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Econ-UA 323

Answers to Sample Examination 3

No calculators, Ipads, laptops, etc., allowed. Put them away, please.

Points 65. Time 75 minutes. The first question carries 30 points; and the second and third 16 points each. 3 points are reserved for extra credit, presentation and clarity. You'll have to grade yourself on it all, including the last!

Guide for Time Allocation*: The questions in (1) should take no more than 5 minutes each to answer; total 30 minutes. Questions (2) and (3) should take you no more than 15 minutes each. This schedule will allow you to finish the exam in 60 minutes. If you are stuck with a question, move on to the next one and plan to come back later. Keep your answers brief and to the point.*

(1) (**30 points, 6 points per part, 5 parts**) Are the following statements true, false, or uncertain? In each case, back up your answer with a brief, but precise explanation.

[**a**] In the tea plantation application studied in class with piece rates, the observed behavior agreed with the predictions of the estimated model immediately after the contract, but not as time went by.

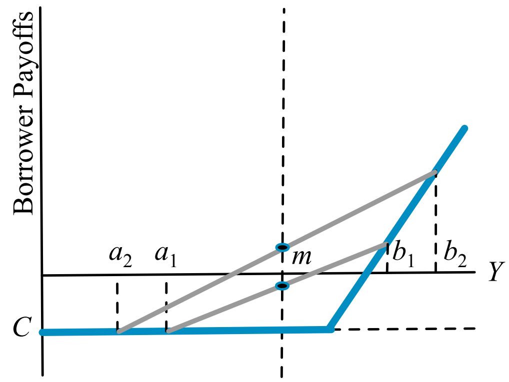
False. The estimated model used a standard utility function, linear in income and convex in effort, which predicted a decline in effort after the new contract. Instead, effort jumped immediately afterwards and stayed that way for the first 10-12 weeks after the contract, and then declined over the 12-16 week period. You can draw here a quick impressionistic view of the predicted and actual densities following the contract change.

[**b**] In credit markets with adverse selection, borrower 1 with a risky project but the same mean as a safer borrower 2 could *strictly* prefer to take a loan at the going rate of interest, while borrower 2 does not.

True. The easiest thing here is to draw a diagram showing the payoffs to the two borrowers. See diagram below. In this diagram, we have drawn the payoff to a borrower as a function of his output when the repayment asked for is R, collateral is C, and the loan is B. It is given by the formula:

$$\pi(Y, r) \equiv \max\{Y - (1 + r)B, -C\},$$

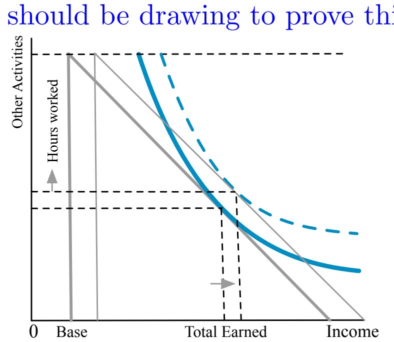
and is shown in the diagram:



Borrower 1 has a project with two possible returns, (a_1, b_1), while borrower 2 has a project with riskier returns (a_2, b_2) but with the same average, given by m. Notice how the expected return of borrower 1 is strictly negative, while that of borrower 2 is strictly positive. The intuitive reason for this discrepancy is that neither borrower has to fully pay the loan on the downside (they are protected by limited liability), while on the upside the riskier borrower earns a higher net return.

[c] In a casual labor market with a base wage and piece rates, an increase in the base wage will *increase* labor hours, assuming that her "alternative activity" (e.g., leisure) is a normal good.

False. This is the diagram you should be drawing to prove this result wrong:



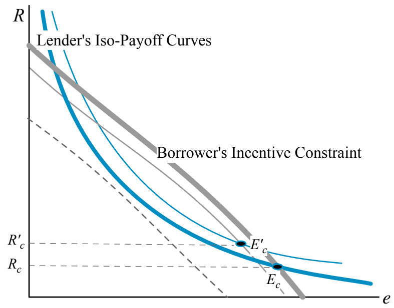
Notice that as the base wage goes up, the consumption of "alternative activities" rises, which means that hours worked falls. This argument of course depends on those activities being normal, otherwise the opposite result would hold.

