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Econ-UA 323

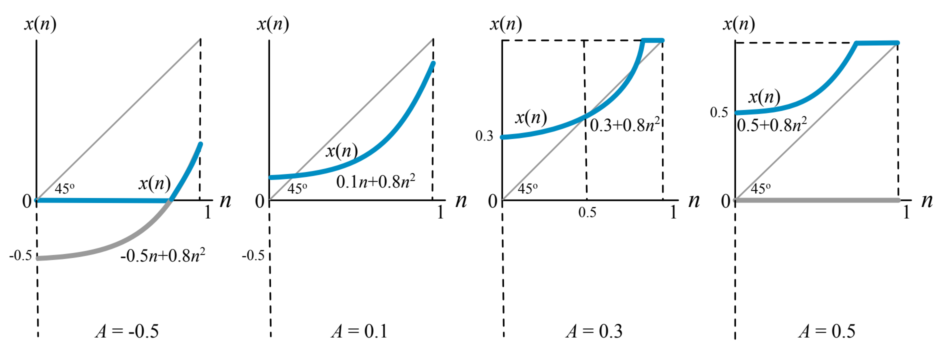
Development Economics

Answers to Problem Set 3

(1) Suppose that fax machines are made newly available in NeverNeverLand. Companies are deciding whether or not to install a machine. This decision partly depends on how many *other* companies are expected to install fax machines. Think of a graph that describes how many companies *will* install fax machines as a function of how many companies are *expected* to install fax machines.

We will work with a particular complementarity map $x(n)$. It follows the equation $A+(0.8)n^2$ whenever this value is between 0 and 1. If this value crosses 1, we set $x(n) = 1$. If this value is negative, we set $x(n) = 0$.

(a) Draw and describe this graph for three values of A: $A = -0.5$, $A = 0.1$, $A = 0.3$, and $A = 0.5$. Pay particular attention to the value of $x(n)$ at the edges 0 and 1, and to the points where it hits the diagonal of the graph. Also, in the region where $x(n)$ lies strictly between 0 and 1, make sure you explain intuitively why an increase in n is increasing $x(n)$.



The number of companies x who want to install (as a function of the number of companies n who are expected to install) is given by the equation:

$$x = A + (0.8)n^2$$

as long as x, calculated by this equation, lies between zero and 1. Specifically, $x(n) = 0$ for all n such that x as defined above is negative, $x(n)$ is given by the above equation when it generates values between zero and 1, and once the above equation generates larger numbers than 1, we set $x(n) = 1$. The diagram above captures this for the four cases $A = -0.5$, $A = 0.1$, $A = 0.3$, and $A = 0.5$. Each panel plots the function $A + (0.8)n^2$, but "truncates" it at the bottom by 0 and at the top by 1. So, for instance, when $A = -0.5$, the resulting $x(n)$ curve is flat and then follows the $A + (0.8)n^2$ function once that becomes positive. And when $A = 0.3$, the function starts positive anyway so $x(n)$ faithfully follows it, but once it rises above 1 we truncate $x(n)$ at the top.

In the strictly increasing segment, the idea is that if more companies are using fax machines, it becomes easier to communicate by fax, so $x(n)$, the fraction of companies that *want* to use those machines, goes up as well.

(b) Analyze the equilibrium adoption of fax machines in NeverNeverLand as A varies. For which ranges of A does a *unique* equilibrium exist? When multiple equilibria exist, what do they look like? Provide some intuition for your answer.

Intuitively, when A is low, it should be optimal *never* to install fax machines. And when A is large, it should optimal to *always* install fax machines, so there should be a unique equilibrium for both low and high A. To go along with this intuition, in the first panel of the diagram, A is low (negative in fact) and there is only one contact between $x(n)$ and the diagonal, and that is at $n = 0$. That is the unique equilibrium, with zero adoption of fax machines. In the second panel, there is again a unique contact with the diagonal, but this time at a strictly positive value as shown. This situation will also have a unique equilibrium, but in that equilibrium there will be some positive adoption. In the third panel, there are three intersections, one with low adoption, one with high, and one with intermediate values of adoption (see the intersection point of $x(n)$ with the diagonal for between 0.5 and 1). This last one you should check is "unstable". Finally, in the fourth panel, there is again just one contact, and that is at $n = 1$.

