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ECON-UA 323

Examination 1, Fall 2023

NO CALCULATORS, IPADS, LAPTOPS, ETC., ALLOWED. PUT THEM AWAY, PLEASE.

Points 65. Time 75 minutes. The first question carries 30 points; the second 17 points, and the third 14 points. 4 points are reserved for extra credit, presentation and clarity.

Guide for Time Allocation*: The questions in (1) should take no more than 5 minutes each to answer; total 30 minutes. Questions (2) and (3) should take you no more than 15 minutes each. This schedule will allow you to finish the exam in 60 minutes. If you are stuck with a question, move on to the next one and plan to come back later. Keep your answers brief and to the point.*

(1) (**30 points, 6 points per part, 5 parts**) Are the following statements true or false? It is *not* enough to just guess one or the other. You need to provide an argument for or against, and only then will any credit be awarded.

[a] If a production function defined on three inputs (say physical capital, human capital and unskilled labor) exhibits diminishing returns to *each one* of these three inputs, then it must have decreasing returns to scale.

[b] The Harrod-Domar model is capable of generating persistent growth in the *per-capita* capital stock (and therefore in per-capita income), even without any technical progress.

[c] In the Solow model with technical change, per-capita growth could be higher than the rate of technical progress, depending on the initial income a country starts from.

[d] A country which has been growing steadily at 2% per year and now has a per-capita income of $60,000 would have a per-capita income of *approximately* $3,750, about 140 years ago.

[e] This is a possible mobility matrix across starting and ending relative income categories, with *every* category populated by *at least some* countries before and after.

	1	2	3	4
1	80%	20%	0%	0%
2	0%	0%	65%	35%
3	0%	50%	50%	0%
4	0%	0%	0	100%

(2) (17 points) The town of Maskdown has been invaded by a nasty viral disease. The chances of catching it depends on whether you are wearing a mask or not. The probability of being infected is 80% (or 4/5) if you don't wear a mask and 40% (or 2/5) if you do. Getting the disease is costly to you: denote that cost by $C > 0$.

But there are other costs as well — because there's an ongoing debate in Maskdown about wearing masks. Some will yell at you if do, and others will shame you if you don't. Let's model these "shame costs." At any point of time, say a share n wears a mask, while $1-n$ don't. The shame cost of *not* wearing a mask increases in the share of people that *are* wearing a mask, and the cost is given by $200n$. There is another "shame cost" of *wearing* a mask that increases in the share of people that are *not* wearing a mask, and the cost is given by $150(1-n)$. Alas, no matter which action you take, you are going to incur one cost or the other. That's just life in Maskdown, as in many real-world cities ...

Each person is interested in *taking the action that minimizes her overall expected cost* — that from getting the disease plus any shame cost that she incurs from her action.

(i) [4 points] Show that no matter how small C is, it is always an equilibrium for people in Maskdown to all wear masks.

(ii) [5 points] For what values of C are there also equilibria in which no one wears masks?

(iii) [4 points] In the second case, describe the "unstable equilibrium" in which some fraction of the population (which I want you to calculate) wear masks, and explain why it is unstable.

(iv) [4 points] Now suppose that in addition to all the costs described above, there is a "loss of personal freedom" cost to wearing a mask; call it F. Find a condition on C and F for which universal mask-wearing is *not an equilibrium*, in contrast to (i).

(3) (14 points) The country of Rápido has a production function which depends on capital and labor, and at any date t, it is given by

$$Y = 9K^{1/3}[e(t)L]^{2/3}.$$

where $e(t) = (1+\pi)^t$ is the exogenous productivity of labor at time t, growing at a constant rate π.

(a) [4 points] Find a formula for per-effective-capita production $\hat{y}$ as a function of the per-effective-capita capital stock $\hat{k}$, and explain the steps to get there.

(b) [5 points] In Rápido, capital has depreciation rate δ, population growth n, and savings rate s. Show that steady state path of output per-capita is given by

$$y^*(t) = 27\left(\frac{s}{\delta+\pi+n}\right)^{1/2}(1+\pi)^t,$$

describing precisely all the steps that lead to this conclusion.

(c) [5 points] Now set technical change and depreciation both equal to zero in Rápido. Show that the coefficient on log savings rate is equal in magnitude to, but has the opposite sign of, the coefficient on log population growth, in any regression of steady state per-capita output on these parameters. Discuss this intuitively.