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THE EFFICIENCY PRINCIPLE IN NON-COOPERATIVE COALITIONAL BARGAINING*

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Applying the non-cooperative theory of coalitional bargaining, I examine a widely held view in economic literature that an efficient outcome can be agreed on in voluntary bargaining among rational agents in the absence of transaction costs. While this view is not always true, owing to the strategic formation of subcoalitions, I show that it can hold under the possibility of successive renegotiations of agreements. Renegotiation may, however, motivate bargainers to form a subcoalition first and to exploit the first-mover rent. This strategic behaviour in the process of renegotiation may distort the equity of an agreement.
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1. Introduction

According to a widely held view in economics, a Pareto-efficient allocation of resources can be attained through voluntary bargaining among rational agents in a world where there is neither private information nor bargaining costs. This view may be called the efficiency principle, and it has been argued as the primary part of the so-called Coase Theorem (see Coase, 1960, and Cooter, 1989).[1] The same view on efficiency has prevailed in cooperative game theory. Some classical solutions, such as the core, presume the Pareto efficiency of a payoff allocation. The premiss of the cooperative game theory is that cooperation takes place whenever it is beneficial to all agents involved.

Recent literature on non-cooperative coalitional bargaining theory (Chatterjee *et al.*, 1993; Moldovanu and Winter, 1995; etc.) shows that this common view on efficiency is not always true under the strategic behaviour of coalition-forming. An inefficient allocation of payoffs may arise from the formation of subcoalitions in a certain institutional setup of the bargaining procedure. The aim of this paper is to re-examine the efficiency principle supporting the Coase Theorem by means of non-cooperative bargaining theory in an economic environment where agents are free to form coalitions seeking higher rewards.

I consider a coalitional bargaining situation described by an n-person game in coalitional form with transferable utility. In the game, a real number is assigned to

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1) Milgrom and Roberts (1992, p. 24) describe the efficiency principle as follows: "If people are able to bargain together effectively and can effectively implement and enforce their decisions, then the outcomes of economic activity will tend to be efficient (at least for the parties to the bargain)."

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every feasible coalition of players, representing the total utility of members in the coalition. To analyse the problem of coalition formation, I specify a bargaining procedure that describes how an agreement can be made. I employ the "random proposers" bargaining procedure (Okada, 1996).

The bargaining rule is simple. In the first round, one player is randomly selected as a proposer with equal probability among all players. The player can propose both a coalition and a feasible payoff allocation for the coalition. All other players in the coalition either accept or reject the proposal sequentially in a predetermined order. If all accept the proposal, they can then reach agreement on forming the coalition with payoff allocation, and the game ends; otherwise, the game goes on to the next round and the same process is repeated with new proposers randomly selected until some agreement is reached. The discounting of future payoffs is assumed. It has been proved in Okada (1996) that, when the discount factor is sufficiently large, equal allocation in the grand coalition is agreed on in the first round in a stationary subgame-perfect equilibrium point if and only if the grand coalition has the largest value per member. Restating the latter condition, equal allocation is in the core of the underlying game. Chatterjee *et al*. (1993) prove that the same result holds in a "fixed-protocol" model, in which the first proposal is selected in a fixed order and the first rejector becomes the next proposer. These results show that, in a simple and natural procedure of n-person bargaining with coalition formation, an efficient allocation of payoffs in the grand coalition is not always agreed on. In particular, when the per capita value of the grand coalition is not the highest among all possible coalitions, or, equivalently, when equal allocation is not in the core of the game, a subcoalition may be formed and thus the final allocation may be inefficient.[2)] The following example depicts an inefficient agreement in a three-person super-additive game.

