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Econ-UA 323

Examination 2, Fall 2023

No calculators, Ipads, laptops, etc., allowed. Put them away, please.

Points 65. Time 75 minutes. The first question carries 30 points; and the second and third 16 points each. 3 points are reserved for extra credit, presentation and clarity.

Guide for Time Allocation: *The questions in (1) should take no more than 5 minutes each to answer; total 30 minutes. Questions (2) and (3) should take you no more than 15 minutes each. This schedule will allow you to finish the exam in 60 minutes. If you are stuck with a question, move on to the next one and plan to come back later. Keep your answers brief and to the point.*

(1) (**30 points, 6 points per part, 5 parts**) Are the following statements true or false? It is *not* enough to just guess one or the other. You need to provide an argument for or against, and only then will any credit be awarded.

[**a**] Consider the regression $Y = A + bX + \epsilon$, and an exogenous variable I. If I is to serve as a good instrument for X, it should be correlated with X and uncorrelated (or at least weakly uncorrelated) with Y.

There could be two equally acceptable answers, if you give the right reasoning:

A. False. It should be uncorrelated (or weakly correlated) with Y *after controlling* for the channel via X. I is certainly allowed to be — in fact will be — nicely correlated with Y via the independent variable X.

B. True. But I am interpreting the statement "I is uncorrelated (or at least weakly uncorrelated) with Y" to mean "apart from any correlations generated via X."

[**b**] Alesina, Guiliano and Nunn's paper on the effects of the plough use the actual production of plough-positive crops as an instrument for plough use.

False. The actual production of plough-positive crops is just as endogenous as the use of the plough. For instance, if the plough was adopted owing to bad gender norms in the community, then the chances of growing crops suitable to the plough would obviously go up, leading to reverse causality — and a failure of the exclusion restriction. Instead, AGN use a measure of land suitability for plough-positive crops which is arguably far more exogenous.

[**c**] A project for a risk-neutral population of 100, that yields \$5 to a winner and \$-3 to a loser, with 60 winners overall, where it is also known that exactly 40 of the winners fully know their identity, but no one else does, must fail a majority vote for adoption.

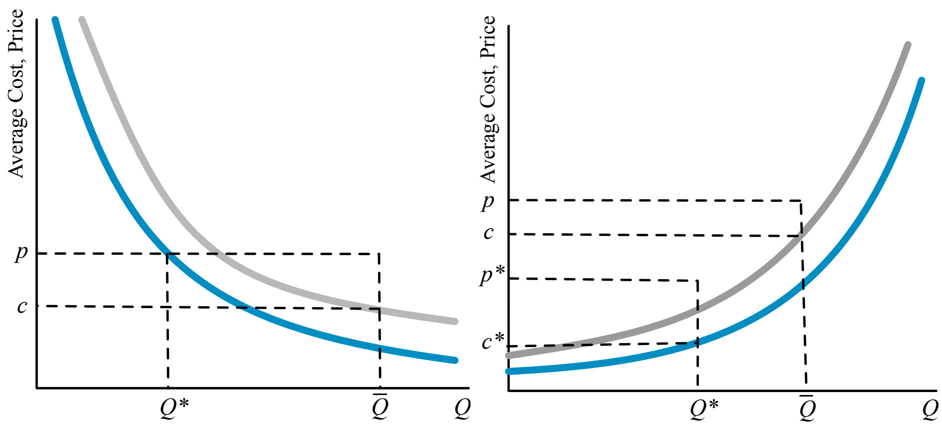
True. If 40 winners know their identity they will all vote yes, But of the 60, the probability of being a winner is

$$p = \frac{60-40}{100-40} = 1/3,$$

and so the expected value for the other 60 voters is negative. So the project fails.

[**d**] All other things being equal, it is easier to displace a product using a more efficiently produced newer version if the new product is produced under conditions of *decreasing* rather than *increasing* returns to scale.

True. This requires you to draw diagrams at the very least. The idea would be to show that under increasing returns to scale, you will might initially need to bear a loss even if the other firm has a worse technology:



If the old firm is charging p, the first digram shows that for every current quantity of the rival that is *below* Q^*, the rival must make a loss in the current period. Of course, as news of the rival's product spreads then the rival will ultimately make a profit, but must bear the initial losses. If credit markets are imperfect, that may not be possible. In contrast, the second diagram shows that the rival can make profits from day one if there is decreasing returns to scale.

[**e**] The coefficient of variation satisfies the relative income principle.

True. You will need to write the formula for the coeffficient of variation:

$$C = \frac{1}{\mu}\sqrt{\sum_{i=1}^{m} n_i(y_i-\mu)^2}.$$

and tell us what the variables are. Then argue that if all incomes are multiplied by the same scalar $\lambda > 0$, then so is the mean, and C remains unchanged.

