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ECON-UA 323
Development Economics

Problem Set 8

(1) A farming family owns land of size a (acres), and farms it with labor ℓ, using the production function

$$Y = 100\ell^{1/2}a^{1/2}.$$

The farm has access to 4 total units of labor (you can think of 4 as the family size), which it divides as finely as it wishes (fractionally if needed) between working *on* the farm and *off* the farm. Off-farm employment yields a wage of 100 per unit.

The farm can also hire *in* labor, again at the wage cost of 100 per unit. But unlike family labor, hired labor has to be supervised, and for this the farm has to hire a supervisor at a cost of 225. Once paid, the supervisor can costlessly supervise all hired labor.

(a) Prove that if the family has less than 16 acres of land ($a \leq 16$), it will devote family labor equal to $a/4$ to the farm, hire in no additional labor, and hire out the remainder $4 - (a/4)$ for off-farm employment.

(b) Prove that if the family has between 16 and 49 acres of land, it will continue to operate as a full family farm, with all its family members working full time on it, but will not hire in any labor. Above 49 acres, it hires a supervisor and at this threshold, its hiring of outside labor jumps up from 0 to slightly over 8 units of hired labor, and then keeps climbing as a continues to rise.

Now suppose that the family has the additional option of leasing out some or all of its land at a fixed rental rate of R per unit. But assume that it cannot lease *in* any land.

(c) Calculate a threshold for R such that above this threshold, the family *never* farms any land, no matter how much or how little land it owns, and leases it all out. [Hint: work out the implicit return to land on the family farm after subtracting the imputed costs of family labor.]

(d) Can you work out what would happen for lower values of the land rental? For instance, can you show values of R such that for small values of land, the family leases out nothing, then leases out some land, and then again goes back to leasing out no land as its holdings get large?

(e) How would your answers to parts (b)-(d) change (if at all) if there were no fixed costs to supervision, and if hired labor costs 25 *per unit* to supervise instead?

(2) Kumar, a small farmer, has his own plot of land (call it A) and leases another plot (call it B) from a large landowner, Malini. These are separate plots and he must farm them separately by allocating his endowment of one unit of effort to the two plots, in the form of e_A and e_B. There is no cost of effort — the opportunity cost on one plot is just the cost of not using his effort on the other plot. The production functions on the two plots are

$$Y_A = A\sqrt{e_A} \text{ and } Y_B = B\sqrt{e_B},$$

so that total output is $Y_A + Y_B$, and of course, $e_A + e_B = 1$.

(a) In the hypothetical case in which Kumar owns both plots of land, show that his effort allocation is given by

$$e_A = \frac{A^2}{A^2 + B^2} \text{ and } e_B = \frac{B^2}{A^2 + B^2},$$

with total output equal to $\sqrt{A^2 + B^2}$.

Assume that if Kumar does not rent Malini's plot, he simply farms his own plot.

(b) With part (a) in mind, show that Malini can extract a total of $\sqrt{A^2 + B^2} - A$ in rent, and demonstrate how she can do that using a fixed rent contract.

(c) Suppose that output is uncertain (we won't formally model this here, though see question 3) and that Malini can only take a *share* σ of Y_B as rent. Find an expression for the rent that Malini can get out of Kumar, expressed as a function of σ and the other exogenous parameters A and B of the model. (You will need to solve out for Kumar's effort level on the plot for each σ, the answer will be similar to that in part (a).)

(d) Without doing any further calculations, try and use your intuition to argue why Malini's rent must now be lower compared to what she gets in part (b).

(3) Problem 2 might leave us wondering why on earth Malini would choose to sharecrop if fixed rent is better. We kind of waved our hands and said that otherwise the situation is too risky for Kumar. We are now going to try and formalize this argument in a very simple setting. In the previous problem, let us say that that when Kumar farms the land, $A = B = 1$ with probability 1/2, or $A = B = \lambda > 1$ with probability 1/2. (Good and bad outcomes are perfectly correlated across the two plots.) In other words,

$$Y_A^+ = \lambda\sqrt{e_A} \text{ and } Y_B^+ = \lambda\sqrt{e_B},$$

with probability 1/2, while

$$Y_A^- = \sqrt{e_A} \text{ and } Y_B^- = \sqrt{e_B},$$

again with probability 1/2, where I have used the signs "+" and "-" to distinguish good and bad outputs. Kumar's next-best alternative (if he does not rent) is just to farm his own plot. And total labor endowment equals 1, as before, and supplied at zero cost.

