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ECON-UA 323
Development Economics

Problem Set 6

(Practice for Exam 2; does not need to be handed in.)

(1) Scrooge has an initial wealth of W_t at the start of every date, and earns capital income on that wealth at the rate of return r. He also earns a labor income of y at every date.

(a) Write down a formula for Scrooge's *total* income at date t.

(b) Now suppose that Scrooge always saves his entire existing wealth as well as a fraction s of his total income; that is, his capital income plus his labor income. Write down a formula that connects W_t to W_{t+1}.

(c) Suppose that Scrooge's labor income grows exogenously at the rate of g from period to period. Find the growth rate in Scrooge's total wealth in the long run. (Hint: you will need to look at two cases, one in which $sr > g$, and the other in which $sr < g$.)

(d) Suppose that the rate of growth of overall per-capita income is 2% per year. Suppose, also, that Scrooge is a pure capital income earner ($y = 0$), and saves 25% of his income ($s = 0.25$). Find the rate of return on capital that will allow Scrooge to double his income relative to national income in 20 years.

(2) We are going to study wealth inequality in the land of Diferencia. We start from some initial, unequal distribution of strictly positive wealths.

(a) If everyone in Diferencia earns a common rate of return on their wealth, saves the same fraction of this capital income, and adds this savings to their initial wealth, show that their next period's wealth distribution has exactly the same Lorenz curve as that of the starting distribution.

(b) If, today, everyone receives an equal addition to their wealth from the Government of Diferencia, show that any Lorenz-consistent inequality measure must register a fall in inequality relative to the starting distribution.

(c) Now forget about the government transfer and go back to part (a). Modify that part by allowing for either wealth-dependent savings rates or wealth-dependent rates of return on capital, but no other transfers to, from or across people. Carefully describe one scenario in which next period's wealth inequality is higher than initial inequality according to any Lorenz-consistent inequality measure.

(3) The land of Oily is filled with inhabitants that only hold shares of the national oil resource, but do so unequally. Number the individuals in $[0, 1]$ from poorest to richest, so

that x_i is the amount of oil held by person i, where $x_i \leq x_j$ if $i \leq j$. The international price of oil is given by p, so for each person i,

$$\text{Wealth } = w_i = px_i$$

(a) Show that no matter how oil prices p move around at any date t, wealth inequality is unchanged in Oily.

Now we complicate matters a bit by giving everyone a fixed handout $y > 0$ over and above their oil holdings, so that for each person i, wealth is now given by

$$\text{Wealth } = w_i = px_i + y$$

(b) Show that if oil prices p rise, inequality must go up in Oily. [Try and do this in two ways for practice: by showing how the Lorenz curves change, or deriving the result from the axioms of inequality measurement. Either approach, if done right, would be valid for full credit.]

(c) How would your answer to b change if y were a tax instead, so that it is negative, not positive? How would you answer change if the tax or the handout were to be exactly proportional to oil holdings?

(d) Set $y = 0$ again. Now assume a foreign power comes and kills off 10% of the *richest* inhabitants of Oily and seizes the oil shares of all those killed. Compare the new inequality in Oily with the old. Use simple numerical examples to illustrate.

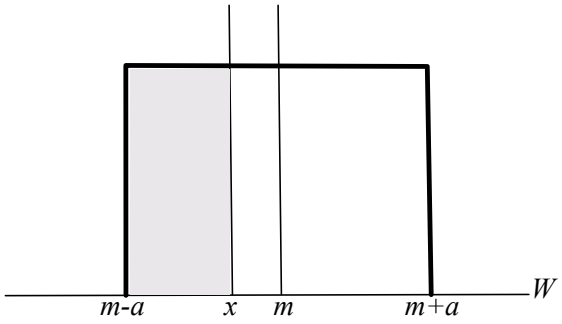
(4) Pooh is a country with only two occupations. You can work as a laborer or you can become an entrepreneur, who hires labor and makes profits. To become an entrepreneur, you need a loan of 20,000 pahs (a pah is the unit of currency in Pooh). With 20,000 pahs you can set up a factory that hires ten workers, each of whom you must pay an income of w pahs over the year. Together, the workers produce for you an output of 30,000 pahs. At the end of the year, you must sell your factory (for 20,000 pahs) and repay the loan. The rate of interest in Pooh is 10% per year.