(2) (16 points) Doublin has four equal-sized groups of people, with individual wealth identical within a group at time t: $W^1(t) < W^2(t) < \mu(t) < W^3(t) < W^4(t)$, where $\mu(t)$ is the mean wealth. At the start of every year, a person with initial wealth W earns a rate of return of r, and then receives x from the government. A fraction s of total income (capital income plus the x) is saved and added to W to get her end-of-year wealth.

Now, Doublin is a most peculiar country in which at the end of the year, each person reproduces asexually to have two offspring. End-of-year wealth is divided equally among the two offspring, and the parent exits gracefully.

We are going to study the inequality across wealth at the start of every year. You are allowed to assume all the principles we studied, with the word "income" replaced by "wealth" everywhere in those principles.

(a) [4 points] For a parent with starting wealth $W(t)$, rate of return r and government income $x(t)$ in year t, prove that the starting wealth $W(t+1)$ of *each* offspring next year is given by

$$W(t+1) = \frac{1}{2}\left(W(t) + s[rW(t) + x]\right).$$

The parent's end-of-year wealth in year t is given by $W_{\text{end}}(t) = W(t) + s[rW(t) + x]$, and so the starting wealth of each offspring at $t+1$ is given by

$$W(t+1) = \frac{1}{2}W_{\text{end}}(t) = \frac{1}{2}\left(W(t) + s[rW(t) + x]\right).$$

(b) [4 points] Prove that if x is proportional to parental wealth W; that is, if $x = aW$ for some constant $a > 0$, then the inequality of starting wealth must be unchanged from year to year in Doublin.

Consider a parent with wealth $W(t)$ and her two children, each with wealth $W(t+1)$. At date $t+1$, "unclone" each pair of offspring back to just one offspring. (Make one of each pair disappear along with their wealth.) By the population principle, that new artificial society at $t+1$ must have the same inequality as the real society at $t+1$, and also the same number of people as the parental society. Moreover, using proportionality, each offspring wealth $W(t+1)$ is linked to parental wealth by the equation

$$W(t+1) = \frac{1}{2}\left(W(t) + s[rW(t) + x]\right) = \frac{1}{2}\left(1 + s[r + a]\right)W(t).$$

This maps each parent's wealth to that of their child by multiplying by $\frac{1}{2}\left(1 + (1-c)[r + a]\right)$. By the relative income (wealth) principle, inequality must be unchanged between date t and the artificial society at $t+1$. Combining the above arguments, we are done.

(c) [4 points] How does your answer to part (b) change if x is the same *irrespective* of wealth?

In this case inequality must fall from one period to the next. Here is a detailed answer. The steps are broken down clearly to show where partial credit could be given.

Again, using the population principle, it suffices to compare period t with the uncloned artificial society at date $t+1$. Because

$$W(t+1) = \frac{1}{2}W_{\text{end}}(t) = \frac{1}{2}\left(W(t) + (1-c)[rW(t) + x]\right) = \frac{1-(1-c)r}{2}\left[W(t) + \frac{1-c}{1-(1-c)r}x\right],$$

the comparison is the same as comparing

$$\{W^1(t), W^2(t), W^3(t), W^4(t)\} \text{ with } \{W^1(t)+z, W^2(t)+z, W^3(t)+z, W^4(t)+z\},$$

by the relative income (wealth) principle, where $z \equiv \frac{1-c}{1-(1-c)r}x$. Moreover, because there is equal population in every group, we can again use the population principle to presume that there is one person in every group. Let $\mu(t)$ be the mean of the first distribution; then obviously $\mu(t)+z$ is the mean of the second. Use the relative income principle again to say that the above comparison is the same as
(1)
$$\{W^1(t), W^2(t), W^3(t), W^4(t)\} \text{ with } \left\{\frac{\mu(t)[W^1(t)+z]}{\mu(t)+z}, \frac{\mu(t)[W^2(t)+z]}{\mu(t)+z}, \frac{\mu(t)[W^3(t)+z]}{\mu(t)+z}, \frac{\mu(t)[W^4(t)+z]}{\mu(t)+z}\right\},$$

These two distributions have the same mean $\mu(t)$. Additionally, in the second distribution we can check that
(2)
$$\frac{\mu(t)[W^i(t)+z]}{\mu(t)+z} > W^i(t) \text{ for groups} i = 1,2 \text{ and } \frac{\mu(t)[W^i(t)+z]}{\mu(t)+z} > W^i(t) \text{ for groups} i = 1,2.$$

because, by assumption, $W^i(t) < \mu(t)$ for $i = 1,2$ and $W^i(t) > \mu(t)$ for $i = 3,4$. You can show any one of these. For instance,

$$\frac{\mu(t)[W^1(t)+z]}{\mu(t)+z} > W^1(t) \text{ if, cross-multiplying, } \mu(t)W^1(t) + \mu(t)z > \mu(t)W^1(t) + W^1(t)z,$$

or equivalently, if $\mu(t) > W^1(t)$, which is precisely what we have.