Example 1

The player set is $\{1, 2, 3\}$. The values of feasible coalitions are: $v(\{i\}) = 0$ for $i = 1, 2, 3$; $v(\{1, 2\}) = v(\{2, 3\}) = v(\{1, 3\}) = 0.9$; $v(\{1, 2, 3\}) = 1$. Consider the following strategies for players. Player 1 proposes $(\{1, 2\}, (0.6, 0.3, 0))$ and accepts any proposal if he is offered a payoff equal to or greater than 0.3. Player 2 proposes $(\{2, 3\}, (0, 0.6, 0.3))$ and employs the same response rule as player 1. Player 3 proposes $(\{1, 3\}, (0.3, 0, 0.6))$ and employs the same response rule as player 1. When this strategy is used, every player receives the same expected payoff, 0.3. When the discount factor of future payoffs is almost one, it can be seen without much difficulty that the strategy constructed is a stationary subgame-perfect equilibrium point in the bargaining procedure of random proposers. In this equilibrium, all two-person coalitions may be formed with equal probability. The payoff allocation is inefficient and favours a proposer. One may wonder why player 1 does not propose any other allocation—say, (0.64, 0.32, 0.04)—in which all players are better off than in the equilibrium allocation (0.6, 0.3, 0) when player 1 becomes the proposer. If he were to do so, player 3 would reject the proposal in equilibrium because he could obtain the expected payoff 0.3 (much higher than 0.04) in the next bargaining round. Thus, player 1 would lose the

2) Aivazian and Callen (1981) discuss, without any specific procedure of bargaining, the possibility that the Coase Theorem does not always hold in n-person coalitional-form games with an empty core. See also comments by Coase (1981) and Hurwicz (1995) on this issue.

advantage of the proposer and his payoff would decrease from 0.6 to 0.3. Notice that the same inefficient equilibrium point as above exists for any discount factor close to one if the two-person coalition value is greater than $3/4$.

Do players have no options after their negotiations result in an inefficient allocation of payoffs? Since there exists some other allocation in which all are better off, it may be natural to assume that they will attempt to "renegotiate" their initial agreement in the direction of a Pareto-improving new agreement. The possibility of renegotiation may, however, change the strategic character of coalitional bargaining. Players can anticipate the outcome of renegotiation, which may affect the nature of the initial agreement.

In this paper I am concerned primarily with whether the possibility of renegotiation contributes to the efficiency of a final agreement, and with how it affects the equity of a payoff allocation. To investigate these problems of renegotiation, I incorporate a renegotiation procedure into the "random proposers" bargaining model and analyse the extended model as a whole non-cooperative game.

The main results of the paper are as follows. The possibility of successive renegotiations necessarily leads to an efficient allocation of payoffs when the initial (or prevailing) agreement is considered as the threat-point of renegotiation; that is, the efficiency principle holds true under some suitable process of renegotiation. The renegotiation may, however, have a negative effect in distorting the equity of a final allocation by creating "vested interests" in the bargaining process. When the discount factor of future payoffs is large, the anticipation of renegotiation motivates bargainers to propose subcoalitions first in order to exploit the first-mover rent.

Seidmann and Winter (1998) present a renegotiation model of coalition formation, closely related to my own, incorporating renegotiations into the fixed-protocol model of Chatterjee *et al*. (1993).[3] Similar to the results reported below, they show that the grand coalition can be eventually formed in their model of renegotiation. However, the result of a fixed-protocol bargaining model such as Seidmann and Winter's is sensitive to the selection of the first proposer. As Chatterjee *et al*. point out, a delay on agreement may occur, unlike in our random-proposers model. Seidmann and Winter show that a coalition will gradually form if the underlying cooperative game has an empty core, irrespective of whether renegotiations are allowed. In contrast, I show that the prospect of renegotiations may itself lead to the gradual formation of coalitions culminating in the grand coalition, regardless of whether the core is empty.

The paper is organized as follows. Section 2 presents an n-person non-cooperative coalitional bargaining model with renegotiations and defines our solution concept. Section 3 proves the main results. Section 4 presents concluding remarks.

2. The coalitional bargaining model with renegotiations

A multilateral bargaining situation is described by an n-person game (N, Σ, v) in coalitional form with transferable utility. Here $N = \{1, \ldots, n\}$ is the set of players and Σ is a class of subsets of N. An element S in Σ represents a feasible coalition of players. For convenience, we assume that Σ includes the null coalition $\varnothing$. The

3) Seidmann and Winter (1998) call their model a "reversible actions" game.

characteristic function v of the game is a real-valued function on Σ with $v(\emptyset) = 0$. The value $v(S)$ of each coalition $S \in \Sigma$ is interpreted as a sum of money that the members of coalition S can distribute among themselves in any way if such an agreement is made. In what follows, we shall assume that $N \in \Sigma$ and $\{i\} \in \Sigma$ for $i \in N$.