In class, we are particularly interested in cases such as those depicted by the third panel, where there may be multiple steady state outcomes to the same underlying system, as in the case of QWERTY and Dvorak.

Which values of A generate these multiple solutions? You can easily see this in your mind's eye by sliding A up continuously as you move from the first to the fourth panel. At intermediate values of A, the complementarity generates two stable solutions: one with low adoption of fax machines, the other high. And in those cases, there is always an unstable solution in between.

You can do all this diagrammatically. What follows is not needed for this question but if you took this path, you are really understanding the material more deeply.

To find the range of A, we first ask the question: when does the equation

$$A + (0.8)n^2 = n$$

have one solution or more? This will help build our answer. The theory of quadratic equations says that the roots of the equation are given by

$$n = \frac{1 \pm \sqrt{1 - 4 \times (0.8) \times A}}{2 \times 0.8} = \frac{1 \pm \sqrt{1 - (3.2) \times A}}{1.6}$$

Notice that if $A > 1/3.2 \simeq 0.312$, then there is no solution (all roots are imaginary). This is the threshold beyond which you have a picture like Panel 4 of the Figure above. In that case, there is a unique equilibrium $n^* = 1$. Everyone adopts fax machines.

For every value of A smaller than $1/3.2$, there are indeed two real solutions, but we have to see if they both lie between zero and 1. The larger solution is

$$n = \frac{1 + \sqrt{1 - (3.2) \times A}}{1.6}$$

and you can easily check that if this is *larger* than 1, then we have a situation like the first or the second panel with just one unique equilibrium. That is the same as the condition that $\sqrt{1 - (3.2) \times A} > 0.6$, or equivalently,

$$1 - (3.2) \times A > 0.36, \text{ or } A < 0.2.$$

In all such cases, and as long as $A > 0$, there is just one solution to $A + (0.8)n^2 = n$ that lies between 0 and 1, and that is the lower root

$$n = \frac{1 - \sqrt{1 - (3.2) \times A}}{1.6}.$$

which corresponds to the situation in Panel 2. And once A goes below 0, this lower root turns negative. We still have a unique equilibrium, but now at $n^* = 0$, as in Panel 1.

Panel 3 — which is the case of multiple equilibria — corresponds to the remaining intermediate range of A, which is $0.2 \leq A \leq 1/3.2$.

(2) Suppose that each of N citizens in the country of Taxland needs to pay a tax of T every year to the government. Each citizen may decide to pay or to evade the tax. Assume that each citizen is risk-neutral and simply seeks to maximize expected income net of any taxes or fines.

If an evader is caught, Taxland imposes a fine of amount F, where $F > T$. However, the Government of Taxland has limited resources to detect evaders. Assume that out of all the people who evade taxes, *the government has the capacity to audit a maximum of m of them, where $1 \leq m < N$.*

Distinguish between two cases: In Case 1, m persons are randomly chosen from the population to be audited. If any of these is an evader, then conditional on being audited, he will be fined for sure. In Case 2, each evader — and no one else — has an anomaly on his tax return which alerts the government. Up to m of them will be randomly audited — and fined.

(a) Write down the expected payoffs in each case from evasion, and show that there are no complementarities in Case 1, while there are complementarities in Case 2.

If I am an evader, then in Case 1, I will be caught with probability $p = m/N$, where N is the total population. In Case 2, the probability of being audited is m/n, where n is now the number of actual evaders (counting me). Well, it is slightly more complicated than that, because if $m \geq n$, then I will be audited with probability 1, and otherwise with probability m/n, so in the case the exact description is that $p = \min\{m/n, 1\}$. E.g., if $n = 3$ and $m = 1$, then there are three evaders and the chance of my getting caught is one out of three or $1/3$. If $n = 2$ and $m = 3$, then I will be audited for sure.)

