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Econ-UA 323

Examination 2, Fall 2023

No calculators, Ipads, laptops, etc., allowed. Put them away, please.

Points 65. Time 75 minutes. The first question carries 30 points; and the second and third 16 points each. 3 points are reserved for extra credit, presentation and clarity.

Guide for Time Allocation: *The questions in (1) should take no more than 5 minutes each to answer; total 30 minutes. Questions (2) and (3) should take you no more than 15 minutes each. This schedule will allow you to finish the exam in 60 minutes. If you are stuck with a question, move on to the next one and plan to come back later. Keep your answers brief and to the point.*

(1) (**30 points, 6 points per part, 5 parts**) Are the following statements true or false? It is *not* enough to just guess one or the other. You need to provide an argument for or against, and only then will any credit be awarded.

[**a**] Consider the regression $Y = A + bX + \epsilon$, and an exogenous variable I. If I is to serve as a good instrument for X, it should be correlated with X and uncorrelated (or at least weakly uncorrelated) with Y.

[**b**] Alesina, Guiliano and Nunn's paper on the effects of the plough use the actual production of plough-positive crops as an instrument for plough use.

[**c**] A project for a risk-neutral population of 100, that yields \$5 to a winner and \$-3 to a loser, with 60 winners overall, where it is also known that exactly 40 of the winners fully know their identity, but no one else does, must fail a majority vote for adoption.

[**d**] All other things being equal, it is easier to displace a product using a more efficiently produced newer version if the new product is produced under conditions of *decreasing* rather than *increasing* returns to scale.

[**e**] The coefficient of variation satisfies the relative income principle.

(2) (16 points) Doublin has four equal-sized groups of people, with individual wealth identical within a group at time t: $W^1(t) < W^2(t) < \mu(t) < W^3(t) < W^4(t)$, where $\mu(t)$ is the mean wealth. At the start of every year, a person with initial wealth W earns a rate of return of r, and then receives x from the government. A fraction s of total income (capital income plus the x) is saved and added to W to get her end-of-year wealth.

Now, Doublin is a most peculiar country in which at the end of the year, each person reproduces asexually to have two offspring. End-of-year wealth is divided equally among the two offspring, and the parent exits gracefully.

We are going to study the inequality across wealth at the start of every year. You are allowed to assume all the principles we studied, with the word "income" replaced by "wealth" everywhere in those principles.

(a) [4 points] For a parent with starting wealth $W(t)$, rate of return r and government income $x(t)$ in year t, prove that the starting wealth $W(t+1)$ of *each* offspring next year is given by

$$W(t+1) = \frac{1}{2}\left(W(t) + s[rW(t) + x]\right).$$

(b) [4 points] Prove that if x is proportional to parental wealth W; that is, if $x = aW$ for some constant $a > 0$, then the inequality of starting wealth must be unchanged from year to year in Doublin.

(c) [4 points] How does your answer to part (b) change if x is the same *irrespective* of wealth?

(d) [4 points] Return to part (b), where inequality in Doublin is unchanging over time. How do you think inequality will change between t and $t+1$ if *the poorer half* of all families had two children each (with wealth split as before), and the other (richer) half had one child each? You can assume that there are only two families at date t, which morph into three families at $t+1$, that $r = 0$, that $s = 1$, and that $x = 0$.

(3) (16 points) There is a population of mass N living in the country of Nobank. Everyone chooses to be either a worker or a capitalist. A worker earns a wage of w, and a capitalist earns a profit of

$$2\sqrt{\ell} - w\ell - S,$$

where $\sqrt{\ell}$ is a production function based on laborers ℓ, w is the wage, and S is a setup cost.

(a) [4 points] If a capitalist can freely choose how much labor to hire to maximize her profits, show that

$$\ell = \frac{1}{w^2},$$

and that her profit is given by

$$\frac{1}{w} - S.$$

(In what follows you can assume these formulae, even if you could not solve this part.)

(b) [4 points] Suppose that a mass M between 0 and N of individuals get to be capitalists (and the rest, $N - M$, are workers). Show that the wage rate that equates supply to demand in the labor market must solve

$$w = \sqrt{\frac{M}{N-M}}.$$

(Again, you can assume all these formulae below, even if you could not solve this part.)

Now we suppose that a mass $\bar{M}$ of people in Nobank have high wealth and can afford the startup cost S out of their own pocket, while the remainder cannot, and they can't borrow. There are no banks in Nobank after all!

(c) [4 points] Show that the high wealth individuals will indeed *choose* to be capitalists only if

$$\sqrt{\frac{1-\bar{m}}{\bar{m}}} - S \geq \sqrt{\frac{\bar{m}}{1-\bar{m}}}.$$

where we have defined $\bar{m} = \bar{M}/N$. Interpret this result. Does the inequality above mean a small value of $\bar{m}$ or a large value of $\bar{m}$? And what does that mean from an economic standpoint?

(d) [4 points] Describe what happens if the expression in part (c) fails to hold? Does everyone with access still become a capitalist? Why or why not? And what would determine the equilibrium wage?