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ECON-UA 323

Sample Examination 1

NO CALCULATORS, IPADS, LAPTOPS, ETC., ALLOWED. PUT THEM AWAY, PLEASE.

Points 90. Time 75 minutes. The first question carries 50 points; and the second and third 20 points each. Presentation and clarity will get you some extra credit.

Guide for Time Allocation*: The questions in (1) should take no more than 5 minutes each to answer; total 30 minutes. Questions (2) and (3) should take you no more than 15 minutes each. This schedule will allow you to finish the exam in 60 minutes. If you are stuck with a question, move on to the next one and plan to come back later.*

(1) (**50 points, 10 points per part, 5 parts**) Are the following statements true or false? It is *not* enough to just guess one or the other. You need to provide an argument for or against, and only then will any credit be awarded.

[a] In the Solow model, *faster* technical progress must *lower* steady state output per effective unit of labor, but increases output per capita.

[b] The larger the share of capital in national income, the larger is the variation in steady state incomes that can be explained by differences in savings rates in the Solow model.

[c] A Cobb-Douglas production function that has increasing returns to scale must also have increasing returns to at least one of its inputs.

[d] A complementarity map with an unstable equilibrium *must* have at least *two* stable equilibria elsewhere (it may have more).

[e] This *could be* a conceivable mobility matrix, with 4 relative income categories on the rows and columns, showing country transitions from one category to another category.

	1	2	3	4
1	80%	10%	10%	0%
2	20 %	70%	10%	10%
3	10%	10%	70%	20%
4	0%	10%	10%	80%

(2) (20 points) Verticalia is a new community in the town of Flatland that is inviting residents to move in. The cool thing about Verticalia is that everyone will live in a mega highrise and get to hang out with everyone else. This "hanging-out" gives rise to *networking payoffs*, given by

$$P = An,$$

where $A > 0$ is a positive parameter and n is the fraction of people in Flatland that move into Verticalia. But the problem is that more people moving in will also raise per-person rents in Verticalia because of congestion. Rent is given by the function

$$R = 10 + 5n.$$

Assume that the net payoff a person receives in Verticalia is networking payoff minus rent, and that there are no other costs or benefits other than these.

Meanwhile, living in Flatland *outside* Verticalia has its own payoff. We simply assume that payoff to be equal to zero for every resident.

(i) [4 points] Show that *no one* living in Verticalia; i.e., $n = 0$, is always an equilibrium, no matter how small or large the value of A is.

(ii) [4 points] Describe precisely the values of A for which $n = 0$ is the *only* equilibrium.

(iii) [7 points] If the conditions in part (ii) are not met, show that there are multiple equilibria and describe all of them, including the unstable one.

(iv) [5 points] Now change the rent function to $R = 1 + 36n^2$, and assume that $A = 20$. Show that there are now just two stable equilibria, one in which no-one lives in Verticalia, and another in which just half of them do.

(3) (20 points)

(a) [5 points]Derive the per-capita capital accumulation equation in the Solow model with Cobb-Douglas constant returns to scale production function and *without* technical progress, and show that it leads to a unique steady state.

(b) [5 points] Adapt the technology so that you can move to the Harrod-Domar model. Use a version of the Solow steady state diagram to show that (barring some knife-edge values of the parameters) either per-capita capital grows without bound or it shrinks all the way down to zero.

(c) [5 points] Define $g(t)$ to be the growth rate of per-capita capital, and prove that

$$sA = (1 + n)(1 + g) - (1 - \delta),$$

where the parameters in the equation have their usual meaning.

(d) [5 points] Describe any one way in which the Harrod-Domar model yields different results from the Solow model.