Anyway, denote the audit probability in either Case by p. If I am not caught, then I pay nothing. But if I am caught, then I pay a fine of F. Thus my expected payout is p times

F. As a potential evader, I will compare this loss with the sure payment of T (if I do not evade), and take the course of action that creates smaller losses. It's a matter of comparing T to pF, and choosing the smaller expected payment. Notice that p could be endogenous and that is what this question is all about.

Notice that in Case 1, this value is independent of the number of evaders n, so that there are no complementarities (the net payoff from evasion is constant). But in Case 2, *p falls with the number of evaders n, at least when $n \geq m$.* This is a case of complementarities: if one person becomes an evader, she makes it easier for other people to evade. This is because the probability of getting caught comes down, so that the expected losses from evasion come down as well.

(b) Contrast Case 1 and Case 2. In Case 1 show that there is typically a unique equilibrium. In Case 2, show that it is always an equilibrium for nobody in society to evade taxes. Are there another equilibria as well? Describe parametric configurations of (m, n, T, F) for which such equilibria will exist, and also describe the equilibria.

In Case 1 there is a unique equilibrium, yielding full evasion or no evasion depending on whether $T - (mF/N)$ is positive or negative. (There are only multiple equilibrium outcomes in the knife-edge case in which $T = mF/n$.) In Case 2, to see that "no evasion" is an equilibrium, suppose that nobody in the economy is evading. You are a potential evader. If you pay your taxes you will pay T. If you evade, then $n = 1$, while $m \geq n$, so p *has* to be 1, which is just another way of saying that you will be caught for sure. So that your expected loss is simply F. But $F > T$ by assumption. It follows that if nobody else is evading, you won't evade either. The same mental calculation holds for everybody, so that "no evasion" all around is an equilibrium.

What about everybody evading? Suppose that this is indeed happening around you, and you are considering evasion. If you do evade, then $n = N$, so that your expected losses are mF/N. It follows that if $mF/N < T$, you will jump on the bandwagon and evade as well. Thus "widespread evasion" is *also* an equilibrium provided that the condition $T > mF/n$ holds.

(3) The country of Skillover has the good fortune of generating spillovers in skills! Each citizen in Skillover simultaneously decides whether to acquire "high skill" at cost $c > 0$, or remain low-skilled. Let y_h and y_ℓ denote the incomes earned by high- and low-skilled workers. Now here comes the spillover:

$$y_h = y_h^0 + nH \text{ and } y_\ell = y_\ell^0 + nL,$$

where $y_h^0 > y_\ell^0$ are baseline values for the two incomes, H and L are positive constants, and n is the fraction of the population that chooses to become high skilled. Thus in Skillover, a person's productivity in both kinds of jobs is positively linked not only to her own skills, but also to the skills of her fellow workers.

(a) There are always positive spillovers ($H > 0$ and $L > 0$) but when exactly is there a *complementarity*?

Our formulation captures the following idea: a person's productivity is positively linked not only to his own skills, but also to that of his fellow workers. But more than that is true: note that $y_h - y_\ell = (y_h^0 - y_\ell^0) + n(H - L)$, which means that the difference between the incomes from low and high skills *widens* with more people acquiring high skills, as long as $H > L$. In this case, it follows that whenever a person chooses to acquire skills, she increases the return to skill acquisition by everybody else. This is precisely the complementarity that underlies any coordination problem. So the answer is: there is a complementarity if $H > L$.

If $H \leq L$, there is still the spillover created by education but it does not generate a complementarity. If H is smaller than L, then the gap between high-skill and low-skill income actually *narrows* as n goes up, leading to people wanting to take less education as the education around them gets larger.

For the next question, define $\Delta \equiv y_h^0 - y_\ell^0$ to be the "baseline difference" between skilled and unskilled incomes.

(b) Show that if $\Delta < c < \Delta + (H - L)$, there are three possible equilibria: one in which everybody acquires skills, one in which nobody does, and a third in which only a fraction of the population becomes high-skilled. Give an algebraic expression for this fraction in the last case. Explain why this equilibrium is "unstable" and is likely to give way to one of the two extreme cases.

