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ECON-UA 323

Sample Examination 3

NO CALCULATORS, IPADS, LAPTOPS, ETC., ALLOWED. PUT THEM AWAY, PLEASE.

Points 65. Time 75 minutes. The first question carries 30 points; and the second and third 16 points each. 3 points are reserved for extra credit, presentation and clarity. You'll have to grade yourself on it all, including the last!

Guide for Time Allocation: The questions in (1) should take no more than 5 minutes each to answer; total 30 minutes. Questions (2) and (3) should take you no more than 15 minutes each. This schedule will allow you to finish the exam in 60 minutes. If you are stuck with a question, move on to the next one and plan to come back later. Keep your answers brief and to the point.

(1) (30 points, 6 points per part, 5 parts) Are the following statements true, false, or uncertain? In each case, back up your answer with a brief, but precise explanation.

[a] In the tea plantation application studied in class with piece rates, the observed behavior agreed with the predictions of the estimated model immediately after the contract, but not as time went by.

[b] In credit markets with adverse selection, borrower 1 with a risky project but the same mean as a safer borrower 2 could *strictly* prefer to take a loan at the going rate of interest, while borrower 2 does not.

[c] In a casual labor market with a base wage and piece rates, an increase in the base wage will *increase* labor hours, assuming that her "alternative activity" (e.g., leisure) is a normal good.

[d] In the moral hazard model of borrowing, with competitive lenders, a decrease in borrower collateral must decrease the social surplus.

[e] In the model of permanent labor, an increase in the casual wage everywhere in the market must give rise to an increase in the permanent wage.

(2) (16 points) João and José are two brothers who each borrow an amount 100 from a microfinance lender, at an interest rate of r. They have identical projects. Each project succeeds or fails with *independent* probability, which they control by means of their independent efforts. They have no collateral for their loans. All parties are risk-neutral.

For each brother João or José, the situation is as follows: Their projects generate a return of 220 with probability p, and fail completely with probability 1 - p. The value of p is

independently chosen by each brother for their project. But higher p is costly; the cost is given by $60p^2$.

(a) [4 points] Consider either brother, and carefully describe his maximization problem at interest rate r. You can assume that the interest rate is not too high, so that the loan plus interest can be fully repaid in the event of success. Show that the brother's optimal choice of success probability is given by

$$p = 1 - \frac{5}{6}r,$$

and explain intuitively why p is declining in r.

(b) [4 points] Assuming the answer from part (a), explain what a profit-maximizing microfinance company will choose to maximize, and show that its optimal rate of interest r is given by 10%. Explain intuitively the pros and cons to the company of setting a higher interest rate.

(c) [4 points] Now suppose that the company lends the startup 100 each to João and José, but makes them *partly liable* for each other's loan. For instance, from João's perspective, we presume that he repays his loan if his project is successful, and in addition must repay half of José's loan if José's project is unsuccessful. (Likewise, he will get partly bailed out by José if his project fails but José's project is successful.) Again assume that the interest rate is never so high that each brother can't pay their debts when successful; they can.

If João's probability of success is denoted by p_1 and José's probability by p_2 , write down a new formula for João's expected return, and show that

$$p_1 = \frac{11 - 5(1+r)[1 + \frac{1}{2}(1-p_2)]}{6}$$

(d) [4 points] Assuming the answer from part (c), describe how p_1 behaves as a function of p_2 and explain your answer intuitively. Does this mean that the microfinance company is worse off by introducing "group liability"?

(3) (16 points) Pedro leases a plot of land from his landlord, Manisha, who offers him a sharecropping contract: she gets a share σ of output and Pedro keeps the remaining share $1 - \sigma$. Pedro farms the plot using his labor e, which has an opportunity cost of w per unit, and with tractor input t, which he leases in at r per unit. The production function is given by

$$y = f(e, t) = 2[e^{1/2} + t^{1/2}].$$

(a) [5 points] For any share σ between 0 and 1, describe Pedro's maximization problem, and show that

$$e = \left(\frac{1-\sigma}{w}\right)^2$$
 and $t = \left(\frac{1-\sigma}{r}\right)^2$,

so that

$$y = 2\left[\frac{1-\sigma}{w} + \frac{1-\sigma}{r}
ight].$$

(b) [6 points] Using the information from part (a), show that Manisha would ideally like to set $\sigma = 1/2$.

(c) [5 points] Suppose that Manisha can share the costs of *both* Pedro's labor and his tractor use, if she wishes. Suppose that Pedro must be given some minimum net income in order to participate. Study what kind of contract you think Manisha will now offer.

You can end the exam here.

Two more optional variations on question (3): you might want to look at them.

(d) Suppose that Manisha is required by law to pay a fraction 0 < s < 1 of the tractor rental fee. Show that her income from leasing out the plot is given by

(1)
$$2\sigma(1-\sigma)\left[\frac{1}{w} + \frac{1}{r(1-s)}\right] - \frac{s(1-\sigma)^2}{r(1-s)^2}$$

(e) Using the previous answers, prove that Manisha's new optimal share strictly exceeds 1/2.

[Hint: You do not need to compute the exact optimum share to answer this question. From Manisha's point of view, every variable in equation (1) is a parameter except for σ , so we can write her profit as $A\sigma(1-\sigma) - B(1-\sigma)^2$, where A and B are positive constants.]