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Econ-UA 323

Development Economics

Problem Set 7

[1] This problem illustrates adverse selection in credit markets. The land of NoGood has a large number of producers of different types, all indexed by their probability p of successful production. Producer p produces quantity $y(p) = 110/p$ if successful, and with probability $1 - p$ their output is zero. No producer has collateral.

(a) Show that every type of producer has the same value of *expected* output.

Suppose that a loan of 100 is needed to start production. It is offered at interest rate r. (In what follows, we are going to move r around.) Notice that the safest producer in NoGood *just* breaks even when the loan size is 100. (Use part (a).)

(b) If only producers with *strictly positive* profits apply for a loan, describe the set of producers who will want to take on the loan of 100 at an interest rate r.

(c) If producers have their probabilities distributed uniformly on $[0, 1]$, calculate the expected return to the bank from choosing the interest rate r, and show that on average, the bank can never recover the loan.

(d) Discuss this in the context of adverse selection more generally, and understand just how the market breaks down in this case.

(2) Consider a moneylender who faces two types of potential customers: call them the *safe type* and the *risky type*. Each type of borrower needs a loan of (the same) size B to invest in some project or activity. The borrower can repay only if the investment produces sufficient returns to cover the repayment.

Suppose that the safe type is always able to obtain a secure return of m ($m > B$) from his investment. On the other hand, the risky type is an uncertain prospect; he *can* obtain a higher return M (where $M > m$), but only with probability p. With probability $1 - p$, his investment backfires and he gets a return of 0.

Suppose that the lender has enough funds *to lend to just one applicant*, and that there are two of them (one risky, one safe).

(a) What is the highest interest rate r_1 for which the safe borrower wants the loan? What is the highest interest rate r_2 for which the risky borrower wants the loan? Calculate these thresholds, notice that $r_2 > r_1$, and explain that finding in words.

Assume the answer of part (a). Notice that If the lender charges r_1 or below, both borrowers will apply for the loan. If the lender cannot tell them apart, he has to give the loan randomly to one of them. On the other hand, if a rate slightly higher than r_1 is charged, the first borrower drops out and excess demand for the loan disappears. The lender may then go all the way up to r_2 without fear of losing the second customer.

(b) The lender's choice is then really between the two interest rates r_1 and r_2. Which should he charge? Show that if

$$p < \frac{m}{2M - m}.$$

then the lender will charge r_1 instead of r_2.

(c) Notice that under the condition of part (b), we have credit rationing *in equilibrium*: out of two customers demanding a single available loan, only one will get it; the other will be disappointed. Explain why the lender does not want to raise his price.

(3) Here is an extension of the previous question. Suppose that a moneylender is lending one dollar to a risky borrower. There is a probability of default (through some involuntary or strategic source that we do not model here). If that happens, assume that the entire loan is lost, principal and interest.

(a) If the interest rate is r, and the repayment probability is p, write down the net expected payoff to the moneylender from making the loan.

Suppose that for one or more of the reasons discussed in the text, p declines with the lender's choice of r. Specifically, assume that $p = 1 - ar$ for some constant $a > 0$. Of course, the above formula only makes sense if r is no bigger than $1/a$, after which p just stays at zero.

(b) Solve for the interest rate r that maximizes the lender's payoff. Interpret the results when a is very close to 1 or very close to zero.

(c) If i is the safe rate of interest on some other option, find a threshold value for i above which the lender will never lend to the risky borrower.

(4) A loan L is made every period, and repayment R charged. The loan produces output $F(L)$ for the borrower. The borrower decides to default or repay at the end of the period. If she defaults, she keeps the R but is banned by the lender and can only get some outside option v every period thereafter. She also loses her collateral C.

(a) Just as in class, derive the no-default constraint.

Next, assume that lenders must recover their opportunity rate of interest i on loans.

(b) Following the same method as in class, describe what happens to loan size and the interest rate on loans as the collateral offered by the borrower goes up. You can assume throughout that the no-default condition binds and holds with equality, so that the problem of potential default is always present.

(5) Now for a small variation on the previous question in which we unpack the outside option.

A borrower takes a loan of L in every period and produces output AL^{α}, where $A > 0$ and $0 < \alpha < 1$. He makes a repayment of R. He has no collateral. At any date, he can default on the repayment. Thereafter, he gains access to new terms forever after on a secondary market; the loan size there is L^* and the repayment required is R^*. Assume that he cannot default at all on the secondary market. He has a discount factor of δ between 0 and 1.

