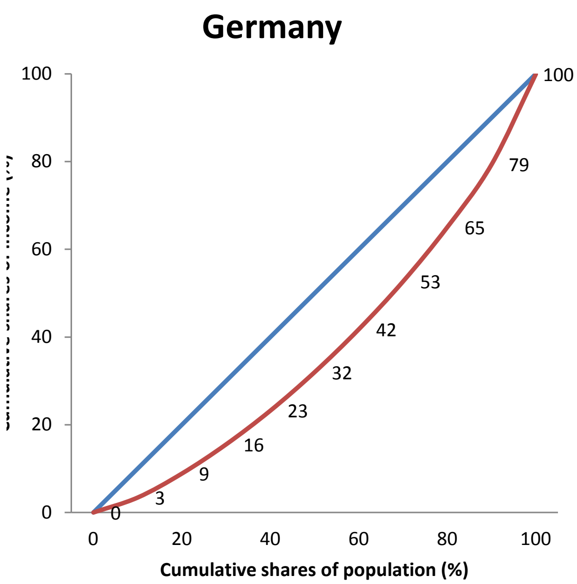
## The Lorenz Criterion

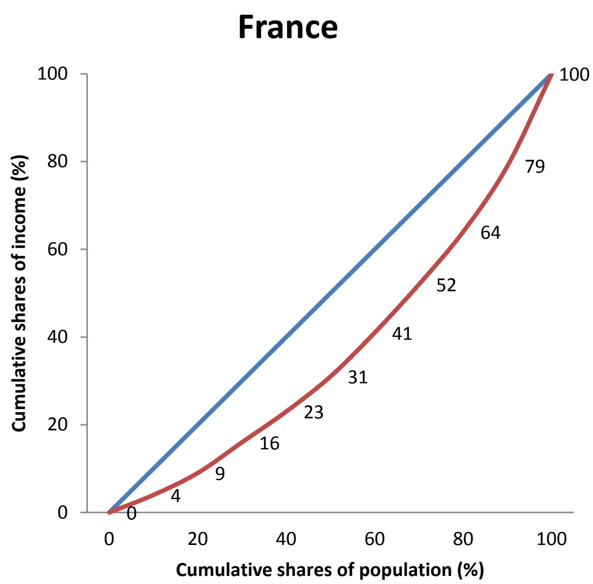



- Nice: the axioms and the Lorenz criterion are the same thing..
  - But also tells us that the axioms don't cover all cases, **because Lorenz curves can cross**.

- Example. Compare $(75, 125, 200, 600)$ and $(25, 175, 400, 400)$.
  - (Draw the corresponding Lorenz curves.)
- Example. Modern sector enlargement (from Fields, 1980).
  - 10 people, rural wage equals 50, urban wage equals 100
  - rural-urban migration

## Some Lorenz Curves

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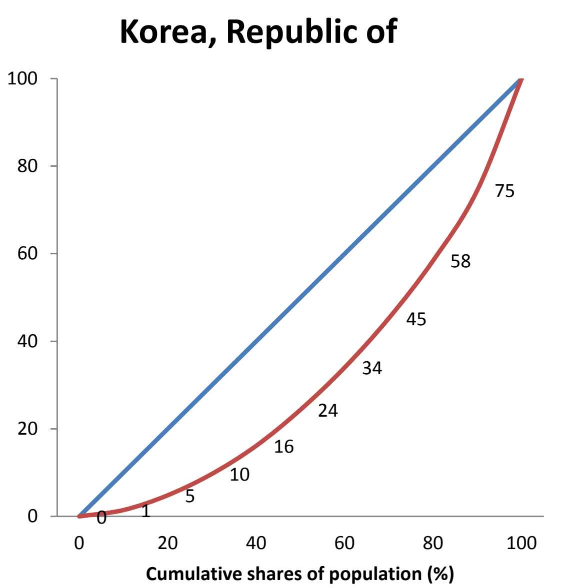