Equation (2) means that the first distribution can be obtained from the second by a sequence of regressive transfers. The first two wealths are higher, so reduce those wealths back to $W^1(t)$ and $W^2(t)$ and "give that money" to wealths 3 and 4 to bring them back up to the old level. We can therefore conclude that the second distribution in (1) is more equal than the first, and we are done.

This looks long because I have written down every step but the intuition should be very clear along the way. The addition of a constant amount independent of wealth makes the *relative* wealths more equal.

(d) [4 points] Return to part (b), where inequality in Doublin is unchanging over time. How do you think inequality will change between t and $t+1$ if *the poorer half* of all families had two children each (with wealth split as before), and the other (richer) half had one child each? You can assume that there are only two families at date t, which morph into three families at $t+1$, that $r = 0$, that $s = 1$, and that $x = 0$.

Inequality would rise. There are several levels at which you can give answers. The most intuitive is that the two halves would have unchanged inequality within them, but inequality across the two halves would change (one "half" now being twice as numerous as the other "half"). That would get you some credit.

To go further, you would have to recognize that the answer would depend on *which* families are getting split. If all the lower income families get split, then definitely income inequality would rise. (If all the richer families are getting split, that is no longer true.) That would get you more partial credit.

I reserve a full credit answer for where you can exactly show that inequality rises under the assumptions of the question. Let A be the family with one child and B the family with two children, with wealths (W^A, W^B) changing to wealths $(W^A, W^B/2, W^B/2)$. By the population principle, comparing these two is the same as comparing

$$(\underbrace{W^A, W^A, W^A}_{3\text{ times}}, \underbrace{W^B, W^B, W^B}_{3\text{ times}}) \text{ with } (\underbrace{W^A, W^A}_{2\text{ times}}, \underbrace{\frac{W^B}{2}, \frac{W^B}{2}, \frac{W^B}{2}, \frac{W^B}{2}}_{4\text{ times}}).$$

Total wealth in the first case is $3W^A + 3W^B$, and in the second case it is $2W^A + 4W^B$. By the relative income principle, the above comparison is the same as comparing

$$\left(\underbrace{\frac{W^A}{3W^A+3W^B}, \frac{W^A}{3W^A+3W^B}, \frac{W^A}{3W^A+3W^B}}_{3\text{ times}}, \underbrace{\frac{W^B}{3W^A+3W^B}, \frac{W^B}{3W^A+3W^B}, \frac{W^B}{3W^A+3W^B}}_{3\text{ times}}\right)$$

with

$$\left(\underbrace{\frac{W^A}{2W^A+4W^B}, \frac{W^A}{2W^A+4W^B}}_{2\text{ times}}, \underbrace{\frac{W^B/2}{2W^A+4W^B}, \frac{W^B/2}{2W^A+4W^B}, \frac{W^B/2}{2W^A+4W^B}, \frac{W^B/2}{2W^A+4W^B}}_{4\text{ times}}\right)$$

(the red entry will be explained below). This looks complicated but it isn't at all; I just had to write out the wealths.

In the original setting, if the *poorer* family splits as assumed, then $W^A > W^B$ and so $3W^A + 3W^B > 2W^A + 4W^B$. With that, you can forget about the original setting and just focus on the situation above. We are comparing two 6-person societies, with the same total income.

In the first two entries above, which are the rich to start with, wealths go up from the first to the second distribution. In the last three entries, which are the poor to start with, wealth goes down. So we can view the last three as making a transfer to the first two. Person 3 (in red) is ambiguously located, but either his wealth has gone up, in which case we can think of the last three as making a transfer to him as well, or his wealth has gone down, in which case we can think of him as having made an additional transfer to the first two. In either case, we have only regressive transfers, so inequality must rise going from the first distribution to the second.

(3) (16 points) There is a population of mass N living in the country of Nobank. Everyone chooses to be either a worker or a capitalist. A worker earns a wage of w, and a capitalist earns a profit of

$$2\sqrt{\ell} - w\ell - S,$$

where $\sqrt{\ell}$ is a production function based on laborers ℓ, w is the wage, and S is a setup cost.

(a) [4 points] If a capitalist can freely choose how much labor to hire to maximize her profits, show that

$$\ell = \frac{1}{w^2},$$

and that her profit is given by

$$\frac{1}{w} - S.$$

(In what follows you can assume these formulae, even if you could not solve this part.)

