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Econ-UA 323

Development Economics

Problem Set 2

(1) This question involves detailed numerical calculations and will not be a typical exam question. But it will help you understand the basic growth model.

The economy of Ping Pong produces its output using capital and labor. The labor force is growing at 2% per year. At the same time, there is "labor-augmenting" technical progress at the rate of 3% per year, so that each unit of labor is becoming more productive.

(a) How fast is the effective labor force growing?

(b) Now let's look at production possibilities in Ping Pong. We are going to plot a graph with capital *per unit of effective labor* ($\hat{k}$) on the horizontal axis and output *per effective unit of labor* ($\hat{y}$) on the vertical axis. Here is a description of the "production function" that relates $\hat{y}$ to $\hat{k}$. As long as $\hat{k}$ is between 0 and 3, output ($\hat{y}$) is given by $\hat{y} = (1/2)\hat{k}$. After $\hat{k}$ crosses the level 3, an *additional* unit of $\hat{k}$ only yields one-seventh *additional* units of $\hat{y}$. This happens until $\hat{k}$ reaches 10. After that, each additional unit of $\hat{k}$ produces only one-tenth additional units of $\hat{y}$. (To draw this graph, you may want to measure the $\hat{y}$ axis in larger units than the $\hat{k}$ axis; otherwise, the graph is going to look way too flat.) On a graph, plot this production function. What are the capital–output ratios at $\hat{k} = 2$, 6, and 12? Note that the answers you get in the case $\hat{k} = 6$ and 12 are different from what happens at the margin (when you increase capital by one unit). Think about why this is happening.

Note: in class, we talked about the output-capital ratio rather than the capital-output ratio. These are just flip sides of the same coin! If θ is the output-capital ratio, the capital-output ratio is nothing but its reciprocal, which is $1/\theta$.

(c) Now let us suppose that Ping Pong saves 20% of its output and that the capital stock is perfectly durable and does not depreciate from year to year. If you are told what $\hat{k}(t)$ is, describe precisely how you would calculate $\hat{k}(t+1)$. In your formula, note two things: (i) convert all percentages to fractions (e.g., 3% = 0.03) before inserting them into the formula and (ii) remember that the capital–output ratio *depends* on what the going value $\hat{k}(t)$ is, so that you may want to use the symbol $1/\theta$ for the capital-output ratio, to be replaced by the appropriate number once you know the value of $\hat{k}(t)$ (as in the next question).

(d) Now, using a calculator if you need to and starting from the point $\hat{k}(t) = 3$ at time t, calculate the value of $\hat{k}(t+1)$. Likewise do so if $\hat{k}(t) = 10$. From these answers, can you guess in what range the long-run value of $\hat{k}$ in Ping Pong must lie?

(e) Calculate the long-run value of $\hat{k}$ in Ping Pong. (Hint: You can do this by playing with different values or, more quickly, by setting up an equation that tells you how to find this value.)

(2) Let's study the production function $Y = AK^aL^b$ in more detail.

(a) We say that a production function satisfies *increasing returns to scale* if a proportional increase in all inputs raises output by *more* than that proportion. Prove carefully that this happens if and only if $a + b > 1$.

(b) Using basic calculus, find a formula for the marginal product of labor. Show that if there is constant returns to scale (so $b = 1 - a$), the marginal product depends *only* on the ratio of capital to labor. But this is no longer true if $a + b \neq 1$. Explain why intuitively.

(c) Notice that if the production function has constant returns to scale, it *automatically* generates diminishing returns to each input. Show this with algebra but then also try to explain it intuitively.

(3) (This could be part of a typical exam question.) Suppose that the country of Xanadu saves 20% of its income and has a constant capital–output ratio of 4. Assume capital does not depreciate.

(a) Using the Harrod–Domar model, calculate the rate of growth of total GNP in Xanadu.

(b) If population growth were 3% per year and Xanadu wanted to achieve a growth rate *per capita* of 4% per year, what would its savings rate have to be to get to this growth rate?

(c) Now go back to the case where the savings rate is 20% and the capital–output ratio is 4. Imagine, now, that the economy of Xanadu suffers violent labor strikes every year, so that whatever the capital stock is in any given year, a quarter of it goes unused because of these labor disputes. If population growth is 2% per year, calculate the rate of per capita income growth in Xanadu under this new scenario.

(4) (Again, a possible exam question, this time on the Solow model.) A country has a production function which depends on capital and labor, and it is given by

$$Y = 100K^{1/3}L^{2/3}.$$

(a) Describe per-capita production y as a function of the per-capita capital stock k.

(b) Assume that capital does not depreciate at all, and that there is no technical progress. Let the savings rate be given by s and the rate of growth of population (=labor) be given by n. Show that steady state output per-capita is given by

$$y^* = 1000\sqrt{s/n}, \tag{1}$$

describing precisely all the steps that lead to this conclusion.