The characteristic function v is assumed to have the following properties: (i) (zero normalization): $v(\{i\}) = 0$ for any $i \in N$; (ii) (monotonicity): $v(S) \geqslant v(T)$ for any S and T in Σ with $T \subseteq S$; and (iii) (profitability): $v(N) > 0$. All properties are standard in cooperative game theory. For $S \in \Sigma$, define

$$X(S) = \left\{ x \in R^n | x = (x_i: i \in N), \sum_{i \in S} x_i = v(S) \qquad \text{and } x_j = 0 (\forall j \in N - S) \right\}, \qquad (1)$$

where R^n is the n-dimensional Euclidean space.

In this paper analysis is restricted to the simple case where one and only one profitable coalition ($v(S) > 0$) can be formed. Examples of bargaining situations in which this restriction may be appropriate include the following.

(1) There are n business firms that are interested in organizing a joint venture for a government public project. The public project is contracted by only one joint venture. Benefits from the joint venture depend on technologies and resources possessed by member firms. The joint venture can be created through negotiations among the firms. No single firm has the ability to carry out the project.
(2) There is a production economy in which n individuals can jointly produce consumption goods from their initial endowments. The production technology is accessible to only one group of individuals. No single individual can utilize the production technology by himself: at least one partner is needed for production. The more individuals who cooperate, the more goods they can produce.

Although the largest benefits can be attained by a group of all individuals, it is not always the case, as Example 1 shows, that such an efficient outcome is agreed upon in the strategic bargaining of coalition formation. The main purpose of the present analysis is to investigate how effective the possibility of renegotiation is in attaining an efficient outcome. I employ a variant of the non-cooperative bargaining model of random proposers (Okada, 1996). A new feature of the model is the possibility that players may renegotiate their (possibly inefficient) agreements.

The bargaining procedure is defined inductively as follows.

(1) In round 1, one player is selected as a proposer among n players with equal probability. Let player $i_1 \in N$ be selected. Player i_1 proposes (a) a coalition S_1 with $i_1 \in S_1 \in \Sigma$, and (b) a payoff vector $x^1 \in X(S_1)$. All other players in S_1 either accept or reject the proposal sequentially, according to a predetermined order over N. The order of responders does not affect the results of the model in any crucial way. If all accept it, proposal (S_1, x^1) becomes the "current" agreement. When $v(S_1) < v(N)$, the game goes to the second round. When $v(S_1) = v(N)$, proposal (S_1, x^1) becomes the "final" agreement and the game ends. If any one responder rejects the proposal, no agreement is made in the first round, and the game goes to the second round. In this case, the null allocation $(\emptyset, 0)$ is set as the "current" agreement for convenience.

(2) In round t ($\geqslant 2$), one player is selected as a proposer among n players with equal probability. Let player $i_t \in N$ be selected. Let (S, x^S) be the "current" agreement at the beginning of round t. Player i_t proposes (a) a coalition S_t such that

$i_t \in S_t \in \Sigma$ and $S \subset S_t$, and (*b*) a payoff vector $x^t \in X(S_t)$. All other players in S_t either accept or reject the proposal sequentially. If all accept it, proposal (S_t, x^t) becomes the new "current" agreement, replacing (S, x^S). When $v(S_t) < v(N)$, the game goes to the next round $t+1$. When $v(S_t) = v(N)$, the new agreement (S_t, x^t) becomes the "final" agreement, and the game ends. If any one responder rejects it, no new agreement is made in round t. In this case, the game goes to the next round $t+1$, where (S, x^S) remains the "current" agreement. The same process is repeated until a coalition with the highest value is formed.

The bargaining rule can be interpreted as follows. When some inefficient agreement (S, x) with $v(S) < v(N)$ is reached in round t, all players are allowed to "renegotiate" it in the next round $t+1$ under the same rule as in round t, except that the threat-point of renegotiation is determined by the "current" agreement (S, x). If renegotiations fail in round $t+1$, the same renegotiation process will be repeated in future rounds.