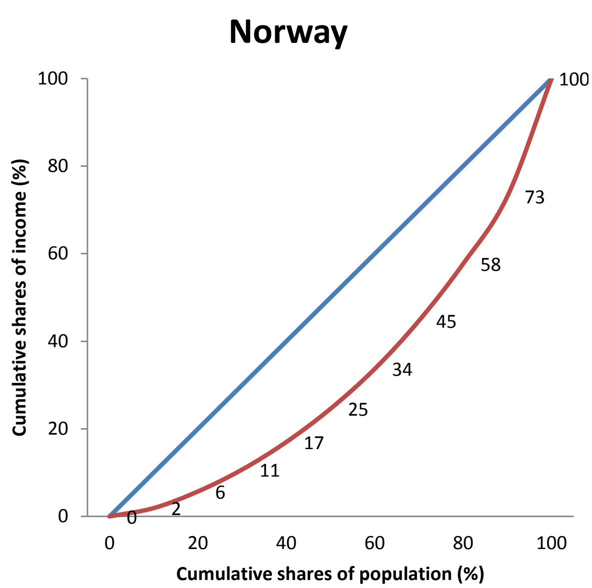

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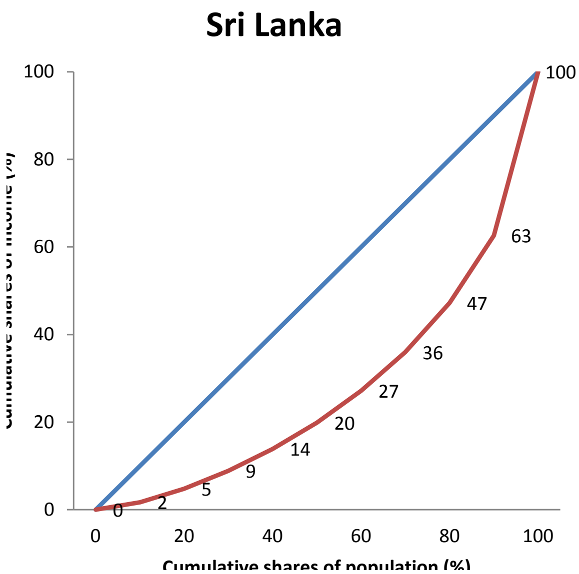


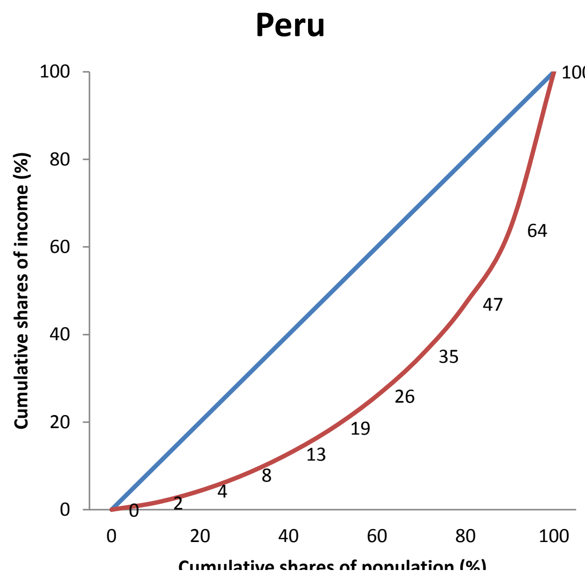


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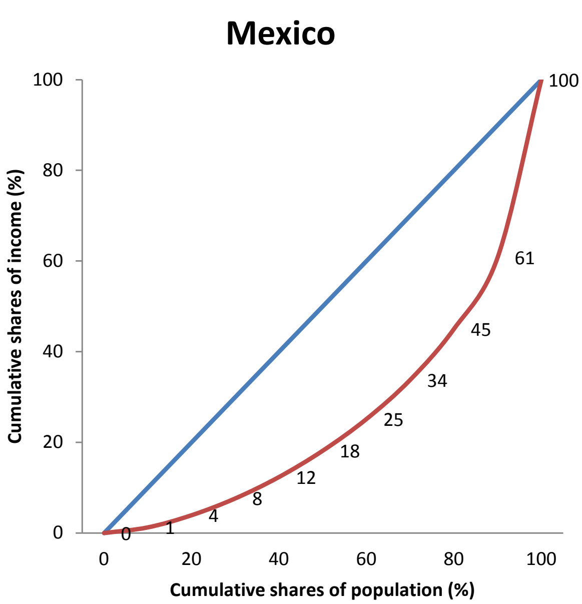


