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Econ-UA 323

Examination 1, Fall 2023

(1) (**30 points, 6 points per part, 5 parts**) Are the following statements true or false? It is *not* enough to just guess one or the other. You need to provide an argument for or against, and only then will any credit be awarded.

[**a**] If a production function defined on three inputs (say physical capital, human capital and unskilled labor) exhibits diminishing returns to *each one* of these three inputs, then it must have decreasing returns to scale.

False. Example: Cobb Douglas on three inputs:

$$Y = AK^a L^b H^c$$

will have diminishing returns to each input if $0 < a, b, c < 1$ but could have constant or even increasing returns to scale if $a+b+c \geq 1$. This answer gives you full credit if you additionally explain why $a+b+c > 1$ means you have increasing returns or why $a+b+c = 1$ means you have constant returns.

[**b**] The Harrod-Domar model is capable of generating persistent growth in the *per-capita* capital stock (and therefore in per-capita income), even without any technical progress.

True. In the Harrod-Domar model, we have the growth equation in per-capita form as

$$(1+n)k(t+1) = (1-\delta)k(t) + sAk(t)^a$$

with a set equal to 1. (You can start this answer even "earlier" with the aggregate capital stock and then dividing by population to get the equation above.) In this limit case, we have the simpler form

$$(1+n)k(t+1) = (1-\delta)k(t) + sAk(t)$$

which tells us that

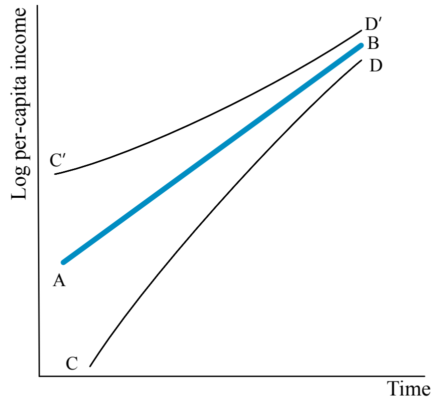
$$\frac{k(t+1)-k(t)}{k(t)} = \frac{sA-n-\delta}{1+n}$$

And of course, income growth occurs at the same rate as per-capita capital growth, as y and k are linearly connected by $y = Ak$. That is,

$$\frac{y(t+1)-y(t)}{y(t)} = \frac{k(t+1)-k(t)}{k(t)}.$$

[**c**] In the Solow model with technical change, per-capita growth could be higher than the rate of technical progress, depending on the initial income a country starts from.

True. In the Solow model, it is only at the *steady state* that the economy grows per-capita at the rate of technical progress. If the initial capital stock relative to effective labor is not at the steady state value $\hat{k}^*$, then growth could be faster or even slower. The diagram below illustrates this.



The blue line indicates the path of per-capita income in steady state, growing at the constant rate of technical progress. But nobody magically decreed that the economy has to start *right on* that blue path. It could start at a point like C, for instance, in which case the trajectory CD moves up faster than the steady state path AB. That faster movement has to translate into a growth rate of per capita capital that's higher than the steady state growth rate. (Of course, the growth rate slows down as the steady state is approached.)

Indeed, the economy could even grow slower than steady state: imagine starting at a point like C' and converging towards the steady state path along the trajectory $C'D'$.

[d] A country which has been growing steadily at 2% per year and now has a per-capita income of \$60,000 would have a per-capita income of *approximately* \$3,750, about 140 years ago.

True. At a 2% rate of growth the doubling time is given by approximately 70/2, which is 35 years. If 140 years ago income was y, then it would double in 35 years, and then three more times over the next 35+35+35 years. So its total income would be $2^4 = 16$ times as large, and so income today is given by

$$16y = 64,000$$

which gives $y = 4,000$.

[e] This is a possible mobility matrix across starting and ending relative income categories, with every category populated by at least some countries at the ending stage, assuming that all countries were equally divided at the starting stage.

	1	2	3	4
1	80%	20%	0%	0%
2	0%	0%	65%	35%
3	0%	50%	50%	0%
4	0%	0%	0	100%

True. It is possible. First you should check that this is indeed a valid mobility matrix, for which you must check that all the rows sum to 100%. They do. Now notice that if you start with countries in each initial category, then every category at the end of the process will have some countries in it. For instance, 80% of the countries in Category 1 remain in Category 2. 50% of the countries from Category 3 move to Category 2. 65% of the countries from Category 2 move to Category 3. And finally, 100% of the countries from Category 4 remain in Category 4.

(2) (17 points) The town of Maskdown has been invaded by a nasty viral disease. The chances of catching it depends on whether you are wearing a mask or not. The probability of being infected is 80% (or 4/5) if you don't wear a mask and 40% (or 2/5) if you do. Getting the disease is costly to you: denote that cost by $C > 0$.

