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ECON-UA 323

Development Economics

Outline of Answers to Problem Set 1

(1) A traded good is one that can be bought and sold through the international market. A nontraded good cannot. Of course, these are extreme descriptions of reality, and some goods may be partially but not fully tradable.

Equilibrium exchange rates are determined by the supply of and demand for a country's currency. The supply of a country's currency is determined by that country's purchases of imports on the world market. The demand for its currency is determined by the purchase of that country's exports by the world. The exchange rate acts to equalize these two (thereby creating trade balance).

(a) If all goods are perfectly traded, then the domestic prices of all goods would be equal to the world prices. Income differences expressed in currency units would perfectly mirror the inequality of real income.

(b) This is no longer true with non-traded goods because their prices will not equalize around the world. The exchange rate across two countries is only brought into line using the supply of and demand for *traded* goods. There is no reason why that should equalize the prices of non-traded goods.

But we can say more. A rich and productive country is likely to have a stronger currency and a higher income. The higher income, in turn, pulls up the prices of those products within the country that are nontraded. Thus measured in terms of exchange rate income, a rich country looks richer than it really is, because, we are not accounting for the fact that it faces (on average) higher domestic prices for the nontraded goods. This is why PPP measurements typically bring down the relative income of a rich country, and pull up the relative income of a poor country.

(c) The price of a Big Mac is, to a large extent, determined by the prices in competing restaurants. Thus Big Macs will sell for a higher price in rich countries (where nontraded restaurant prices are likely to be higher). Thus Big Mac prices incorporate, to some extent — and often to a better extent than exchange rates do — the "true" cost of living within a country. Therefore, using the relative prices of Big Macs to create "exchange rates" across country currencies will often serve as a good approximation of relative PPP income.

(d) The effect of inequality on the prices of nontraded goods is complex and ambiguous. If, in a very unequal society, those goods are produced by relatively poorer people (such as haircuts, or basic services such as housecleaning or tourism), these prices are going to be lower, because the people who supply those goods earn a lower income relative to the average population. But for those nontraded goods provided by relatively rich people — high-end restaurants, housing, or certain specialized non-traded services, prices are likely to be bumped up in a high-inequality society.

On the whole, the net effect will depend on exactly which goods are included in the PPP correction.

(2) The setting up of infrastructure or industrial standards involves a "sunk cost" and a "variable cost". The sunk cost is the cost of the entire infrastructure: in the example of tv systems, this would be all kinds of interconnected senders and receivers that broadcast at a particular type and refinement of resolution. The variable cost refers to the individual purchase of tv sets. Now given that the infrastructure is not set up for it, it might make little sense for customers to buy high-definition tv sets. Moreover, given the huge sunk costs, countries which have already invested in a particular standard may not want to tear up this standard and start all over again, unless the new technology is much, much better. However, a country that is starting afresh will obviously want to use the latest technology: there is no past, no sunk cost, to be borne in mind. This is why countries which have been early innovators are often saddled with older systems, and newcomers can leapfrog over them with the newer technology. Television and mobile phone systems are only two examples of these.

In the problem that accompanies the question, the value of the existing technology is xT (without any discounting). The value of the new technology is yT, and net of cost it is $yT - C$. Therefore the new technology will be installed if $yT - C > xT$. Clearly, the larger the current value of x the less likely it is that this inequality will hold, so it is entirely possible that a country with $x = 0$ today will adopt while a country with $x > 0$ today will not.

Here is the specific example that I am looking for. Let's suppose that the life of the investment is 10 years, that $y = 10,000$ per year, and the setup cost is 60,000. Then a country which is *already* generating $x = y/2$ will already have 5000 a year in social value. Then that country will not want to switch to the new technology, but a country that is generating a small value, say $x = 100$, will.

(3) (a) You will have to know how to do the compounding, which gives us $(1.3)^{12} = 23.29$, or 2329% per year

(b) For the country growing at 5%: It doubles its income in t years, where $2 = (1 + 0.05)^t$. Solve to get $t = 14$. (For an exam you won't be able to do this without a calculator, so you should know the doubling time formula done in class.)