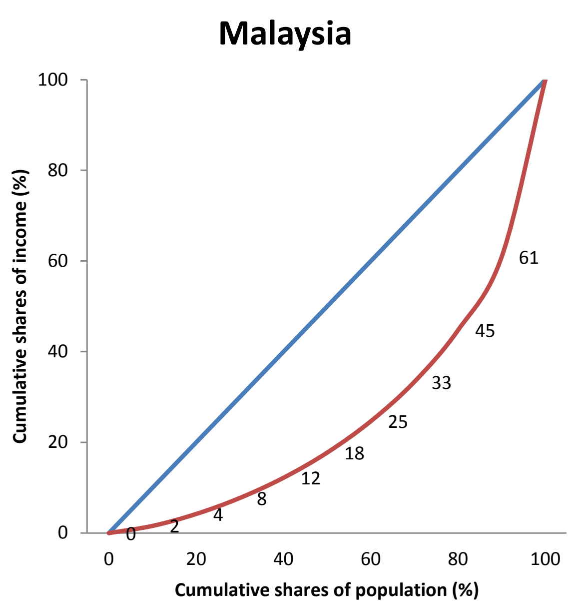

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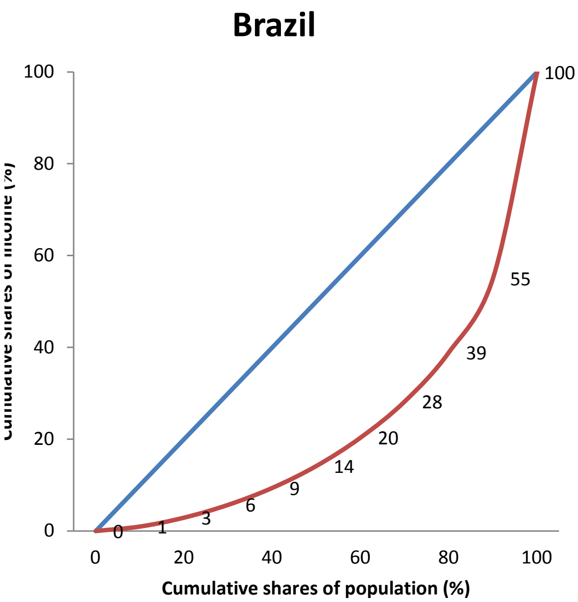
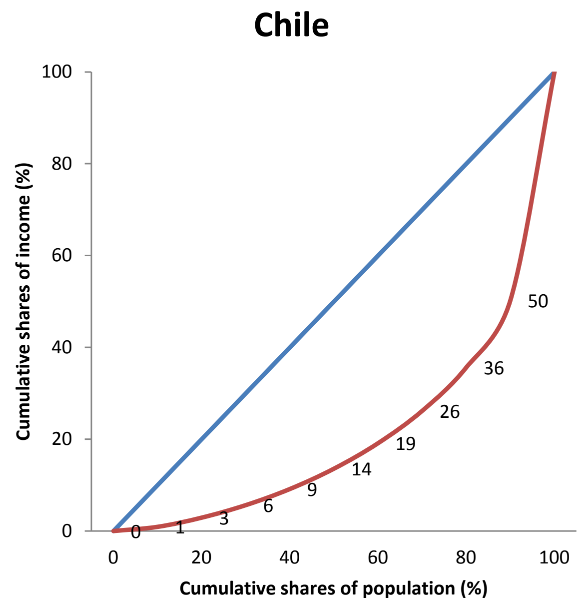
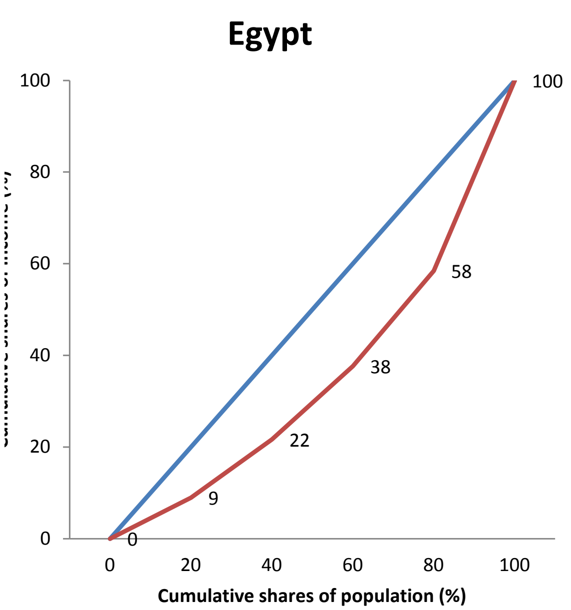
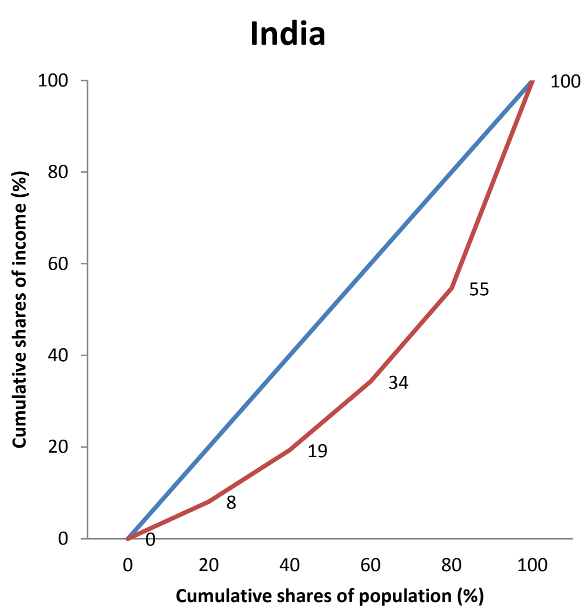