[d] In the moral hazard model of borrowing, with competitive lenders, a decrease in borrower collateral must decrease the social surplus.

True. First let us recall the definition of the social surplus, which is

$$S \equiv \underbrace{p(e)(Y - R) - (1 - p(e))C - e}_{\text{Borrower payoff}} + \underbrace{p(e)R + (1 - p(e))C}_{\text{Lender payoff}} = p(e)Y - e.$$

Observe how the size of the collateral does not directly go into the formula for the social surplus, as it only appears as a transfer between borrower and lender and therefore washes out. But of course, it indirectly affects the total effort supplied to the project. This diagram is useful:



As collateral falls, the lender's iso-payoff line (for the same value of profit) must shift up, because more e and/or R is needed to get him to the same expected profit, C being lower. At the same time, the borrower's incemntive constraint shifts inward, because effort is governed by the first order condition

$$p'(e)[Y - R + C] = 1$$

and therefore falls for each R as C goes up. The new competitive equilibrium shifts to the left as shown in the diagram, so borrower effort falls. That brings down the social surplus $p(e)Y - e$, because it increases in e up to the first best effort e^*, and equilibrium effort is always below e^*.

[**e**] In the model of permanent labor, an increase in the casual wage everywhere in the market must give rise to an increase in the permanent wage.

True. After all, the permanent wage is set so as to make the self-enforcement constraint

$$x = \delta(1 - q)[w_p - w_c]$$

hold with equality. (You should explain what the variables are in this equation, and say something briefly about how it is derived.) Therefore an increase in w_c must provoke an equal increase in w_p, so as to ensure that the equation above continues to hold.

(2) (16 points) João and José are two brothers who each borrow an amount 100 from a microfinance lender, at an interest rate of r. They have identical projects. Each project succeeds or fails with *independent* probability, which they control by means of their independent efforts. They have no collateral for their loans. All parties are risk-neutral.

For each brother João or José, the situation is as follows: Their projects generate a return of 220 with probability p, and fail completely with probability $1 - p$. The value of p is independently chosen by each brother for their project. But higher p is costly; the cost is given by $60p^2$.

(a) [4 points] Consider either brother, and carefully describe his maximization problem at interest rate r. You can assume that the interest rate is not too high, so that the loan plus interest can be fully repaid in the event of success. Show that the brother's optimal choice of success probability is given by

$$p = 1 - \frac{5}{6}r,$$

and explain intuitively why p is declining in r.

Given an interest rate r on the loan, the borrower will choose p to maximize

$$[220 - 100(1+r)]p - 60p^2.$$

The first-order condition to this maximization problem shows that

$$220 - 100(1+r) = 120p$$

and simplifying this yields our answer.

(b) [4 points] Assuming the answer from part (a), explain what a profit-maximizing microfinance company will choose to maximize, and show that its optimal rate of interest r is given by 10%. Explain intuitively the pros and cons to the company of setting a higher interest rate.

Given this behavior, a profit-maximizing microfinance company, whose payoff is given by

$$100(1+r)p,$$

will therefore act as if it maximizing

$$(1+r)p = (1+r)\left(1 - \frac{5r}{6}\right).$$

Writing down the first-order conditions, we see that

$$-\frac{5}{6}(1+r) + \left(1 - \frac{5r}{6}\right) = 0,$$

and simplifying, this tells us that $r = 1/10$ or 10%.

(c) [4 points] Now suppose that the company lends the startup 100 each to João and José, but makes them *partly liable* for each other's loan. For instance, from João's perspective, we presume that he repays his loan if his project is successful, and in addition must repay half of José's loan if José's project is unsuccessful. (Likewise, he will get partly bailed out by José if his project fails but José's project is successful.) Again assume that the interest rate is never so high that each brother can't pay their debts when successful; they can.