The country quadruples its income in $t = 28$.

Similarly for the country growing at 10%: it doubles, using our formula, in $70/10 = 7$ years and quadruples in $t = 14$ years.

(c) If the population growth rate comes down by 1% a year, but overall income continues to grow at the same rate, the country's per-capita growth increases from 5% per year to 6% per year for 20 years.

Now, at 5% a year for 20 years the country's income would swell to a factor of $1.05^{20} = 2.65$. At 6% it would be $1.06^{20} = 3.21$. So it would be richer by a factor of 3.21/2.65, which is about 1.21 or 21% richer.

(4) (a) If growth has been steady at 2% per year over the last t years and has arrived at 50,000 today, then income t years ago must have been y, where y solves the equation

$$y\left(1+\frac{2}{100}\right)^t = 50,000, \text{ or } y = \frac{50,000}{1.02^t}.$$

Setting $t = 100$, we see that income was

Income t years ago given by $50,000(1-\frac{2}{100})^t$, or $50,000)(\frac{98}{100})^t$. 100 years ago, it is 6902, 200 years ago 953, and 300 years ago it would be just 131! That proves how recent the phenomenon of economic growth is, even at a relatively low rate of 2% per year.

(b) If a country grows at 2% one year and shrinks by 2% the next it would seem to have constant income over time. But no, per-capita income will fall over the long run. Over every pair of years,

$$y_{t+2} = y_t(1.02)(0.98) = (0.9996)y_t$$

so there is a slight drop every pair of years. Over time this will amplify; e.g., a country that did this for 1000 years would then shrink by a factor of 0.9996^{1000}, which is a loss of 1/3 of its starting income! So fluctuations are bad for you.

On the stock market, for instance, if you were an ace trader making 100% in one year and then losing "only" 50% the next year, you would be barely breaking even over the long run. Don't let those hotshots fool you!

(c) A country that grows at 2% a year would have a growth factor of 1.02^t in t years, while one that grows and shrinks as describes would have a factor of $[(1.07).(0.97)]^{t/2}$. So the ratio of the steady to the fluctuating country's incomes is given by

$$\frac{1.02^t}{[(1.07).(0.97)]^{t/2}} = \left(\frac{1.02}{\sqrt{(1.07)(0.97)}}\right)^t.$$

In 100 years, the steady country would be over 13% richer.

(5) No mobility

	1	2	3	4
1	100%			
2		100%		
3			100%	
4				100%

Full Mobility

Poor countries grow, on average, faster than rich countries

	1	2	3	4
1	25%	25%	25%	25%
2	25%	25%	25%	25%
3	25%	25%	25%	25%
4	25%	25%	25%	25%

	1	2	3	4
1	10%	45%	30%	5%
2	10%	40%	40%	10%
3	10%	15%	50%	25%
4	5%	10%	45%	40%

(6) If x is the share of people in manufacturing, then total income per person is given by

$$10x + 5(1-x),$$

so that the share of the top 20% is given by

$$\begin{aligned} s_{20}(x) &= \frac{10x + 5[0.2 - x]}{10x + 5(1-x)} \text{ when } x < 0.2 \text{ (or 20\%)} \\ &= \frac{10 * (0.2)}{10x + 5(1-x)} \text{ when } x \geq 0.2. \end{aligned}$$

If you plot this you will see that the share of the top 20% rises and then falls. Similarly, you should write down the share of the bottom 40%, which is

$$\begin{aligned} s_{40}(x) &= \frac{5 * 0.4}{10x + 5(1-x)} \text{ when } x < 0.6 \text{ (or 60\%)} \\ &= \frac{10[x - 0.6] + 5(1-x)}{10x + 5(1-x)} \text{ when } x \geq 0.6. \end{aligned}$$

and you can show the opposite pattern: this share falls and then rises with x.