## Some Lorenz Curves: Consumption Data



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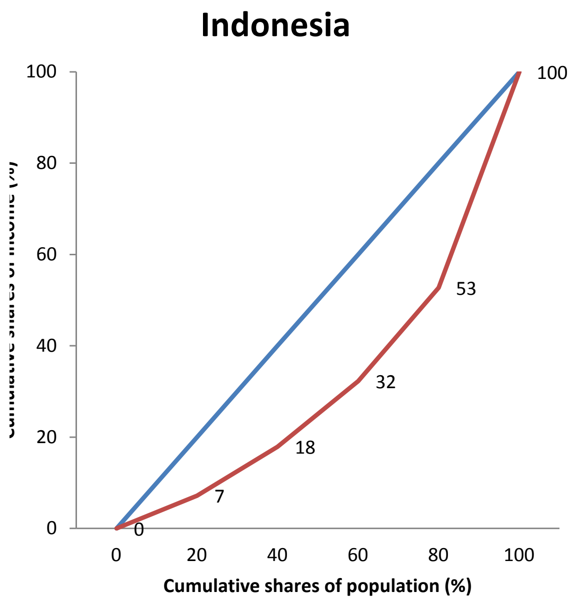
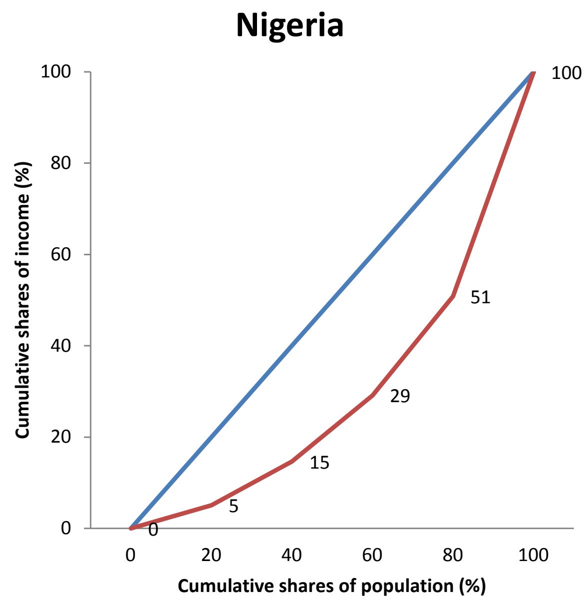