(c) Provide intuitive economic reasoning to explain why equal percentage increases in savings rates and population growth rates appear to nullify each other in equation (1), in the sense of leaving per-capita income unchanged.

(5) In class, we discussed how the Solow model fails to adequately account for per-capita income differences across countries. There is a related problem: the model also appears to predict too high a discrepancy between the rates of return to capital across a developed and developing country. To appreciate this problem, suppose that the production function in both countries is given by

$$Y = K^a L^{1-a}.$$

(a) Show, using calculus and a little algebra, that the rate of return r to capital is given by

$$r = ak^{a-1} = ay^{(a-1)/a},$$

where k and y have the usual meanings.

(b) Show that if one country is 15 times richer than another and $a = 0.3$, then the poorer country will have a rate of return to capital over 200 times that of the richer country.

(c) Suggest ways of resolving this puzzle. That is, what forces could still allow the 15-times gap in incomes and yet give you a smaller gap between the two rates of return on physical capital? Question (8) at the end is an optional question which is *not* required for this problem set, but it bears on this discussion and I encourage you to try.

(6) (These could be typical true-false questions in exam.) *Discuss* whether the following statements are true or false. In each case, just saying "true" or "false" is not enough. Provide an argument for truth, or simply a counterexample if you think it's false.

(a) The Harrod–Domar model states that a country's per capita growth rate *depends* on its rate of savings, whereas the Solow model states that it does not.

(b) According to the Harrod–Domar model, if the capital–output ratio in a country is high, that country will grow faster.

(c) To understand if there is convergence in the world economy, we must study countries that are currently rich.

(d) Middle-income countries are more likely to change their relative position in world rankings of GNP than poor or rich countries.

(e) In the Solow model, a change in the population growth rate has no effect on the long-run rate of per capita growth.

(f) In the Solow model, output per head goes down as capital per head increases, because of diminishing returns.

(g) A Cobb-Douglas production function that has increasing returns to scale must also have increasing returns to at least one of its inputs.

(h) A country which has been growing steadily at 10% per year and now has a per-capita income of \$100,000 would have a per-capita income of \$12,500 approximately 22 years ago.

Optional Practice Problems (do not need to be handed in)

(7) This is a review question and all it does is encourage you to go over the class notes and the text material.

A. Review of Solow. Suppose that the production function is given by $Y(t) = AK(t)^a L(t)^{1-a}$, where A is a fixed technological parameter. There is no technical change. Assume a fixed rate of depreciation δ and a constant rate of growth of population n. Explicitly solve for the *steady-state* value of the per capita capital stock and per capita income. How do these values change in response to a rise in (a) the technological parameter A, (b) the rate of saving s, (c) a, (d) δ, the depreciation rate, and (e) the population growth rate n?

B. Review of Harrod-Domar. Recall the basic accumulation equation

$$(1+n)k(t+1) = sy(t) + (1-\delta)k(t)$$

In this problem we're interested in looking at a case in which there is *no* diminishing returns in production, so that

$$y = Ak.$$

(a) Draw a diagram to convince yourself that in this case, there is no positive limit capital stock as in the Solow model: either k grows without bound or it shrinks all the way down to zero.

(b) Define $g(t)$ to be the growth rate of per-capita capital; that is

$$g(t) = [k(t+1)/k(t)] - 1.$$

Prove that this is also the growth rate of per-capita output at date t.

(c) Show that $g(t)$ is the same value at every date, and that

$$sA = (1+n)(1+g) - (1-\delta) \simeq n + g + \delta.$$

This is the *Harrod-Domar equation*. See text for more details.

(d) Note that in the Solow model, the savings rate only affects the limit value of per-capita output, *but does not affect the rate of growth of that output in the long run*. But in the Harrod-Domar variant, it does affect the growth rate. Discuss why.

In the Harrod-Domar model, the value A is known as the *output-capital ratio*. It is the amount of flow output generated by one fixed unit of the capital stock. Its reciprocal, $1/A$, is more familiarly known as the *capital-output ratio*. The best way to think about the Harrod–Domar equations is to attach some numbers to them, as we did in Problem 3.

(8) Think of the three-input model with unskilled and skilled labor (as well as physical capital) mentioned in class: $Y(t) = AK(t)^a H(t)^b U(t)^{1-a-b}$, where H is skilled labor and U is unskilled labor. One useful feature of this model is that it simultaneously explains how rates of return to physical capital as well as the wage rate for unskilled labor might be low for developing countries. But there is a problem with this argument.

(a) Using the Cobb-Douglas with three inputs instead of two, show that such a model predicts that the rate of payment to *human capital* must be higher in developing countries.

(b) Adapt the Cobb-Douglas specification in part (a) to allow for differences in technology across developed and developing countries. Now it is possible to generate situations in which the return to every input is lower in developing countries. Which input is likely to have the lowest return (in a relative sense)?