(a) Derive the borrower's no-default constraint for the current loan contract (L, R).

(b) Show that if the borrower chooses not to default, she must enjoy a higher payoff from (L, R) than she does from (L^*, R^*). Prove this using the no-default constraint and also explain your answer intuitively.

(c) Suppose that after a default, the borrower has to wait several periods before access to *any* other market (getting a payoff of zero meanwhile), and then gets access to the secondary market at (L^*, R^*). Would part (b) still be true?

(6) Another variation. There is a large population of borrowers and lenders. Each active borrower takes a loan L from a lender at every date, produces $F(L)$ and repays R. If a borrower defaults on a loan, the lender tries to spread the word that the borrower has defaulted. Just how successful he is in doing so is described as follows:

When a defaulting borrower appears to a new lender, the lender tries to remember if this borrower has been complained about. But with probability q between 0 and 1 he doesn't know the old lender, so the new lender may not have heard about the default. In that case the lender starts up a relationship with the borrower, and the past slate is wiped clean.

If the lender has heard about this borrower's past behavior, then he does not lend to the borrower. The borrower waits one period, getting zero, and then meets a lender again the day after, with exactly the same story repeating itself.

[a] If every borrower borrows on the same terms from every lender, show that the lifetime value to a borrower who has just defaulted — but does not plan to default again — is given by

$$V = \frac{q[F(L) - R]}{(1 - \delta)[1 - \delta(1 - q)]}.$$

[Hint: it must be the case that for a defaulting borrower who does not plan to default again,

$$V = q\frac{F(L) - R}{1 - \delta} + (1 - q)\delta V.$$

First explain very clearly what this equation is, and then use it.]

[b] Write down the no-default constraint using the solution from part (a), and show that after some simplification, it is given by

$$F(L) - R \geq [1 - \delta(1 - q)]F(L).$$

[c] Using this formula, prove that if q is close to 1, then the market breaks down because no lender will be unable to recover his loan.

[d] What can you say about the possibility of an active market for credit if q is close to zero?

[e] What does q mean? Explain using three examples: (i) a modern credit market in a developed country, (ii) a credit market in a very traditional rural society where everyone knows everyone, and (iii) a credit market in a fast-changing developing society without modern computer systems.

(7) Indebtia is a country in big trouble. It owes a huge amount of money to the ROW (the rest of the world), which is putting pressure on Indebtia to repay. ROW wants the Government of Indebtia to invest in a new project that will pay off the debt. The project will yield an amount 100 with probability p and nothing with probability $1 - p$. The government can put in effort $e \geq 0$ to influence this probability, and we suppose that

$$p(e) = e/200.$$

The cost of the effort to the government is $(1/400)e^2$.

The debt is massive, bigger than 100. ROW is trying to figure out how best to collect some part of it. That is, if R is the repayment asked for, and if Indebtia can pay only in the event of success, ROW is trying to maximize $p(e)R$. The problem is that e will depend on how much R is demanded. (And of course, R cannot exceed 100.)

(a) What is the net payoff to the Government of Indebtia if it incurs effort e and the repayment demanded is R?

(b) Maximize this payoff with respect to e to show that

$$e = 100 - R.$$

(c) Knowing this, solve for the repayment that ROW should demand from Indebtia.

(d) ROW can invest an additional amount x in Indebtia and double the size of the project from 100 to 200 (with the same probabilities of success as before). If x could be invested safely elsewhere by ROW at an interest of r, calculate the maximum value of x for which ROW will invest in Indebtia, knowing that it can adjust its repayment demand if it makes the investment.

Extra Problems on Credit Markets.

These do not need to be handed in as part of the problem set.

(8) Are the following statements true or false? It is *not* enough to just guess one or the other. You need to provide an argument for or against, and only then will any credit be awarded.

[**a**] Default risks can be adequately covered by the choice of a suitably high interest rate compared to the safe rate.

[**b**] Borrowers and lenders that perfectly agree on maximizing expected returns might still disagree on the choice of project.

[c] Individuals with less collateral will want to take on riskier projects, all other things being equal.

[d] Collateral that's valuable for the borrower but completely valueless to the lender cannot be useful in enforcing the repayment of a debt.