If João's probability of success is denoted by p_1 and José's probability by p_2, write down a new formula for João's expected return, and show that

$$p_1 = \frac{11 - 5(1+r)[1 + \frac{1}{2}(1-p_2)]}{6}.$$

Given an interest rate r on the loan, and given that José's probability of success is p_2, João will choose p_1 to maximize his expected return, which is now given by

$$\left(220 - 100(1+r)[1 + \frac{1}{2}(1-p_2)]\right)p_1 - 60p_1^2.$$

Why is this the expected return? Well, with probability p_1, João produces 220, and pays $100(1+r)$ to cover his own debt. But he will also be responsible for half of José's debt in case the latter fails to repay, and that will happen with probability $1-p_2$. So all in all, the expected return is as written in the formula. Maximizing this with respect to p_1 and writing down the first-order condition to this maximization problem, we see that

$$220 - 100(1+r)[1 + \frac{1}{2}(1-p_2)] = 120p_1,$$

and simplifying this yields our answer.

(d) [4 points] Assuming the answer from part (c), describe how p_1 behaves as a function of p_2 and explain your answer intuitively. Does this mean that the microfinance company is worse off by introducing "group liability"?

It is clear from the equation above that as p_2 goes up, so does p_1. And the same would be true of p_2 as a function of p_1 — that would come from the other brother's behavior. Intuitively, as the other brother behaves "more responsibly," his sibling responds with more effort, because the debt overhang for the sibling becomes smaller.

You could say a bit more at this point. For instance, you might observe that efforts to repay can be viewed as complements: more of one generates more of the other. You might go still further to investigate the equilibrium of p_1 and p_2, which would be amazing, but would not be required.

Finally, this shows that group liability has a negative effect because it increases the debt overhang and makes each borrower a bit more reluctant to put in effort. But does this mean that the microfinance company is worse off by introducing "group liability"? Not necessarily. It is true that the borrowers put in less effort. But now there is a chance of collecting from one of the brothers if the other brother defaults, an option that had been missing under individual liability. So group liability might be chosen by the lender in some situations.

(3) (16 points) Pedro leases a plot of land from his landlord, Manisha, who offers him a sharecropping contract: she gets a share σ of output and Pedro keeps the remaining share $1-\sigma$. Pedro farms the plot using his labor e, which has an opportunity cost of w per unit, and with tractor input t, which he leases in at r per unit. The production function is given by

$$y = f(e,t) = 2[e^{1/2} + t^{1/2}].$$

(a) [5 points] For any share σ between 0 and 1, describe Pedro's maximization problem, and show that

$$e = \left(\frac{1-\sigma}{w}\right)^2 \text{ and } t = \left(\frac{1-\sigma}{r}\right)^2,$$

so that

$$y = 2\left[\frac{1-\sigma}{w} + \frac{1-\sigma}{r}\right].$$

Pedro seeks to maximize his net income, which is given by

$$2(1-\sigma)[e^{1/2} + t^{1/2}] - we - rt.$$

Setting the share-weighted marginal product of each input to its marginal cost, we see that

$$(1-\sigma)e^{-1/2} = w \text{ and } (1-\sigma)t^{-1/2} = r.$$

Transposing terms and squaring, we get

$$e = \left(\frac{1-\sigma}{w}\right)^2 \text{ and } t = \left(\frac{1-\sigma}{r}\right)^2,$$

Substituting these expressions into the production function, we must conclude that

$$y = 2\left[\frac{1-\sigma}{w} + \frac{1-\sigma}{r}\right].$$

(b) [6 points] Using the information from part (a), show that Manisha would ideally like to set $\sigma = 1/2$.

Manisha's income from this arrangement is

$$\sigma y = 2\sigma\left[\frac{1-\sigma}{w} + \frac{1-\sigma}{r}\right] = 2\sigma(1-\sigma)\left[\frac{1}{w} + \frac{1}{r}\right].$$

Use the fact that $\sigma(1-\sigma)$ is maximized at $\sigma = 1/2$ to obtain the desired answer.