Assume that $\Delta < c < \Delta + (H - L)$. First let us see if "no skill acquisition" can be an equilibrium. To this end, suppose that no one in society is acquiring skills: then $n = 0$. If you are thinking of becoming high-skilled, then the gain in your income is $y_h - y_\ell$, which is just Δ (because $n = 0$). If $\Delta < c$ (which is assumed, see above), then it is not worthwhile for you to acquire skills. We have thus shown that if everybody believes that nobody else will acquire skills, then no one will indeed acquire skills. Such beliefs therefore form a self-fulfilling prophecy.

Now let us see if universal skill acquisition can be an equilibrium. Suppose that you believe that everybody else will acquire skills: then $n = 1$. Therefore, if you are thinking of becoming high-skilled, then the gain in your income is $y_h - y_\ell$, which is $\Delta + (H - L)$ (because $n = 1$). If $\Delta + (H - L) > c$ (which is assumed — see above) then it is worthwhile for you to acquire skills. We have thus shown that if everybody believes that everybody else will acquire skills, then everyone will acquire skills. These beliefs also form a self-fulfilling prophecy.

Finally, there is a third equilibrium in which just the right amount of people invest in skill acquisition so that everybody is indifferent between acquiring or not acquiring skills. This is given by a fraction of skilled people n^* such that $\Delta + n^*(H - L) = c$, This is an equilibrium because no one is doing anything suboptimal given his or her beliefs. But you can intuitively see why this equilibrium must be unstable. If for some reason the fraction of skilled people exceeds n^* even by a tiny amount, then it becomes strictly preferable for everyone else to acquire skills, so that we rapidly move to the universal skills equilibrium. If on the other hand, n falls below n^* (if only by a tiny amount), everyone will desist from acquiring skills, so that we move towards the no skills equilibrium.

(c) Describe the set of equilibria when $H < L$.

When $L > H$, this means that low-skill income is more responsive to the fraction of high-skilled people, than high-skill income. The condition $y_h - y_\ell = [y_h^0 - y_\ell^0] + n(H - L) = \Delta + +n(H - L)$ is now seen to yield the opposite conclusion: that there will be a unique equilibrium. To see this, first suppose that even when everyone is skilled, the income gap exceeds the cost; that is:

Case (i). $\Delta + (H - L) \geq c$. Then, remembering that $H - L < 0$, it must be that for every $0 \leq n < 1$,

$$y_h - y_\ell = \Delta + n(H - L) > c,$$

and so the unique equilibrium is that everyone acquires skills.

Now consider the opposite situation, that even when no one is skilled, the income gap between high and low skilled people is too low:

Case (ii). $\Delta \leq c$. Then, again remembering that $H - L < 0$, it must be that for every $0 < n \leq 1$,

$$y_h - y_\ell = \Delta + n(H - L) < c,$$

and so the unique equilibrium is that no one acquires skills.

Finally, look at the intermediate case:

Case (iii). $\Delta > c > \Delta + (H - L)$. Then it is easy to see that there is once again a unique equilibrium at n^* skilled people, where n^* is such that

$$c = \Delta + n^*(H - L).$$

In this case there cannot be any multiple equilibrium, for exactly the same reason as the traffic congestion example in the text cannot exhibit multiple equilibria.

(d) Consider another variation, this time with no skill spillover onto incomes. Specifically, suppose that $y_h = y_h^0$ and $y_\ell = y_\ell^0$ are both fixed and independent of n (that is, $H = L = 0$). However, suppose that the *cost of individual education* is indeed affected by the presence of skilled people $c(n) = (1 - n)/n$. (The idea here is that it is easier to learn if there are more educated people around). Describe the set of equilibria.