Formally, the bargaining procedure is represented as an infinite-length extensive game with perfect information. A *play* of the game is either a finite or an infinite path of actions from the start of the game tree. When a play consists of a finite path of actions, the game stops at some terminal node. Every possible play of the game is associated with a sequence

$$\alpha = ((S_{t_1}, x^{t_1}), \ldots, (S_{t_m}, x^{t_m})),\ t_1 < \ldots < t_m \tag{2}$$

of all agreements on the play where (S_t, x^t), $x^t \in X(S_t)$, $t = t_1, \ldots, t_m$, is the agreement made in round t. The bargaining rule implies: $S_{t_1} \subset \ldots \subset S_{t_m}$. When $v(S_{t_m}) = v(N)$, it means that the game ends at round t_m with the final agreement (S_{t_m}, x^{t_m}). When $v(S_{t_m}) < v(N)$, it means that the game does not stop without any new agreements after round t_m. If no agreement is made in the whole play, we simply set $\alpha = \varnothing$ for notational convenience. We call (2) the *agreement history* in bargaining. When the game is played with an agreement history (2), we assume that every player $i \in N$ receives discounted payoff u_i, i.e.,

$$u_i = \delta^{t_1-1} x_i^{t_1} + \delta^{t_2-1}(x_i^{t_2} - x_i^{t_1}) + \ldots + \delta^{t_m-1}(x_i^{t_m} - x_i^{t_{m-1}}), \tag{3}$$

where $\delta (0 \leqslant \delta < 1)$ is the discount factor of future payoffs. Note that (3) assigns to every player $j \in N - S_{t_m} u_j = 0$ (see (1)). When no agreement is made ($\alpha = \varnothing$), every player receives zero payoff.

Equation (3) can be interpreted as follows. When agreement (S_{t_1}, x^{t_1}) is made in round t_1, it is assumed to be enforceable, and becomes the threat-point of renegotiations in future rounds; that is, players will negotiate for how much they should gain over the agreed payoff x^{t_1}. Even if they fail in renegotiations, they can receive the payoff x^{t_1}. Evaluating his future payoffs at the beginning of negotiations (i.e. in round 1), player i discounts the payoff x^{t_1} agreed on in round t_1 by δ^{t_1-1}. If a new agreement (S_{t_2}, x^{t_2}) is reached in round t_2, the gain $(x_i^{t_2} - x_i^{t_1})$ is discounted by δ^{t_2-1}. The same discounting rule is applied to new agreements in future rounds.

The specific form (3) of players' discounted payoffs can be justified in two ways. The first justification is given as the (normalized) sum of discounted payoffs in a standard framework of infinitely repeated games. Suppose that a payoff allocation is implemented according to a "current" agreement at the end of each bargaining round, and moreover that players evaluate the payoffs received in future rounds by a discount factor δ. In this "repeated-game" interpretation, the agreement history (2) gives player i the (normalized) sum of discounted payoffs as

$$(1-\delta)\{0+\ldots+0+\delta^{t_1-1}x_i^{t_1}+\ldots+\delta^{t_2-2}x_i^{t_1}+\delta^{t_2-1}x_i^{t_2}+\ldots \\ +\delta^{t_m-2}x_i^{t_{m-1}}+\delta^{t_m-1}x_i^{t_m}+\delta^{t_m}x_i^{t_m}+\ldots\}. \quad (4)$$

It can be easily seen that (4) is equal to payoff u_i in (3). In the repeated-game framework, we interpret the worth $v(S)$ of coalition S as the total utility available to S in each of an infinite number of rounds. This framework is employed by Seidmann and Winter (1998).

The second justification for (3) is given in terms of the expected payoff for a player if the bargaining rule is modified as follows. There is a random move at the end of every round that determines whether negotiations can continue in the next round. Whenever the game stops, the current agreement is implemented, and players can receive their respective payoffs. Let $\delta(0 \leqslant \delta < 1)$ be reinterpreted as the probability that negotiations can continue in the next round. When the agreement history (2) is realized on some play of the game, the following possibilities are conceivable. Agreement (S_{t_1}, x^{t_1}) may be implemented in round t_1 with probability $\delta^{t_1-1}(1-\delta)$, and may also be implemented in round t_1+1 with probability $\delta^{t_1}(1-\delta), \ldots$; agreement (S_{t_2}, x^{t_2}) may be implemented in round t_2 with probability $\delta^{t_2-1}(1-\delta)$; and so on. Then, the whole expected payoff of player i is given by (4), which is equal to u_i in (3).

