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Econ-UA 323

Development Economics

Problem Set 3

(1) A *fax machine* is a pre-email system of communication. (It still exists! You need to buy one to receive and send letters. The more people who use it, the bigger the incentive to have one, so this is a good example of complementarities.)

Suppose that fax machines are made newly available in NeverNeverLand. Companies are deciding whether or not to install a machine. This decision partly depends on how many *other* companies are expected to install fax machines. Think of a graph that describes how many companies *will* install fax machines as a function of how many companies are *expected* to install fax machines.

We will work with a particular complementarity map $x(n)$. It follows the equation $A+(0.8)n^2$ whenever this value is between 0 and 1. If this value crosses 1, we set $x(n) = 1$. If this value is negative, we set $x(n) = 0$.

(a) Draw and describe this graph for four values of A: $A = -0.5$, $A = 0.1$, $A = 0.3$, and $A = 0.5$. Pay particular attention to the value of $x(n)$ at the edges 0 and 1, and to the points where it hits the diagonal of the graph. Also, in the region where $x(n)$ lies strictly between 0 and 1, make sure you explain intuitively why an increase in n is increasing $x(n)$.

(b) Analyze the equilibrium adoption of fax machines in NeverNeverLand as A varies. For which ranges of A does a *unique* equilibrium exist? When multiple equilibria exist, what do they look like? Provide some intuition for your answer.

(2) Suppose that each of N citizens in the country of Taxland needs to pay a tax of T every year to the government. Each citizen may decide to pay or to evade the tax. Assume that each citizen is risk-neutral and simply seeks to maximize expected income net of any taxes or fines.

If an evader is caught, Taxland imposes a fine of amount F, where $F > T$. However, the Government of Taxland has limited resources to detect evaders. Assume that out of all the people who evade taxes, *the government has the capacity to audit a maximum of* m *of them, where* $1 \leq m < N$.

Distinguish between two cases: In Case 1, m persons are randomly chosen from the population to be audited. If any of these is an evader, then conditional on being audited, he will be fined for sure. In Case 2, each evader — and no one else — has an anomaly on his tax return which alerts the government. Up to m of them will be randomly audited — and fined.

(a) Write down the expected payoffs in each case from evasion, and show that there are no complementarities in Case 1, while there are complementarities in Case 2.

(b) Contrast Case 1 and Case 2. In Case 1 show that there is typically a unique equilibrium. In Case 2, show that it is always an equilibrium for nobody in society to evade taxes. Are there another equilibria as well? Describe parametric configurations of (m, n, T, F) for which such equilibria will exist, and also describe the equilibria.

(3) The country of Skillover has the good fortune of generating spillovers in skills! Each citizen in Skillover simultaneously decides whether to acquire "high skill" at cost $c > 0$, or remain low-skilled. Let y_h and y_ℓ denote the incomes earned by high- and low-skilled workers. Now here comes the spillover:

$$y_h = y_h^0 + nH \text{ and } y_\ell = y_\ell^0 + nL,$$

where $y_h^0 > y_\ell^0$ are baseline values for the two incomes, H and L are positive constants, and n is the fraction of the population that chooses to become high skilled. Thus in Skillover, a person's productivity in both kinds of jobs is positively linked not only to her own skills, but also to the skills of her fellow workers.

(a) There are always positive spillovers ($H > 0$ and $L > 0$) but when exactly is there a *complementarity*?

For the next question, define $\Delta \equiv y_h^0 - y_\ell^0$ to be the "baseline difference" between skilled and unskilled incomes.

(b) Show that if $\Delta < c < \Delta + (H - L)$, there are three possible equilibria: one in which everybody acquires skills, one in which nobody does, and a third in which only a fraction of the population becomes high-skilled. Give an algebraic expression for this fraction in the last case. Explain why this equilibrium is "unstable" and is likely to give way to one of the two extreme cases.

(c) Describe the set of equilibria when $H < L$.