In this case, note that the cost of acquiring skills becomes infinitely high as n becomes close to zero, while the cost declines to near zero as n approaches one. Thus we see again that there are three equilibria. In the first, there is no skill acquisition because everyone, expecting that there is no skill acquisition, feels that the cost of acquiring high skills will be very high, and so desists from doing so. At the same time, the expectation that everyone acquires skills is also a self-fulfilling prophecy, because in this case the cost of education is very low. And there is a third equilibrium where people are indifferent between the two options. Just as in part (a), this equilibrium must be described by the condition that $y_h - y_\ell = y_h^0 - y_\ell^0 = \frac{1-n^*}{n^*}$.

(4) Multiplania is a community that wants to recycle its glass and plastic, and encourages its residents to do so. But you have to take the trash pretty far to find recycling bins, so it is inconvenient to do so. Everyone has a benefit $b > 0$ from just throwing all glass and plastic into the garbage.

On the other hand, anybody who doesn't recycle can be *seen* not doing so, and there is an individual stigma or shame attached to it, a cost $s(n)$, where n is the overall fraction of the community who actually recycle. Assume that $s(n) = a + cn$, where $a \geq 0$ and $c > 0$.

The payoff from not recycling is the benefit minus the shame. The payoff from recycling is just taken to be zero.

(i) Interpret the function $s(n)$. In particular, why should c be positive? What about a?

The function $s(n)$ is increasing in n if $c > 0$. (You could draw a picture.) It means that if more people recycle, the (fewer) people who don't would stand out more easily and in the eyes of the community there would be a bigger sense of shame. If everyone is flouting the norm, then the sense of shame would be pretty small. We have still allowed for people to feel a "baseline" sense of shame a, which may or may not be zero.

(ii) Describe the range of values of (a, b, c) for which there is a unique equilibrium.

For there to be a unique equilibrium, the function that maps expected number of recyclers n into the *actual* number of recyclers x must lie everywhere above or everywhere below the 45^0 line. For the first, we need

$$b < s(n) = a + cn$$

for all n. In that case, $x = 1$ no matter what the value of n, and everyone recycles. But for the above to hold for *every* n, it is necessary and sufficient that it holds at $n = 0$. That implies $a > b$, which is the required condition.

For the second case, we need

$$b > s(n) = a + cn$$

for all n. In that case, $x = 0$ no matter what the value of n, and no one recycles. But for the above to hold for *every* n, it is necessary and sufficient that it holds at $n = 1$. That means $b > a + c$, which is the required condition.

(iii) Describe the range of values of (a, b, c) for which there is are multiple equilibria, and describe all the equilibria, including the unstable one.

In the remaining region; that is, whenever

$$a \leq b \leq a + c,$$

there are multiple equilibria. Now there is one equilibrium where no one recycles — after all, if $n = 0$, then x is also equal to zero because $b \geq s(0) = a$. There is another where everyone recycles — after all, if $n = 1$, then x is also equal to 1 because $b \leq s(1) = a + c$. Assuming that strict inequality holds everywhere above (it will get you a bit of extra credit if you make this point clear), there is a third equilibrium located at the point n^* where

$$b = s(n^*) = a + cn^*,$$

or equivalently, at

$$n^* = \frac{b - a}{c}.$$

This equilibrium is always unstable. Draw the picture.

(iv) In the world of part (iii), consider a policy that imposes a fine $F > 0$ on every garbage thrower. Show that the threshold value for the population share of recyclers that tips the society over into the good equilibrium (i.e., the unstable equilibrium value) must fall with the fine.

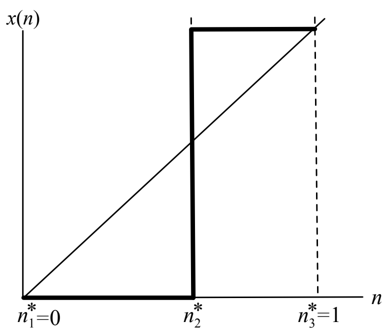
With a fine F, the net payoff from *not* recycling is given by

$$b - s(n) - F = b - a - cn - F.$$

A bad equilibrium in which $n = 0$ can still be supported, therefore, if when $n = 0$, everyone derives a net benefit from not recycling; that is, if

$$b - a - F > 0.$$

So the situation looks as in this diagram:



and the unstable threshold is given by the point n_2^* where the net payoff from recycling is exactly zero; that is, $b - a - cn - F = 0$, or

$$n_2^* = \frac{b - a - F}{c}.$$

It is easy to see from the above equation that as we increase the fine, the threshold population needed for the unstable tip-over to recycling becomes lower.