## Complete Measures of Inequality

- **The Lorenz ordering is partial:**
  - Lorenz-consistent measures complete this ordering in different ways.
  - We also want measures that take full advantage of available information:
  - $m$ groups, $\mathbf{y} = (y_1, \ldots y_m)$, $(N_1, \ldots, N_m)$,
  - $n_i = \frac{N_i}{N}$ is pop share, $\mu = (\sum_{i=1}^{m} N_i y_i)/N$ is mean income.
- Examples of popular crude measures:
  - Range: $(y_m - y_1)/\mu$.
  - Kuznets ratios: (richest $x$%)/(poorest $y$%); e.g., 20%/80%.

## Complete Measures of Inequality

- More seriously:
- **Mean absolute deviation:** $\frac{1}{\mu}\sum_{j=1}^{m}\frac{N_j}{N}|y_j-\mu| = \frac{1}{\mu}\sum_{j=1}^{m}n_j|y_j-\mu|.$
- Is this Lorenz-consistent? Check axioms.
- **Coefficient of variation:** $\frac{1}{\mu}\sqrt{\sum_{j=1}^{m}\frac{N_j}{N}(y_j-\mu)^2} = \frac{1}{\mu}\sqrt{\sum_{j=1}^{m}n_j(y_j-\mu)^2}.$
- Is this Lorenz-consistent? Check axioms.

## Complete Measures of Inequality

- **Gini:** $\frac{1}{2\mu}\sum_{j=1}^{m}\sum_{k=1}^{m}\frac{N_jN_k}{N^2}|y_j-y_k| = \frac{1}{2\mu}\sum_{j=1}^{m}\sum_{k=1}^{m}n_jn_k|y_j-y_k|.$
- Is this Lorenz-consistent? Check axioms.
- A **variant** of the Gini (he had 13 of them!):
- **Gini′:** $\frac{1}{2\mu}\sum_{j=1}^{m}\sum_{k=1}^{m}\frac{N_jN_k}{N(N-1)}|y_j-y_k|.$
- Declares that a two-person society can exhibit perfect inequality.

  Recall debate.
- But fails the population principle.

- **Gini ($G$) compared with coefficient of variation (COV)**
  - They may disagree when Lorenz curves cross.
- Example. Compare $A = (3, 12, 12)$ and $B = (4, 9, 14)$.
  - COV($A$): $\frac{1}{9}\sqrt{\frac{1}{3}(3-9)^2 + \frac{2}{3}(12-9)^2} = \frac{1}{9}\sqrt{18}$.
  - COV($B$): $\frac{1}{9}\sqrt{\frac{1}{3}(4-9)^2 + \frac{1}{3}(14-9)^2} = \frac{1}{9}\sqrt{\frac{50}{3}}$.
  - So the COV falls going from $A$ to $B$.
  - $G(A)$: $\frac{[2|3-12|+2|12-3|]}{2.9^2} = \frac{18}{81}$.
  - $G(B)$: $\frac{[|4-9|+|4-14|+|9-4|+|9-14|+|14-4|+|14-9|]}{2.9^2} = \frac{20}{81}$.
  - So the Gini rises going from $A$ to $B$.