7 (a) Suppose that Scrooge's wealth is W_t in year t. Suppose that he saves a positive fraction s of his total income (capital and labor) in that year, which is $rW_t + y$. Then the addition to his wealth in that year is precisely $s[rW_t + y]$, and it follows that

$$W_{t+1} = W_t + s[rW_t + y] = (1 + sr)W_t + sy. \tag{1}$$

(b) There is a nice formula that will allow us to precisely explain how W_t changes over time. It will take a little patience to do, but bear with me and you will learn something. To begin, just divide on both sides of equation (1) by the value $(1 + sr)^{t+1}$, then you get

$$\frac{W_{t+1}}{(1+sr)^{t+1}} = \frac{W_t}{(1+sr)^t} + \frac{sy}{(1+sr)^{t+1}}.$$

Now we can easily telescope this sum all the way back to $t = 0$ to see that

$$\frac{W_{t+1}}{(1+sr)^{t+1}} = \frac{W_0}{(1+sr)^0} + sy \sum_{k=1}^{t+1} \frac{1}{(1+sr)^k} = W_0 + \frac{y}{r} \left[\frac{(1+sr)^{t+1} - 1}{(1+sr)^{t+1}} \right],$$

where the second equality uses a familiar sum of a geometric series that you should know. Now multiply throughout by $(1 + sr)^{t+1}$ to get

$$W_{t+1} = W_0(1 + sr)^{t+1} + \frac{y}{r}\left[(1 + sr)^{t+1} - 1\right]$$

$$(2) \qquad = \left[W_0 + \frac{y}{r}\right](1 + sr)^{t+1} - \frac{y}{r}.$$

(c) Now we write this same equation for Scrooge and Stooge, but we will distinguish the two using the superscript 1 for Scrooge and 2 for Stooge. Dividing one of these equations by the other, we have

$$(3) \qquad \frac{W^1_{t+1}}{W^2_{t+1}} = \frac{\left[W^1_0 + \frac{y^1}{r^1}\right]\left[(1 + s^1r^1)^{t+1} - \frac{y^1}{r^1}\right]}{\left[W^2_0 + \frac{y^2}{r^2}\right]\left[(1 + s^2r^2)^{t+1} - \frac{y^2}{r^2}\right]}$$

For this part, you are asked to assume that starting wealths at date 0 are the same, incomes are the same, savings rates are the same, but $r^1 \neq r^2$ (say $r_1 > r_2$ for concreteness). Then as time goes on, the geometric term

$$\left[\frac{1 + s^1r^1}{1 + s^2r^2}\right]^{t+1}$$

is exploding to infinity as $t \to \infty$. So, using equation (3), the ratio of Scrooge's wealth to Stooge's wealth must go to infinity in this case. (The extra terms y^1/r^1 and y^2/r^2 in equation (3) become insignificant relative to the geometric terms and do not affect this conclusion.)

(d) In this part, you are asked to only assume that Scrooge has twice the labor income of Stooge. Now the two terms $(1 + s^1r^1)^{t+1} = (1 + s^2r^2)^{t+1}$ *separately* explode to infinity, but of course their ratio remains at 1, because they are equal ($s^1 = s^2$ and $r^1 = r^2$). Again, because the extra terms y^1/r^1 and y^2/r^2 in equation (3) become insignificant relative to the geometric terms, the long-run ratio of W^1_{t+1} to W^2_{t+1} is given approximately by

$$\frac{W^1_{t+1}}{W^2_{t+1}} \simeq \frac{\left[W^1_0 + \frac{y^1}{r^1}\right]}{\left[W^2_0 + \frac{y^2}{r^2}\right]} = \frac{\left[W_0 + \frac{2y}{r}\right]}{\left[W_0 + \frac{y}{r}\right]}$$

using $W^1_0 = W^2_0 = W_0$, $y^2 = y$, $y^1 = 2y$ and $r^1 = r^2$. Of course the last expression above is bigger than 1 but unlike the case of part (c), it does not become unboundedly large.

(e) The difference between the results in (c) and (d) is important. In part (c), the *rate of returns* are different and this difference compounds over time, resulting in an unbounded divergence in wealth ratios. In contrast, in part (d), the rates of return and savings rates are the same, and the incomes are different. But this last difference has only an *additive*, not geometric or compounded, effect on wealths. So while the wealth of the high-income person is (of course) higher, the wealth ratios do not diverge in some unbounded way.