(c) [5 points] Suppose that Manisha can share the costs of *both* Pedro's labor and his tractor use, if she wishes. Suppose that Pedro must be given some minimum net income in order to participate. Study what kind of contract you think Manisha will now offer.

The idea is to recall the class notes, which states that sharecropping along with full input cost sharing at the same rate as the share rate achieves the first best. If they write anything along these lines give them credit, and if they manage to do more give them extra credit. A full answer follows.

The total surplus in the system is

$$2[e^{1/2} + t^{1/2}] - we - rt,$$

so the maximum that Manisha can conceivably get from any land contract is

$$2[e^{1/2} + t^{1/2}] - we - rt - M,$$

where M is the minimum payoff to be given to Pedro. This can be achieved if Manisha can get Pedro to maximize the total surplus and then transfer all the rest (over and above M). This can be done, as we discussed in class, by Manisha charging a tax γ on Pedro's net income and leasing him the land. That is, Pedro keeps

$$(1-\gamma)\left(2[e^{1/2} + t^{1/2}] - we - rt\right).$$

Notice that a profit tax is non-distortionary, so Pedro will choose the first best level of inputs (e^*, t^*), while γ can be chosen so that

$$(1-\gamma)\left(2[e^{*1/2} + t^{*1/2}] - we^* - rt^*\right) = M.$$

This has to be optimal for Manisha; she can't do better. But now interpret γ as the output share for Manisha, and also the input cost subsidy on e and t, and we are done.

You can end the exam here.

Two more optional variations on question (3): you might want to look at them.

(d) Suppose that Manisha is required by law to pay a fraction $0 < s < 1$ of the tractor rental fee. Show that her income from leasing out the plot is given by

$$(1) \qquad 2\sigma(1-\sigma)\left[\frac{1}{w} + \frac{1}{r(1-s)}\right] - \frac{s(1-\sigma)^2}{r(1-s)^2}.$$

If Manisha pays a fraction s of the rental fee r, then the effective cost of tractor rental to Pedro is $r(1-s)$ per unit. Therefore his demand for t is adjusted to

$$t = \left(\frac{1-\sigma}{r(1-s)}\right)^2$$

and Manisha pays $rst = rs\left(\frac{1-\sigma}{r(1-s)}\right)^2$ as a consequence. Moreover, output gets adjusted to

$$y = 2\left[\frac{1-\sigma}{w} + \frac{1-\sigma}{r(1-s)}\right].$$

Manisha's net income is her land rental income net of this payment, which must therefore equal:

$$2\sigma(1-\sigma)\left[\frac{1}{w} + \frac{1}{r(1-s)}\right] - \frac{rs(1-\sigma)^2}{r^2(1-s)^2} = 2\sigma(1-\sigma)\left[\frac{1}{w} + \frac{1}{r(1-s)}\right] - \frac{s(1-\sigma)^2}{r(1-s)^2}.$$

(e) Using the previous answers, prove that Manisha's new optimal share strictly exceeds 1/2.

[**Hint**: You do *not* need to compute the exact optimum share to answer this question. From Manisha's point of view, *every variable* in equation (1) is a parameter except for σ, so we can write her profit as $A\sigma(1-\sigma) - B(1-\sigma)^2$, where A and B are positive constants.]

The expression in part (c) can be written as

$$(2) \qquad A\sigma(1-\sigma) - B(1-\sigma)^2,$$

where A and B collect all the other parameters of the problem; that is:

$$A = 2\left[\frac{1}{w} + \frac{1}{r(1-s)}\right] \text{ and } B = \frac{s}{r(1-s)^2}.$$

Maximize this expression with respect to σ. The first order condition is to set the derivative equal to zero. That condition is given by

$$A - 2A\sigma + 2B(1-\sigma) = 0,$$

so that

$$\sigma = \frac{1}{2}\frac{A+2B}{A+B} > 1/2.$$

The above answer gets full credit. Intuitively, the first term in equation (2) increases in σ all the way up to half, so that the maximum has to be attained above that.