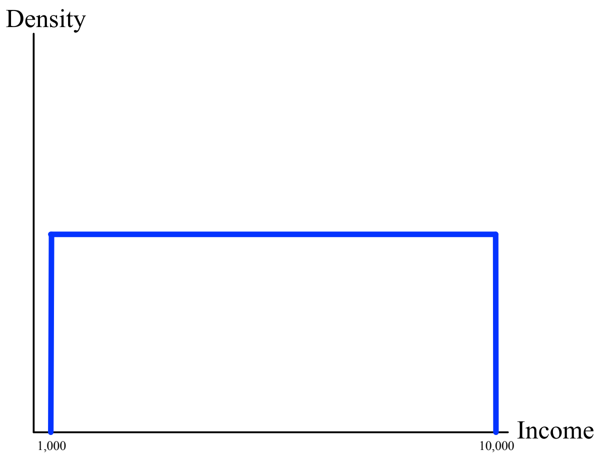
## Gini and the COV: Real-Life Disagreement

| Country | Gini | COV | Richest 5% | Poorest 40% |
|---|---|---|---|---|
| Puerto Rico | | | | |
| 1953 | 0.415 | 1.152 | 23.4 | 15.5 |
| 1963 | ↑0.449 | ↓1.035 | ↓22.0 | ↑13.7 |
| Argentina | | | | |
| 1953 | 0.412 | 1.612 | 27.2 | 18.1 |
| 1959 | ↑0.463 | ↑1.887 | ↑31.8 | ↑16.4 |
| 1961 | ↑0.434 | ↓1.605 | ↑29.4 | ↑17.4 |
| Mexico | | | | |
| 1950 | 0.526 | 2.500 | 40.0 | 14.3 |
| 1957 | ↑0.551 | ↓1.652 | ↓37.0 | ↑11.3 |
| 1963 | ↑0.543 | ↓1.380 | ↓28.8 | ↑10.1 |

- **Inequality in three countries** (Weisskoff (1970), Fields (1980)).
  - Arrows ↑ and ↓ show direction of inequality relative to base year.

## Beyond Economic Inequality

- "As the struggle proceeds, the whole society breaks up more and more into two hostile camps, two great, directly antagonistic classes . . . The classes *polarize*, so that they become internally more homogeneous and more and more sharply distinguished from one another in wealth and power." (Deutsch 1971, p.44)

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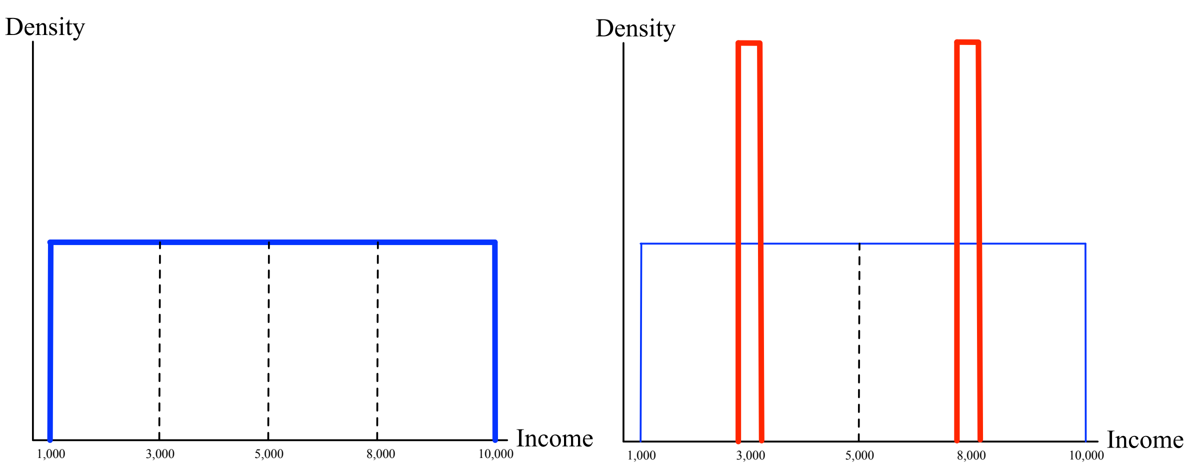

## Polarization

- **Polarization adds up alienations, like the Gini**
  - But individual alienations are themselves determined by group size:
  - $n_i|y_i - y_j|$ instead of $|y_i - y_j|$.
  - **Add these**:

$$P = \frac{1}{\mu}\sum_{i=1}^{m}\sum_{j=1}^{m} n_i^2 n_j |y_i - y_j|.$$

  - Looks very much like the Gini coefficient, but different.
- Can be applied to study the connection between distribution and conflict. [Later in the course.]