But there are other costs as well — because there's an ongoing debate in Maskdown about wearing masks. Some will yell at you if do, and others will shame you if you don't. Let's model these "shame costs." At any point of time, say a share n wears a mask, while $1 - n$ don't. The shame cost of *not* wearing a mask increases in the share of people that *are* wearing a mask, and the cost is given by $200n$. There is another "shame cost" of *wearing* a mask that increases in the share of people that are *not* wearing a mask, and the cost is given by $150(1 - n)$. Alas, no matter which action you take, you are going to incur one cost or the other. That's just life in Maskdown, as in many real-world cities ...

Each person is interested in *taking the action that minimizes her overall expected cost* — that from getting the disease plus any shame cost that she incurs from her action.

(i) [4 points] Show that no matter how small C is, it is always an equilibrium for people in Maskdown to all wear masks.

Suppose that a share n is expected to wear a mask. Then if you wear a mask, your expected cost is

$$(2C/5) + 150(1 - n)$$

which is the expected disease cost + the shame cost coming from those who are not wearing. Likewise, if you don't wear a mask, your expected cost is

$$(4C/5) + 200n,$$

and you take the action with the lower cost. If n is expected to be 1 (everyone wearing), the cost of the former is $2C/5$ while the cost of the latter is $(4C/5) + 200$, and clearly everyone will then *want* to wear a mask. It follows that $n = 1$, or ubiquitous mask-wearing, is always an equilibrium.

(ii) [5 points] For what values of C are there also equilibria in which no one wears masks?

Similarly, try out the above expressions for the case in which everyone expects no one to wear a mask; that is, $n = 0$. Then the cost of the former option is $(2C/5) + 150$ while the cost of the latter option is $4C/5$. For $n = 0$ to be an equilibrium, the latter option must have a lower cost; that is, we need

$$4C/5 \leq (2C/5) + 150, \text{ or } C \leq 375.$$

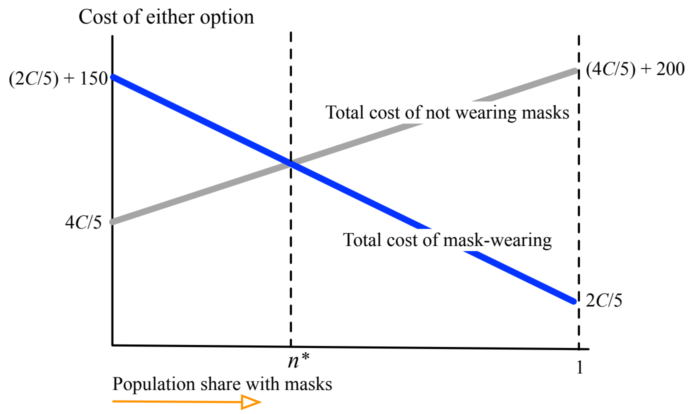
That is, it is also an equilibrium for no one to wear a mask provided the cost of the disease is not too high.

(iii) [4 points] In the second case, describe the "unstable equilibrium" in which some fraction of the population (which I want you to calculate) wear masks, and explain why it is unstable.

In the second case, we have both $n = 0$ and $n = 1$ as equilibria. There is a third equilibrium n^* which is unstable, given by the value of n for which the two costs are exactly equal:

$$(2C/5) + 150(1 - n^*) = (4C/5) + 200n^*, \text{ or } n^* = \frac{150 - (2C/5)}{350}.$$

This is perfectly well-defined (because $C \leq 375$). If n goes above this threshold n^*, the system drifts off to the all-mask equilibrium! If n lies below this threshold, it drifts to the no-mask equilibrium. Note: C might exactly equal 375 in which case the unstable equilibrium coincides with the no-mask equilibrium, otherwise n^* lies strictly between 0 and 1. The diagram below tells you why n^* is unstable.



(iv) [4 points] Now suppose that in addition to all the costs described above, there is a "loss of personal freedom" cost to wearing a mask; call it F. Find a condition on C and F for which universal mask-wearing is *not an equilibrium*, in contrast to (i).

Just redo part 1 with the freedom cost factored in. Suppose that a share n is expected to wear a mask. Then if you wear a mask, your expected cost is

$$(2C/5) + 150(1 - n) + F$$

which is the expected disease cost + the shame cost + the freedom cost. If you don't wear a mask, your expected cost is

$$(4C/5) + 200n,$$

and again, you take the action with the lower cost. If n is expected to be 1 (everyone wearing), the cost of the former is $(2C/5) + F$ while the cost of the latter is $(4C/5) + 100$. This will *fail* to be an equilibrium if the former cost is larger than the latter; that is, if

$$(2C/5) + F > (4C/5) + 200, \text{ or } F > (2C/5) + 200.$$

The above condition will knock out universal mask-wearing as an equilibrium.