(v) Using part (iv), can you discuss a situation where we begin with a bad equilibrium (i.e., one in which no one recycles), impose a fine, then remove the fine after a temporary imposition, with the society now in a good equilibrium with recycling?

Suppose that we are in the situation of part (ii), where we have multiple equilibria but are currently stuck in the bad equilibrium with $n^* = 0$. Choose a fine F such that $F > b$. Now there is no reason to rely on shame, because the fine is higher than the benefit from throwing garbage, so now everyone will recycle. After this is in place, remove the fine. The equilibrium with $n^* = 1$ will stay in place without the fine. Only if many people simultaneously start throwing garbage might society move back to the equilibrium where everyone throws garbage on the street. In this sense, temporary policy interventions can have a permanent effect.

(5) In the question that follows, n refers to the *number* of people rather than a fraction of the population. In the land of Pampa, living in the countryside gives you a fixed payoff of 100 (Pampa has lots of land), while living in a city gives you a payoff that first increases with

the number of people living in the city (agglomeration), and then declines after the number of people goes above a certain threshold (congestion). Let us write this payoff as

$$r = 20n - n^2/2,$$

where n is the number of city dwellers in that particular city.

(a) Let N be the total population in Pampa. If only one city can exist in the entire country, trace out the set of equilibria (i.e., population allocations between countryside and city) as N varies from 0 to infinity.

Suppose there is one city. Notice that r is maximized by setting $n = 20$ (write down the maximization problem for r and set the derivative with respect to n to equal zero). It follows that the maximum value of r is given by $20^2 - 20^2/2 = 200$. But of course, the *actual* value of r will depend on how many people choose to live in Pampa. To find this, look at the intersections of the line $20n - n^2/2$ with the value 100. That means you solve the quadratic equation

$$20n - n^2/2 = 100, \text{ or } n^2/2 - 20n + 100 = 0,$$

which has the pair of solutions

$$n_1 \text{ and } n_2 = 20 \pm \sqrt{400 - 4 * (1/2) * 100}, \text{ or } n_1 = 20 - \sqrt{200} \text{ and } n_1 = 20 + \sqrt{200}.$$

Note that if the population N is less than n_1, *there is just one equilibrium*, everyone in the country. If $N > n_1$, but no bigger than n_2, then there are three equilibria.[1] The two stable ones involve everyone being in the city, or everyone being in the country. There is just one more equilibrium in which the two returns have to be equalized, which means that $n = n_1$ and the the rest, which is $n - n_1$, live in the country. Draw a diagram to show that this is unstable.

If $N > n_2$ then again there are three equilibria. One is everyone in the country and one is unstable (as usual). The third involves n_2 people in the city and $N - n_2$ in the country.

(b) Now suppose that new cities can come up, each yielding exactly the same payoff function as above. Focus on the equilibrium in each case with the maximum possible city dwellers, and explain how this equilibrium will move with the overall population N.

If there is room for several potential cities the additional cities come into play only in the case that $n > n_2$. There are now several equilibria. Let's focus on the one with the most city dwellers. Let M be the largest integer such that $N \geq Mn_2$. If the difference $N - Mn_2$ is smaller than n_1, then there are M cities that will be active in (the maximum-city-dweller) equilibrium, each housing n_2 people and the the rest living in the country. If the difference $N - Mn_2$ is larger than n_1, then there are equilibria with $M + 1$ cities, and no one living in the country.

