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Econ-UA 323

Sample Examination 2

No calculators, Ipads, laptops, etc., allowed. Put them away, please.

Points 65. Time 75 minutes. The first question carries 30 points; the second 17 points, and the third 15 points. 3 points are reserved for extra credit, presentation and clarity. You'll have to grade yourself on it all, including the last!

Guide for Time Allocation*: The questions in (1) should take no more than 5 minutes each to answer; total 30 minutes. Questions (2) and (3) should take you no more than 15 minutes each. This schedule will allow you to finish the exam in 60 minutes. If you are stuck with a question, move on to the next one and plan to come back later. Keep your answers brief and to the point.*

(1) (**30 points, 6 points per part, 5 parts**) Are the following statements true, false, or uncertain? In each case, back up your answer with a brief, but precise explanation.

(a) The Kuznets ratio, given by then income share of the richest 20% divided by the income share of the poorest 40%, is a measure of inequality that fails the Dalton transfer principle.

True. As our objective is to show that the Kuznets ratio fails the transfers principle, all we have to do is find one example. Consider a regressive transfer from a relatively poor person to a relatively rich person, *both* of whom are in the top 20%, such that the person losing income does not fall out of the top 20%. Then the Kuznets ratio as defined will stay unchanged, and so fails the transfers principle. It would be perfect to provide a clear example of this: consider a ten-person income distribution with incomes

$$\{1, 1, 1, 1, 1, 1, 1, 1, 3, 4\}$$

and change it via a regressive transfer to

$$\{1, 1, 1, 1, 1, 1, 1, 1, 2, 5\}$$

Then none of the relevant shares is altered, so the Kuznets ratio is unchanged.

(b) If the Lorenz curve of two situations do not cross, the Gini coefficient and the coefficient of variation cannot disagree.

True. Both G and the COV are Lorenz-consistent. So when the Lorenz curves for a pair of distributions do not cross, both measures *have* to move in the same direction. This is a complete answer. Unless I ask you to, you don't have to spend time showing that the two are Lorenz-consistent. You just have to know that fact — for this question, at least.

(c) Consider the regression $Y = A + bX + \epsilon$, which suffers from endogeneity problems. If a fully exogenous variable I strongly influences Y independently of X, then I is a good instrument for X.

False. The exclusion restriction for an acceptable instrument states that I should *not* influence Y except for its influence via X. The sentence in the question clearly contradicts that requirement.

(d) An omitted variable Z biases the estimate of b in the regression $Y = A + bX + \epsilon$, because its omission makes X correlated with the error term ϵ.

True. If there is an omitted variable Z, then the true regression is

$$Y = A + bX + cZ + \epsilon'$$

instead of $Y = A + bX + \epsilon$. So in the wrong regression, Z will be included as part of the error term ϵ. Now, by definition, an omitted variable is one that's related to both the supposed cause X and the supposed effect Y in the original regression — that is, to both the independent and dependent variables. Therefore, the error term of the wrong regression becomes correlated with X.

(e) In the Acemoglu-Johnson-Robinson study of institutions and per-capita, the presence of malaria is a threat to the exclusion restriction for the instrument they use.

True. AJR use the mortality rate of soldiers, sailors and bishops in the colonies as a proxy for exclusionary institutions, thereby instrumenting for modern institutions. But the outcome variable they are interested in is (modern) per-capita income. The exclusion restriction asks that a good instrument be largely *uncorrelated* with the outcome *except* via the variable that is being instrumented for, which in this case is modern institutions. However, mortality rates were certainly correlated with the disease environment then, which in turn is correlated with the disease environment today, leading to a plausible additional channel of influence on the final outcome. Malaria is a leading example of that disease environment.

(2) (17 points) We are going to study the *inequality of wealth* in the land of Equitania. Suppose that starting from some initial, unequal distribution of strictly positive wealths, everyone saves 20% of their wealth, earns a rate of interest of 10% on that savings, and then earns an income y, which adds to their wealth.