Let $\Gamma^\delta(\Sigma, v)$ denote the bargaining game defined above where δ is the discount factor for future payoffs. A (pure) strategy for player i in $\Gamma^\delta(\Sigma, v)$ is a sequence $\sigma_i = (\sigma_i^t)_{t=1}^\infty$ of round t-strategies σ_i^t where σ_i^t is a function assigning (*a*) a proposal (S, x^S) and (*b*) a response policy to proposals, depending upon every possible history of negotiations before round t. In this paper we consider pure strategies only. For a strategy combination $\sigma = (\sigma_1, \ldots, \sigma_n)$, we can define the expected (discounted) payoff $H_i^\delta(\sigma)$ of player i in $\Gamma^\delta(\Sigma, v)$ in the usual manner.

Our solution concept for the bargaining game $\Gamma^\delta(\Sigma, v)$ is a subgame-perfect equilibrium point satisfying subgame consistency. The notion of subgame consistency is introduced by Harsanyi and Selten (1988). Broadly, it requires that an equilibrium point prescribes the same actions for players in every two "isomorphic" subgames of the bargaining game $\Gamma^\delta(\Sigma, v)$. Recall that the game $\Gamma^\delta(\Sigma, v)$ and all subgames of $\Gamma^\delta(\Sigma, v)$ are extensive games with perfect information.

In this paper, we say that two extensive games with perfect information are isomorphic if the two games have identical game trees up to a positive affine transformation of payoffs. Formally, I define an isomorphism between two extensive games with perfect information as follows (see also Okada and Winter, 1995). Let Γ and Γ' be two extensive games with perfect information, and let K and K' be the set of all nodes in the game trees of Γ and Γ', respectively. An *isomorphism* from Γ and Γ' is a one-to-one mapping g from K onto K' satisfying three properties:

(1) g preserves the tree structures of K and K'; that is, for any two nodes x and y in K, if node x is after node y, then node $g(x)$ is after node $g(y)$ in K'.
(2) If one node x is a decision node for player i in Γ, then node $g(x)$ is a decision node for the same player i in Γ'. The same is true for chance nodes in Γ and Γ'. Every two corresponding edges at chance nodes in Γ and Γ' have the same probabilities.
(3) For any play z in Γ, let $h(z) = (h_1(z), \ldots, h_n(z))$ be a payoff vector for players

associated with z. Let z' denote a play in Γ' uniquely obtained from z by the isomorphism g, and let $h'(z') = (h'_1(z'), \ldots, h'_n(z'))$ denote the payoff vector associated with play z' in Γ'. Then, there exist constants $a_i > 0$ and b_i for every $i \in N$ such that $h'_i(z') = a_i h_i(z) + b_i$.

Let Γ and Γ' be two isomorphic extensive games with perfect information, and let g be an isomorphism from Γ to Γ'. For any pure strategy combination σ in Γ, the isomorphism g determines uniquely a pure strategy combination, denoted by $g(\sigma)$, in Γ'.

I shall now define the subgame consistency of a (Nash) equilibrium point in an extensive game with perfect information.

Definition 1: An equilibrium point σ of an extensive game Γ with perfect information is said to be *subgame-consistent* if, for every two isomorphic subgames Γ' and Γ'' of Γ, $\sigma(\Gamma'') = g(\sigma(\Gamma'))$ for some isomorphism g from Γ' to Γ'' where $\sigma(\Gamma')$ and $\sigma(\Gamma'')$ are strategy combinations induced by σ on Γ' and Γ'', respectively.