Kumar's utility is strictly concave in his own income — he is risk averse. If Kumar earns x, his utility is given by $\log(x)$.

(a) Show that if Kumar only has his own land (plot A) and does not rent (plot B), his expected utility is given by $(1/2)\log(\lambda)$.

(b) Show that if Kumar rents Plot B on fixed rent tenancy with rent R, then he will put equal effort on both plots, with $e_A = e_B = 1/2$, so that his expected utility is given by

$$\frac{1}{2}\log(\sqrt{2} - R) + \frac{1}{2}\log(\lambda\sqrt{2} - R)$$

(c) Using the answers to parts (a) and (b), write down an equation that describes how Malini would pick down the maximum rent that she can extract from fixed rent tenancy. You don't need to explicitly solve this equation (you can try it, though, it's not hard), but prove that R cannot exceed $\sqrt{2}$. You will have to use some standard properties of logarithmic functions.

(d) Now suppose that Malini offers a sharecropping contract with share σ to herself and $1-\sigma$ to Kumar. Using a similar logic to that in the previous question, show that

$$e_B = \frac{(1-\sigma)^2}{1+(1-\sigma)^2}.$$

so that the *expected rent* that Malini receives is given by

$$\text{Sharecropping rent } = \frac{1+\lambda}{2}\frac{\sigma(1-\sigma)}{\sqrt{1+(1-\sigma)^2}}.$$

(e) Use part (c) and the formula in part (d) to show that if λ is large enough, a risk-neutral Malini prefers sharecropping to fixed rent tenancy. Intuitively explain your answer.

(4) Miguel works on a tea plantation as a plucker. (If you are wondering how someone named Miguel could be working on a tea plantation, remember that Argentina is the ninth largest exporter of tea in the world!) He gets paid a basic wage — we will call it b — and an extra incentive payment s for every kilo of tea leaves that he plucks. So his total payment w is given by

$$w = b + sy,$$

where y is the number of kilos of tea that he plucks. Miguel has a utility function given by $u(w) = 100\log w$, and his cost of plucking y kilos of tea is given by y. So Miguel's net utility is given by

$$u(w) - y.$$

(a) For any given b and s, show that Miguel will pluck y kilos of tea, where

$$y = 100 - \frac{b}{s}$$

provided that $b < 100s$, otherwise Miguel won't pluck any tea at all!

Can you explain intuitively why b and s affect the amount of tea plucked in the way they do here? In particular, why does Miguel's effort drop to zero if $b \geq 100s$?

(b) Assume that the minimum wage b is fixed by the government at a strictly positive number, but smaller than 100. The tea plantation cannot tamper with it, but can freely choose the piece rate. The plantation wants to maximize profits from hiring Miguel, which are given by

$$y - w = y - [b + sy] = (1-s)y - b.$$

where we are setting the price of tea equal to 1. Prove that for any $b < 100$, the tea plantation will want to set

$$s = \frac{\sqrt{b}}{10}.$$

assuming that it hires Miguel to begin with. Notice how as b goes up, s goes up as well. Why?

(c) Prove that if the baseline wage b goes above 25, the plantation would rather not hire Miguel to begin with.

(5) An economy has a labor force of 100, and a production function that uses labor to produce output. Output price is fixed at 1, and the production function is given by

$$y = A\ell^{1/2}$$

with labor chosen to maximize profit at the going wage rate. If a worker is unemployed, she obtains the net monetary equivalent of \$30 per day, perhaps doing home tasks or working on the farm. If she is employed, she earns a wage of w (to be determined), but has to work at a minimum pace, which incurs a personal cost of \$27. Each worker has a discount factor of $\delta = 9/10$.

(a) Assume that detection is certain if you work below the minimum required pace, and that no fired worker is ever hired again (which may sound unreasonable but let's do it for practice, and also for a reason that I will reveal below). Then, show that the minimum wage for self-enforcement is \$60.