(a) [3 points] For a person with wealth W_t at date t and income y_{t+1} at date $t+1$, show that new wealth is given by

$$W_{t+1} = \frac{11}{50} W_t + y_{t+1}.$$

New wealth is given by

$$W_{t+1} = (1+r)sW_t + y_{t+1},$$

where r is the rate of interest and s is the savings rate from total wealth. Using $r = 1/10$ and $s = 1/5$, we see that $(1+r)s = 11/50$.

(b) [4 points] Show that if a person's income y_{t+1} is proportional to her wealth W_t in the previous period; i.e., if $y_{t+1} = \lambda W_t$ for some $\lambda > 0$, then any Lorenz-consistent inequality measure defined on the wealth distribution must remain the same over time.

In this case, new wealth is given by

$$W_{t+1} = \frac{11}{50} W_t + y_{t+1} = \left(\lambda + \frac{11}{50}\right) W_t,$$

so all wealths scale up by the same proportion. Because inequality doesn't change when all wealths are scaled by the same amount, it remains constant.

(c) [5 points] Suppose that a house in Equitania costs some amount $H > 0$, and assume that at some date t, everyone has enough wealth to buy one, and *must do so to live in Equitania*. Prove that net financial wealth (not counting the house) must be more unequal than gross wealth (counting house ownership).

The gross distribution of wealth at any date t is $\{W^i_(t)\}$, whereas the net distribution of wealth after paying for the house is $\{W^i(t) - H\}$. Now we use the same argument as in the answers to Problem Set 6, but with the argument flipped because H is something taken away, not added.

The wealth share of the poorest j people (for any j) under the "net distribution" is then given by

$$\frac{(\sum_{i=1}^{j} W_i) - jH}{(\sum_{i=1}^{n} W_i) - NH},$$

where N is the total number of people in Equitania. Compare this with the share of the poorest j people under the "gross distribution", which is

$$\frac{\sum_{i=1}^{j} W_i}{\sum_{i=1}^{n} W_i}$$

Clearly, the former wealth share is smaller than the latter, because in the former, an equal amount of H is subtracted from every wealth while the initial wealth distribution is unequal, so relatively speaking there is relative disequalization. But this is true for every j, which means that the Lorenz curve has shifted down under the net distribution, leading to higher inequality.

Another way to do this is by using the basic principles of inequality measurement and not the Lorenz curve. See and adapt the relevant answer in Problem Set 6.

(d) [5 points] Now assume that y is the same for everyone. Using the variables (y, H) along with the numbers you already have for s and r, describe the conditions under which *everyone* can ultimately afford to buy a house in Equitania, no matter what her (positive) starting wealth W_0 might be.

We know that

$$W_{t+1} = \frac{11}{50} W_t + y$$

for all individuals, so wealth cannot grow forever. Long run wealth is given by W^*, which must therefore solve

$$W^* = \frac{11}{50}W^* + y, \text{ or } W^* = \frac{50y}{39},$$

and this is *irrespective* of initial wealth. So in the long run, everyone can afford a house if

$$50y > 39H!$$

(3) (15 points) In the land of TwoSkill, there are just two skills. You can become an unskilled worker at no cost, earning a wage of w_u, or you can become a skilled worker by paying an education cost of E, and then you receive w_s. These wages are exogenous as far as you are concerned, but are determined endogenously by macroeconomic conditions. That is, there is a production function that produces output as follows:

$$Y = 2\sqrt{US},$$

where U is the amount of unskilled labor in the economy and S is the amount of skilled labor. Normalize the price of output to 1. It is a competitive economy and each factor is paid its marginal product.

(a) [5 points] If U and S are the supplies of skilled and unskilled labor, show that

$$w_u = \sqrt{S/U} \text{ and } w_s = \sqrt{U/S}.$$

For the market to absorb the quantities (S, U), it has to be the case that wages for each type of labor must equal their respective values of marginal products. The value of the marginal product of unskilled labor, with output price equal to 1, equals the partial derivative of the production function with respect to unskilled labor, and so:

$$w_u = \frac{1}{2}2U^{-1/2}S^{1/2} = \sqrt{S/U},$$

and a parallel computation yields the required formula for w_s.