(3) (14 points) The country of Rápido has a production function which depends on capital and labor, and at any date t, it is given by

$$Y = 9K^{1/3}[e(t)L]^{2/3}.$$

where $e(t) = (1+\pi)^t$ is the exogenous productivity of labor at time t, growing at a constant rate π.

(a) [4 points] Find a formula for per-effective-capita production $\hat{y}$ as a function of the per-effective-capita capital stock $\hat{k}$, and explain the steps to get there.

Divide both sides of the production function by effective labor $e(t)L$; then:

$$\frac{Y}{e(t)L} = \frac{9K^{1/3}[e(t)L]^{2/3}}{e(t)L} = 9\left[\frac{K}{e(t)L}\right]^{1/3},$$

so that, defining per-effective-capita magnitudes by $\hat{y} = Y/e(t)L$ and $\hat{k} = K/e(t)L$, we have:

$$\hat{y} = 9\hat{k}^{1/3}.$$

(b) [5 points] In Rápido, capital has depreciation rate δ, population growth n, and savings rate s. Show that steady state path of output per-capita is given by

$$y^*(t) = 27\left(\frac{s}{\delta+\pi+n}\right)^{1/2}(1+\pi)^t,$$

describing precisely all the steps that lead to this conclusion.

Let us recall the Solow equation for capital accumulation, which is:

$$K(t+1) = (1-\delta)K(t) + sY(t),$$

where we use the assumption that there is no depreciation. Dividing through by $e(t)L$ on both sides:

$$\frac{K(t+1)}{e(t)L} = (1-\delta)\frac{K(t)}{e(t)L} + s\frac{Y(t)}{e(t)L} = (1-\delta)\hat{k}(t) + s\hat{y}(t),$$

and because $K(t+1)/e(t)L = [e(t+1)/e(t)][K(t+1)/e(t+1)] = (1+\pi)\hat{k}(t+1)$, we get

$$(1+\pi)(1+n)\hat{k}(t+1) = (1-\delta)\hat{k}(t) + s\hat{y}(t) = (1-\delta)\hat{k}(t) + 9s\hat{k}(t)^{1/3},$$

where the last equality uses part (a). In steady state $\hat{k}^*$, we therefore have

$$(1+\pi)(1+n)\hat{k}^* = (1-\delta)\hat{k}^* + 9s\hat{k}^{*1/3},$$

and solving this system for its positive solution, we have:

$$\hat{k}^* \simeq 9^{3/2}\left(\frac{s}{\delta+\pi+n}\right)^{3/2},$$

where at this stage we use the approximation that $(1+\pi)(1+n) \simeq 1+\pi+n$. It follows that

$$\hat{y}^* = 9\hat{k}^{*1/3} = 9\left[9^{3/2}\left(\frac{s}{\delta+\pi+n}\right)^{3/2}\right]^{1/3} = 9^{3/2}\left(\frac{s}{\delta+\pi+n}\right)^{1/2} = 27\left(\frac{s}{\delta+\pi+n}\right)^{1/2},$$

and opening up $\hat{y}$ to trace out per-capita income y^*, we see that $y^*(t) = \hat{y}(t)(1+\pi)^t$, so that

$$y^*(t) = 27\left(\frac{s}{\delta+\pi+n}\right)^{1/2}(1+\pi)^t,$$

which establishes the equation we want.

(c) [5 points] Now set technical change and depreciation both equal to zero in Rápido. Show that the coefficient on log savings rate is equal in magnitude to, but has the opposite sign of, the coefficient on log population growth, in any regression of steady state per-capita output on these parameters. Discuss this intuitively.

With technical change and depreciation both set to zero, steady state output is given by

$$y^* = 27\left(\frac{s}{n}\right)^{1/2}.$$

Take logs to see that

$$\ln y^* = \ln 27 + \frac{1}{2}\ln s - \frac{1}{2}\ln n.$$

So the predicted regression coefficients on $\ln s$ and $\ln n$ are equal in magnitude, but opposite in sign.

There is good reason for this. An increase in the fraction of savings out of national in come will raise capital accumulation , thereby moving up the Solow steady state. An increase in the population growth rate *lowers* capital accumulation *per capita*. The two effects tend to cancel out when there is no technical progress or depreciation. An increase in the population growth rate by 1% can be exactly nullified by the same percentage increase in the savings rate, so that the resulting growth in capital makes up exactly for the resulting higher population numbers.