[e] In the moral hazard borrowing model, as we move down the incentive constraint for which lower repayment is demanded and higher effort is put into the project, the borrower's payoff is affected in an ambiguous way.

[f] In the model of moral hazard and risk-taking by borrowers, a higher collateral must improve the terms of credit to a borrower.

(9) A t-shirt exporter is trying to break into the market of a developed country. He can produce either high quality or low quality t-shirts. Production of the low quality shirt is costless, but production of the high quality t-shirt entails a cost of $c per shirt.

The low quality has no value on the market, so the retailer tests the exporter by asking him to provide a sample (s units) for free. If the sample is bad, the exporter must start again with another retailer. If the sample is good, the retailer offers an indefinite contract for a high-quality order of $q > s$ units, for which he pays p per shirt *in advance*, where $p > c$. And it remains that way period after period, unless the exporter deviates by sending in low quality.

Every retailer behaves the same way, an exporter can meet a new retailer every period, and no retailer knows about the history of a new exporter who approaches him.

(a) Derive the discounted lifetime value of profits to the exporter starting from the sampling phase. (Of course, in what follows, you can assume this amount is positive.)

(b) Show that the self-enforcement constraint for the exporter during the contract phase is given by

$$\delta(pq + s) \geq q(1 + \delta).$$

(c) Prove that the self-enforcement contract can *never* hold when δ is close to zero, and that it does hold when δ is close enough to 1 provided that $pq + cs > 2cq$. Give some intuition for these results.

(d) There is a second self-enforcement constraint: this is during the sampling phase. Show that this constraint is automatically satisfied as long as the lifetime value in part (a) is positive.

(10) A borrower is about to take a loan of L, on which interest of r will be charged. He has collateral C, which he puts up for the loan. There is no strategic default, but there is moral hazard. The borrower puts in effort e at a cost $c(e)$ to generate a success probability of $p(e)$. The functions $p(e)$ and $c(e)$ are continuously increasing in e.

If there is success, then output is given by $Q = F(L)$ and the loan is repaid. If there is failure, output is zero and the borrower loses the collateral C.

As in class, e is chosen by the borrower in response to the terms of the loan contract and you can assume that there is a unique such optimal choice for every loan contract.

(a) Write down an expression for the net expected payoff of the borrower.

(b) Prove that if the interest rate r rises, or the collateral C falls, the optimally chosen effort put in by the borrower must come down.

(11) SelfHelp is a newly formed credit cooperative. Members of SelfHelp can deposit savings with the cooperative and they can also turn to SelfHelp for a loan if they need one. We summarize the payoff gain from all these activities as $S > 0$ per member, per period of time.

Without access to SelfHelp, payoff is just s per period, and we assume that $S > s$.

If a member misbehaves or defaults on a SelfHelp loan he is punished with a penalty of value F, and excluded from all future dealings with SelfHelp.

(a) If each member has a discount factor of δ between 0 and 1, write down the lifetime discounted value of having access to SelfHelp.

(b) Suppose there is no telling whether SelfHelp will survive at any date; that is, for any date at which SelfHelp is currently alive, denote its conditional probability of survival in the *next* period by p. Now write down the *expected* lifetime discounted value of having access to SelfHelp, conditional on SelfHelp being alive today.

[Hint. Let V be this value. Then

$$V = S + \delta p V + \delta(1-p)\frac{s}{1-\delta}.$$

Understand why this equation is true, and then use it.]

(c) Assume we are in the world of part (b). If a member needs to repay R to SelfHelp, write down the condition describing her incentives to repay.

(d) Assume that p is a function of the percentage n of members who repay. Explain why this might be the case. Write down any one functional form that could link n to p.

(e) Using the functional form you wrote down in part (d), and also using parts (b) and (c), show that there can be two outcomes or equilibria for the *same* parameters: (i) there is no (voluntary or strategic) default and SelfHelp survives with little or no government assistance or (ii) there are high rates of default and SelfHelp survives with low probability.

(12) A *risk-lover* is an individual who prefers a random lottery with some expected value to a fixed amount of money with the value as the mean of a lottery.

(a) Let $u(x)$ be the utility function of a risk-lover. Write down a specific example of $u(x)$ that exhibits risk-loving behavior.

(b) Show that with limited liability, a risk-neutral or even a risk-averse borrower may behave *as if* he is a risk-lover when considering some projects in which he would like to invest.