(a) Suppose you were to contemplate running away instead. Imagine you would be caught, fined 5000 pah, and 20% of your business profits would be seized. You would also lose whatever collateral you put up with the bank (and would have received back, along with a credit for the 10% interest on it). Find a formula that describes how much collateral the bank should ask for (in pah units) before it would advance you a loan. Examine this required collateral for different levels of the wage income: $w = 1000$, 2000, and $2,500$. Does the required collateral go up or down with the wage? Explain your answer.

(b) Let's suppose that the minimum wage in Pooh is fixed by law at 500 pah. Find the collateral required to get a loan if w is at the minimum wage. Consider the following statement: "If more than $\bar{x}$% of Pooh inhabitants are unable to put up this collateral, then some people are unable to get employment, whether as laborers or entrepreneurs." Calculate $\bar{x}$.

(5) Suppose that to set up a business needs a setup cost of S, which has to be paid out of one's own pocket. (There are no credit markets.)

Suppose that existing wealth is distributed with a uniform distribution around a mean of m, starting from some lower bound $m-a$ to an upper bound $m+a$. In what follows, we are going to vary both m and a. To help you, I have drawn the uniform distribution. Because any wealth between $m-a$ and $m+a$ is equally likely under the distribution, the density just equals $1/(2a)$.



(a) Show that the proportion of the population earning wealth between $m-a$ and some value x (smaller than $m+1$) is given by the formula

$$\frac{x-m+a}{2a}.$$

[Look at the diagram!]

Now verify:

(b) As a goes up from 0 to larger values, the resulting sequence of income distributions is more and more unequal in the sense of Lorenz-dominance. [Try to construct the Lorenz curves.]

We are now interested in the effect of higher inequality on the fraction of people who can afford the setup cost S. Of course, if S is larger than $m+a$ then no one can afford S, and if S is below $m-a$ everyone can afford S. So assume that S lies strictly between $m-a$ and $m+a$.

(c) Show that an increase in inequality lowers the fraction of the population who can afford the setup cost if $m > S$, but it *raises* the fraction of the population who can afford the setup cost if $m < S$. Explain this result intuitively, in words.

(6) In the land of TwoOcc, there are just two occupations. You can become a worker at no cost, earning a wage of w, or you can become an entrepreneur. If the latter, you set up a factory at cost S, and then hire outside labor ℓ at the worker wage w, producing

$$y = A\sqrt{l}$$

in total value. Everyone lives for just one round of production, and everyone seeks to maximize their income net of costs.

(a) If the wage is w, show that every profit-maximizing entrepreneur will hire ℓ units of labor, given by

(i) $$\ell = \left(\frac{A}{2w}\right)^2$$

and that each entrepreneurs's net income (her profit) will be given by

(ii) $$\text{Profit } = \frac{A^2}{4w} - S$$

(b) If everyone can freely choose to become a worker or a entrepreneur (and can pay the setup cost up-front), then they *must be indifferent between these two occupations*. Using this fact, show that the equilibrium wage must be given by

(iii) $$w^2 + Sw = \frac{A^2}{4},$$

and using this formula (either graphically or algebraically), show that the equilibrium wage rises with A and falls with S. Intuitively explain these results in words.

(c) Using equation (i), show that the number of entrepreneurs — call it e — must be given by the formula

(iv) $$e = \frac{n}{1 + (A/[2w])^2},$$

where n is the total working population. Using the equations you have so far, prove that the number of entrepreneurs must increase as A increases, and the profits of each entrepreneur must go up. (If you have trouble, try at least giving an intuitive explanation.)

(d) Indicate very briefly which of equations (i–(iv)) might fail to hold if the setup cost must be borrowed and the capital market is imperfect, even if that borrowing is done at a zero rate of interest?

(7) A question to get you to understand how aggregates are affected by inequality:

The following functions describe *total individual* savings S as a function of income Y:

(a) $S = (1/10)Y$, (b) $S = (1/5)Y - 10$, (c) $S = \sqrt{Y} - 1$, (d) $S = Y^2/(1 + Y)$.

For each, describe what happens to the savings *rate* $s = S/Y$ as income changes.

(b) Take any income distribution and suppose that we change it to another by imposing a Dalton transfer from relatively poor to relatively rich. For which of the above savings functions will overall savings in the economy go *up* as a result of the transfer?