(6) Let us suppose that people are arrayed in order of decreasing honesty, and that the "honesty payoff" to person i, where i is an index between 0 and 1, is given by $h(i) = 1 - ai$, where i runs from 0 to 1, and a is a positive parameter. This is the direct payoff person i

[1] We left out the knife-edge case in which N *just* happens to be equal to n_1, in which case there are only two equilibria, either $n = 0$ (stable) and $n = n_1$, which is unstable.

gets from being honest in a particulate situation. But she also gets a cheating payoff equal to bn, where n is the fraction of individuals being dishonest in that situation, and $b > 0$ is a positive parameter. Each person has just two choices, to be honest or dishonest, receiving the honesty payoff if she takes the honest action and the cheating payoff if she takes the dishonest action.

Describe the equilibria of this model for different values of the parameters a and b.

Note that in any equilibrium, by the way we have ordered people in order of decreasing honesty, it must be that if i is honest, so is j, where $j < i$ in the index. Now suppose that the fraction of dishonest people is n; that means from the above argument that $1 - n$ is the index of the "last" honest person. The honesty payoff of that threshold person is $1 - a(1-n)$, and her cheating payoff is bn.

Can we get an equilibrium in which everyone is honest? That is possible if $n = 0$, which requires $1 - a(1 - n) \geq bn$ evaluated at $n = 0$, or $a \leq 1$. (Interpret this.)

Can we get an equilibrium in which everyone is dishonest? That is possible if $n = 1$, which requires $1 - a(1 - n) \leq bn$ evaluated at $n = 1$, or $b \geq 1$. (Interpret this.)

Now you can write down conditions for unique and multiple equilibrium. Remember that there could also be an interior equilibrium, in which $0 < n^* < 1$. That requires the condition $1 - a(1 - n^*) = bn^*$, or $n^* = (1 - a)/(b - a)$. Note that this can only exist if *both* $a < 1$ and $b > 1$ (why?). Argue that this is unstable for the usual reasons.

(7) Suppose that people's attitudes can take three possible positions: L, M, and R, where you can think of L as leftist, R as rightist, and M as middle-of-the-road. Consider a society in which *it is known that* a fraction α are M types, and the remaining fraction $1 - \alpha$ are divided equally between L and R, but no individual is known to be L, M, or R at first sight. Suppose that each individual gets satisfaction S from expressing his or her own true views, but feels a loss ("social disapproval") in not conforming to a middle-of-the-road position. The amount of the loss depends on the fraction α of M types: suppose that it equals the amount $\alpha/(1 - \alpha)$.

(a) Show that there is a threshold value of α such that everybody in society will express their own view if α is less than the threshold, but will all express M-views if α exceeds the threshold.

The gain from being your own self is S. If you are an L-type or an R-type, you will also feel a loss equal to $\frac{\alpha}{1-\alpha}$. Therefore the net gain from being your own type L or R is $S - \frac{\alpha}{1-\alpha}$. This is negative if $\alpha > \frac{S}{1+S}$. Above this threshold, everybody will say that they are type M, and below that everyone will freely report their own view.

(b) What happens if we change the specification somewhat to say that the "social disapproval loss" equals $\beta/(1 - \beta)$, where β is the expected fraction of people who *choose* to express M views (and not necessarily the true fraction of M types)?

In this case, there are two possibilities. First, assume that $\alpha > \frac{S}{1+S}$, the threshold derived in part (a). Note that a fraction α (the true M-types) will always say that they are type M, because they have nothing to gain by stating any other position. But by part (a), the other types will hide their identity, which raises the value of β (the announced M-types) above the value of α. This process can only stop when everybody announces that they are type M. On the other hand, if $\alpha \leq \frac{S}{1+S}$, there is an equilibrium in which everybody announces their true type, and so $\beta = \alpha$. You can check that nobody will want to deviate from their announcements. But at the same time, there is another conformity equilibrium in which everybody announces that they are type M (and in which β takes on the value of one).

(c) Indicate how you would extend the analysis to a case in which there are potential conformist urges attached to each of the views L, M, and R, and not just M.

If there are potential conformist urges attached to each of the views L, M and R (and not just M), then other equilibria appear. There may be conformist equilibria in which everybody announces L, or in which everybody announces R (try and provide a simple algebraic example of this).