(b) [5 points] Assume that the population mass of people is N (that is, $S + U = N$), and that everyone can pay the education cost E out of their own pocket. Let s be the *share* of skilled labor in the economy. Without necessarily solving explicitly for s (though you can attempt it), find a formula that connects s to E, and show that s is declining as E goes up. Explain this result intuitively.

The equilibrium condition is given by

$$w_s - w_u = E, \text{ or equivalently by part (a), } \sqrt{U/S} - \sqrt{S/U} = E.$$

Note that $S/N = s$ and so $U/N = 1 - s$. Using this information above, we see that

$$\sqrt{(1-s)/s} - \sqrt{s/(1-s)} = E, \text{ or equivalently, } x - 1/x = E,$$

where we have defined $x \equiv \sqrt{(1-s)/s}$. The above equation uniquely solves for x, for every $E > 0$, and from that solution we can work out what s must be. To get the answers you have been asked here, you don't have to solve any more explicitly than this. As E increases, it is obvious that to regain the equality above, x must rise, which means that s must fall. That makes sense. When education costs are higher, then you can have fewer skilled people. That also helps to restore the balance between the two choices by raising skilled wages.

if you wanted to solve explicitly for s, note from the last equation above that

$$x^2 - Ex - 1 = 0, \text{ so that } x^* = \frac{1}{2}\left(E + \sqrt{E^2+4}\right),$$

where we have only picked up the positive root x^* for the solution, of course. Remembering that $x = (1-s)/s$, we can then solve explicitly out for s^* as

$$s^* = \frac{1}{1 + \frac{1}{2}\left(E + \sqrt{E^2+4}\right)},$$

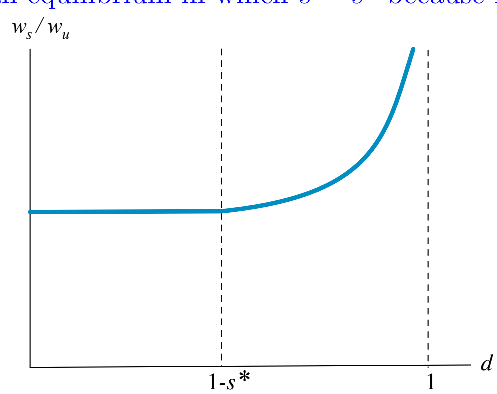
and then the analysis with respect to E is even clearer.

(c) [5 points] Now suppose that a fraction d of the population (that is, dN people) are denied access to education; that is, they cannot afford pay for E while the others can. Describe fully the allocation of labor to skilled and unskilled, as well as the relative wages of skilled to unskilled labor, w_s/w_u as d varies all the way from 0 to 1.

To answer this question, the important point is to understand that *as long as d* is smaller than $1 - s^*$, where s^* is the solution in part (b), *nothing changes*. The reason is that enough people have access to education so that the equilibrium we have in part (b) in automatically implemented. Therefore, for all $d \le 1 - s^*$, the solution stays put at s^*. What about the ratio of w_s/w_u? Using the answer in part (a), we see that this ratio is given by

$$\frac{w_s}{w_u} = \frac{\sqrt{U/S}}{\sqrt{S/U}} = \frac{U}{S} = \frac{1-s^*}{s^*},$$

and stays steady at this value as long as $d \le 1 - s^*$. When d exceeds $1 - s^*$, however, it is impossible to implement an equilibrium in which $s = s^*$ because not that many people have



access to education. The important thing for you to understand is that in this situation, the fraction of skilled labor must settle exactly at $1 - d$. At this fraction, we see that

$$w_s - w_u = \sqrt{U/S} - \sqrt{S/U} = \sqrt{d/(1-d)} - \sqrt{(1-d)/d} > \sqrt{s^*/(1-s^*)} - \sqrt{(1-s^*)/s^*} = E,$$

so that a larger wage differential opens up. But it is still an equilibrium because although at this differential, individuals would *like* to become entrepreneurs, they *cannot*. The ratio of skilled to unskilled wages is just

$$\frac{w_s}{w_u} = \frac{\sqrt{U/S}}{\sqrt{S/U}} = \frac{U}{S} = \frac{d}{1-d},$$

and this ratio keeps rising with d after d crosses $1 - s^*$, rising all the way to infinity as $d \to 1$.