To apply the notion of subgame consistency to the bargaining game $\Gamma^\delta(\Sigma, v)$, we examine the structure of a subgame of $\Gamma^\delta(\Sigma, v)$ starting in every round from the viewpoint of isomorphism. For each round t, suppose that an agreement history, $\alpha_t = ((S_{t_1}, x^{t_1}), \ldots, (S_{t_k}, x^{t_k}))$, $t_1 < \ldots < t_k < t$, is realized before round t. Note that (S_{t_k}, x^{t_k}) is the "current" agreement at the start of round t. Let $\Gamma^{\delta,t}(\Sigma, v|\alpha_t)$ denote a subgame of $\Gamma^\delta(\Sigma, v)$ starting with the initial node (i.e., the random move) in round t after the agreement history α_t. We call $\Gamma^{\delta,t}(\Sigma, v|\alpha_t)$ a *renegotiation subgame* of $\Gamma^\delta(\Sigma, v)$ with agreement history α_t if we want to distinguish it from other subgames of $\Gamma^\delta(\Sigma, v)$. When $t_k < t-1$, no agreements are made in rounds $t_k+1, \ldots, t-1$. In the subgame $\Gamma^{\delta,t}(\Sigma, v|\alpha_t)$, players can propose only those coalitions that include S_{t_k}, by the rule of $\Gamma^\delta(\Sigma, v)$. For each (non-null) coalition $S \in \Sigma$, let us define the class $\Sigma(S)$ of all coalitions T including S by

$$\Sigma(S) = \{T \in \Sigma | S \subset T\} \tag{5}$$

and the characteristic function v^S on $\Sigma(S)$ by

$$v^S(T) = v(T) - v(S) \qquad \text{for } T \in \Sigma(S). \tag{6}$$

For the null coalition $\varnothing$, we let $\Sigma(\varnothing) = \Sigma$ and $v^\varnothing = v$.

Lemma 1: *For each t, let $\alpha_t = ((S_{t_1}, x^{t_1}), \ldots, (S_{t_k}, x^{t_k}))$ be an agreement history before round t in the bargaining game $\Gamma^\delta(\Sigma, v)$. Then, the renegotiation subgame $\Gamma^{\delta,t}(\Sigma, v|\alpha_t)$ of $\Gamma^\delta(\Sigma, v)$ with agreement history α_t is isomorphic to the bargaining game $\Gamma^\delta(\Sigma(S), v^S)$ where $S = S_{t_k}$.*

Proof. By the rule of the bargaining game $\Gamma^\delta(\Sigma, v)$, every feasible proposal (S, x) in the subgame $\Gamma^{\delta,t}(\Sigma, v|\alpha_t)$ satisfies (i) $x \in X(S)$ and (ii) $S \in \Sigma(S_{t_k})$. Define $y = x - x^{t_k}$. Then (S, y) is a feasible proposal in the game $\Gamma^\delta(\Sigma(S_{t_k}), v^{S_{t_k}})$ since

$$\sum_{j \in S} y_j = \sum_{j \in S} x_j - \sum_{j \in S} x_j^{t_k} = v(S) - v(S_{t_k}) = v^{S_{t_k}}(S).$$

This fact implies that there is a natural isomorphism g from $\Gamma^{\delta,t}(\Sigma, v|\alpha_t)$ to $\Gamma^\delta(\Sigma(S_{t_k}), v^{S_{t_k}})$ which maps the proposal (S, x) onto the proposal (S, y). It is enough for us to prove that condition (3) above of an isomorphism holds true for g. Consider any play z in the subgame $\Gamma^{\delta,t}(\Sigma, v|\alpha_t)$ which induces an agreement history $\beta = ((S_{t_{k+1}}, x^{t_{k+1}}), \ldots, (S_{t_m}, x^{t_m}))$, $t \leqslant t_{k+1} < \ldots < t_m$. Given the whole agreement history (α_t, β), every player i receives payoff u_i given by (3) in $\Gamma^{\delta,t}(\Sigma, v|\alpha_t)$. On the other hand, the payoff that player i receives for the play z' corresponding to z under g in the game $\Gamma^{\delta}(\Sigma(S_{t_k}), v^{S_{t_k}})$ is given by

$$u_i' = \delta^{t_{k+1}-t} y_i^{t_{k+1}} + \delta^{t_{k+2}-t}(y_i^{t_{k+2}} - y_i^{t_{k+1}}) + \ldots + \delta^{t_m - t}(y_i^{t_m} - y_i^{t_{m-1}}), \tag{7}$$

where $y_i^{t_j} = x_i^{t_j} - x_i^{t_k}$ for $j = k+1, \ldots, m$. Comparing (3) with (7), we have a positive affine transformation

$$u_i = \delta^{t-1} u_i' + b_i,$$

where $b_i = \delta^{t_1 - 1} x_i^{t_1} + \ldots + \delta^{t_k - 1}(x_i^{t_k} - x_i^{t_{k-1}})$. This proves the lemma. ■