A capitalist chooses ℓ to maximize

$$2\sqrt{\ell} - w\ell - S.$$

The marginal benefit of one more unit of labor is the derivative of the production function, and the marginal cost of that unit is the extra wage income, which is just w. Setting marginal cost equal to marginal benefit, we see that

$$2(\frac{1}{2})\ell^{-1/2} = w,$$

and rearranging terms, we see that

$$\ell = 1/w^2.$$

Substituting this optimal solution into profits, we see that

$$2\sqrt{\ell} - w\ell - S = 2\frac{1}{w} - w\frac{1}{w^2} - S = \frac{1}{w} - S.$$

(b) [4 points] Suppose that a mass M between 0 and N of individuals get to be capitalists (and the rest, $N - M$, are workers). Show that the wage rate that equates supply to demand in the labor market must solve

$$w = \sqrt{\frac{M}{N - M}}.$$

(Again, you can assume all these formulae below, even if you could not solve this part.)

If there are M capitalists and $N - M$ workers, then the number of workers per capitalist is just $(N - M)/M$. This number must be optimally chosen by every firm. For that to happen, the wage must be such so that the maximizing choice of ℓ in part (a) is precisely $(N - M)/M$. Using the first formula derived in part (a), it follows that

$$\frac{N - M}{M} = \frac{1}{w^2},$$

and rearranging terms, we get

$$w = \sqrt{\frac{M}{N - M}}$$

as required.

Now we suppose that a mass $\bar{M}$ of people in Nobank have high wealth and can afford the startup cost S out of their own pocket, while the remainder cannot, and they can't borrow. There are no banks in Nobank after all!

(c) [4 points] Show that the high wealth individuals will indeed *choose* to be capitalists only if

$$\sqrt{\frac{1-\bar{m}}{\bar{m}}} - S \geq \sqrt{\frac{\bar{m}}{1-\bar{m}}}.$$

where we have defined $\bar{m} = \bar{M}/N$. Interpret this result. Does the inequality above mean a small value of $\bar{m}$ or a large value of $\bar{m}$? And what does that mean from an economic standpoint?

All high-wealth individuals will voluntarily choose to be capitalists only if the resulting profit income from all being capitalists is at least as high as the wage, which is their other option. Now profits are given by $(1/w) - S$, as already seen in part (a), and if all $\bar{M}$ are capitalists, then $w = \sqrt{\bar{M}/(N-\bar{M})} = \sqrt{\bar{m}/(1-\bar{m})}$ as already seen in part (b), so combining these two observations, profit income is

$$\frac{1}{w} - S = \sqrt{\frac{1-\bar{m}}{\bar{m}}} - S.$$

So the condition "profit income is at least as high as the wage" is the same as

$$\sqrt{\frac{1-\bar{m}}{\bar{m}}} - S \geq w = \sqrt{\frac{\bar{m}}{1-\bar{m}}}.$$

which is what is required.

Notice that the left-hand side of this expression goes *down* to zero as $\bar{m} \to 1$ and up to infinity as $\bar{m} \to 0$, while the opposite is true of the right-hand side. We can therefore conclude that the expression holds for $\bar{m}$ small enough. That makes sense from an economic point of view. If $\bar{m}$ is small, then there are few entrepreneurs and lots of workers. The equilibrium wage will therefore be low, and profits will be high, so everyone who can be a capitalist will indeed choose to be one.

(d) [4 points] Describe what happens if the expression in part (c) fails to hold? Does everyone with access still become a capitalist? Why or why not? And what would determine the equilibrium wage?

Assume that the expression in part (c) fails. Then if everyone who *can* become a capitalist *does* become a capitalist, the net return to being a capitalist (the left hand side of the expression in (c)) will be lower than the wage (the right hand side). That cannot happen in equilibrium, as an individual would then rather be a worker. So let's imagine that M out of the $\bar{M}$ rich people become capitalists, where $M \leq \bar{M}$. Notice that you can always find *some* value of M, call it M^*, (by bringing it down from $\bar{M}$) such that

$$\sqrt{\frac{1-m^*}{m^*}} - S = \sqrt{\frac{m^*}{1-m^*}},$$

where we have defined $m^* = M^*/N$. At that value, capitalist profit equals the wage, and people with access are indifferent between being a capitalist or a worker. So the equilibrium finally looks like this: (i) all low-wealth people become workers, (ii) M^* out of the $\bar{M}$ who can be capitalists become capitalists, (iii) the remainder become workers along with everyone else who don't have access, so the total mass of workers is $N - M^*$, and (iv) the returns to entrepreneurship and labor are equalized.

Also observe that the expression in part (c) is likely to fail if the value of $\bar{M}$ is relatively high; that is, more people have access to begin with. Everybody who *can* be a capitalist in that case will not choose to be capitalists. Some will be capitalists and some workers, and the returns to both occupations will be equalized. Then we have the standard outcome with socially efficient markets, as the analysis above reveals.