(b) Remember there are 100 units of labor in the whole economy. Show that if the technological coefficient A is smaller than 1200, then there is involuntary unemployment, and the market wage settles at 60. But also show that if A exceeds 1200, the market wage rises *above* the minimum necessarily for self-enforcement, and there is full employment.

The market wage can *only* rise strictly above the self-enforcement constraint if there is full employment. Otherwise an unemployed person could credibly bid that wage down (slightly) and get employed — because the self-enforcement constraint would still hold.

(c) Now I want you to contrast this scenario with the realistic case, in which a fired worker might be re-employed. Let this re-employment probability *exactly equal* the fraction of people who are employed in the economy; i.e., it is equal to the employment rate e. Now show that the self-enforcement constraint is given by:

$$w \geq 30\frac{2-e}{1-e}. \tag{1}$$

(d) This is very different from part (b). Show that there can *never* can be full employment in this scenario, unlike in part (b). In particular, *the self-enforcement constraint always holds with equality.* Carefully explain why things are so different now.

(e) Now we will work to solve for the equilibrium in this labor market. First note that for any wage rate w, the demand for labor is given by

$$\ell = \left(\frac{A}{2w}\right)^2.$$

(you will surely have solved for this earlier, but if not, do so now). Combine this with the self-enforcement constraint (1) that we know to always hold with equality:

$$w = 30\frac{2-e}{1-e}$$

to show that in equilibrium, e must solve the equation

$$600\sqrt{e}\left(\frac{2-e}{1-e}\right) = A,$$

where you will need to use the fact that the employment rate e is the total employment ℓ divided by the total labor force, which is 100. (That is, $e = \ell/100$.)

The next and last question is an optional problem which you do not need to hand in. It builds on the ideas of the lecture. It is great practice.

(6) Consider a production cooperative with just two farmers. Each farmer chooses independently how much labor — ℓ_1 and ℓ_2 — to supply to the cooperative. The cooperative output is given by

$$Y = A(\ell_1 + \ell_2)^\alpha$$

where $A > 0$ and α lies between 0 and 1. Each unit of labor is supplied at an opportunity cost of w, so the total cost of effort supply is $w\ell_1$ for farmer 1 and $w\ell_2$ for farmer 2.

(a) Draw production and total cost as a function of labor input. Find (both diagrammatically and using first order conditions) the amount of labor input that maximizes farm surplus.

(b) Now suppose that each labor is supplied independently by each cooperative member in an effort to maximize her own net profit. Say that a pair of labor allocations (ℓ_1, ℓ_2) forms an *equilibrium* if, given e_2, the choice of e_1 is optimal for farmer 1, and given e_1, the choice of e_2 is optimal for farmer 2.

Show that if total output is divided equally among the farmers, production *must fall short* of the answer in part (a). Describe all possible equilibrium labor allocations in the problem of part (b). Show that there are many of them, but they all have the same output.

(c) Try to intuitively relate this exercise to the problem of inefficiency in sharecropping.

(d) Next, suppose that farmer 1 receives a share $s > 1/2$ of the total output, while farmer 2 gets $1-s$ (everything else is the same as before). Show that there is now a unique equilibrium labor allocation, and describe what it looks like.

(e) Show that if if s is slightly larger than $1/2$, then farmer 1 — who gets the larger share — is actually *worse off* in terms of her *net* payoff.

(f) Parametrically moving s from 1/2 to 1, describe what happens to production and labor efforts. Show that the system maximizes overall social surplus when one share equals 1.

(g) The result in (f) is strange on a number of grounds! First, efficiency is reached when the system is highly unequal. Second, what happened to the double moral hazard problem we discussed in class? Shouldn't that place limits on one side being a residual claimant? Think about this intuitively and now move to the next part.

(h) Change the problem by supposing that the joint production function is of the form

$$Y = A\ell_1^{1/3}\ell_2^{1/3}.$$

Notice that we have diminishing returns in the two inputs jointly ($1/3 + 1/3 < 1$) because land is also an input and it is fixed.

Now which value of the share do you think maximizes social surplus? Explain why the answer is so different from the preceding answer.