(d) Consider another variation, this time with no skill spillover onto incomes. Specifically, suppose that $y_h = y_h^0$ and $y_\ell = y_\ell^0$ are both fixed and independent of n (that is, $H = L = 0$). However, suppose that the *cost of individual education* is indeed affected by the presence of skilled people $c(n) = (1 - n)/n$. (The idea here is that it is easier to learn if there are more educated people around). Describe the set of equilibria.

(4) Multiplania is a community that wants to recycle its glass and plastic, and encourages its residents to do so. But you have to take the trash pretty far to find recycling bins, so it is inconvenient to do so. Everyone has a benefit $b > 0$ from just throwing all glass and plastic into the garbage.

On the other hand, anybody who doesn't recycle can be *seen* not doing so, and there is an individual stigma or shame attached to it, a cost $s(n)$, where n is the overall fraction of the community who actually recycle. Assume that $s(n) = a + cn$, where $a \geq 0$ and $c > 0$.

The payoff from not recycling is the benefit minus the shame. The payoff from recycling is just taken to be zero.

(i) Interpret the function $s(n)$. In particular, why should c be positive? What about a?

(ii) Describe the range of values of (a, b, c) for which there is a unique equilibrium.

(iii) Describe the range of values of (a, b, c) for which there is are multiple equilibria, and describe all the equilibria, including the unstable one.

(iv) In the world of part (iii), consider a policy that imposes a fine $F > 0$ on every garbage thrower. Show that the threshold value for the population share of recyclers that tips the society over into the good equilibrium (i.e., the unstable equilibrium value) must fall with the fine.

(v) Using part (iv), can you discuss a situation where we begin with a bad equilibrium (i.e., one in which no one recycles), impose a fine, then remove the fine after a temporary imposition, with the society now in a good equilibrium with recycling?

(5) In the question that follows, n refers to the *number* of people rather than a fraction of the population. In the land of Pampa, living in the countryside gives you a fixed payoff of 100 (Pampa has lots of land), while living in a city gives you a payoff that first increases with the number of people living in the city (agglomeration), and then declines after the number of people goes above a certain threshold (congestion). Let us write this payoff as

$$r = 20n - n^2/2,$$

where n is the number of city dwellers in that particular city.

(a) Let N be the total population in Pampa. If only one city can exist in the entire country, trace out the set of equilibria (i.e., population allocations between countryside and city) as N varies from 0 to infinity.

(b) Now suppose that new cities can come up, each yielding exactly the same payoff function as above. Focus on the equilibrium in each case with the maximum possible city dwellers, and explain how this equilibrium will move with the overall population N.

(6) Let us suppose that people are arrayed in order of decreasing honesty, and that the "honesty payoff" to person i, where i is an index between 0 and 1, is given by $h(i) = 1 - ai$, where i runs from 0 to 1, and a is a positive parameter. This is the direct payoff person i gets from being honest in a particulate situation. But she also gets a cheating payoff equal to bn, where n is the fraction of individuals being dishonest in that situation, and $b > 0$ is a positive parameter. Each person has just two choices, to be honest or dishonest, receiving the honesty payoff if she takes the honest action and the cheating payoff if she takes the dishonest action.

Describe the equilibria of this model for different values of the parameters a and b.

(7) Suppose that people's attitudes can take three possible positions: L, M, and R, where you can think of L as leftist, R as rightist, and M as middle-of-the-road. Consider a society

in which *it is known that* a fraction α are M types, and the remaining fraction $1 - \alpha$ are divided equally between L and R, but no individual is known to be L, M, or R at first sight. Suppose that each individual gets satisfaction S from expressing his or her own true views, but feels a loss ("social disapproval") in not conforming to a middle-of-the-road position. The amount of the loss depends on the fraction α of M types: suppose that it equals the amount $\alpha/(1 - \alpha)$.

(a) Show that there is a threshold value of α such that everybody in society will express their own view if α is less than the threshold, but will all express M-views if α exceeds the threshold.

(b) What happens if we change the specification somewhat to say that the "social disapproval loss" equals $\beta/(1 - \beta)$, where β is the expected fraction of people who *choose* to express M views (and not necessarily the true fraction of M types)?

(c) Indicate how you would extend the analysis to a case in which there are potential conformist urges attached to each of the views L, M, and R, and not just M.