The lemma shows that every two renegotiation subgames of the bargaining game $\Gamma^{\delta}(\Sigma, v)$ are isomorphic if, when the subgames start, their current agreements involve the same coalitions of players. More precisely, let $\alpha_t = ((S_{t_1}, x^{t_1}), \ldots, (S_{t_k}, x^{t_k}))$, $t_1 < \ldots < t_k < t$, and $\beta_{t'} = ((U_{t_1'}, y^{t_1'}), \ldots, (U_{t_m'}, y^{t_m'}))$, $t_1' < \ldots < t_m' < t'$, be two agreement histories. Then, the two renegotiation subgames $\Gamma^{\delta,t}(\Sigma, v|\alpha_t)$ and $\Gamma^{\delta,t'}(\Sigma, v|\beta_{t'})$ are isomorphic if $S_{t_k} = U_{t_m'}$.

We are now ready to define the notion of our non-cooperative solution for the bargaining game $\Gamma^{\delta}(\Sigma, v)$.

Definition 2: A pure strategy combination $\sigma^* = (\sigma_1^*, \ldots, \sigma_n^*)$ of the bargaining game $\Gamma^{\delta}(\Sigma, v)$ is called a *non-cooperative solution* of $\Gamma^{\delta}(\Sigma, v)$ if two properties hold: (i) (subgame perfection): σ^* is a subgame-perfect equilibrium point of $\Gamma^{\delta}(\Sigma, v)$; and (ii) (subgame consistency): for every t and every agreement history $\alpha_t = ((S_{t_1}, x^{S_{t_1}}), \ldots, (S_{t_k}, x^{S_{t_k}}))$, $t_1 < \ldots < t_k < t$, before round t, the round t-strategy σ_i^{*t} of every player i induced by σ^* on the renegotiation subgame $\Gamma^{\delta,t}(\Sigma, v|\alpha_t)$ depends only on the last coalition S_{t_k} in α_t.

I have omitted the notion of subgame perfection as it is standard in the theory of extensive games. The second property is a restatement of the subgame consistency in Definition 1, when it is applied to the bargaining game $\Gamma^{\delta}(\Sigma, v)$. It follows from Lemma 1 that every two renegotiation subgames of $\Gamma^{\delta}(\Sigma, v)$ are isomorphic if their current agreements involve the same coalitions of players. Therefore, subgame consistency requires that the equilibrium strategy of every player be independent of all histories except the last coalition agreed on before a renegotiation subgame. In the context of bargaining, subgame consistency has an implication of “forgiveness—let bygones be bygones”, in that players do not punish one another even if they have been treated unfavourably in past rounds, as long as payoff-relevant variables of negotiations are identical. It generalizes the notion of a stationary equilibrium point that is used in almost every coalitional bargaining model (see e.g. Selten, 1981; Chatterjee *et al.*, 1993; Okada, 1996; Seidmann and Winter, 1998). It is well known in the literature of non-cooperative coalitional bargaining that, if this kind of restriction is dropped, then the set of all subgame-perfect equilibrium payoffs is very large, often equal to the set of all individually rational payoff allocations, when the discount factor δ converges to one.

Finally, note that subgame consistency is closely related to the notion of a Markov-perfect equilibrium point studied frequently in repeated game models with state variables (see Fudenberg and Tirole, 1991). If we take a coalition in a current agreement as a state variable in each round, our non-cooperative solution can be reformulated as a Markov-perfect equilibrium point of the bargaining game $\Gamma^\delta(\Sigma, v)$.

3. The efficiency principle

We investigate whether the possibility of successive renegotiations is effective for attaining an efficient payoff allocation in the n-person coalitional bargaining. Let $\sigma^* = (\sigma_1^*, \ldots, \sigma_n^*)$ be a non-cooperative solution of the bargaining game $\Gamma^\delta(\Sigma, v)$. I denote by v_i^δ the expected payoff of player $i \in N$ for σ^* in $\Gamma^\delta(\Sigma, v)$, i.e. $H_i^\delta(\sigma^*) = v_i^\delta$. For each $S \in \Sigma$, I also denote by $v_i^{S,\delta}$ player i's expected payoff for σ^* in the bargaining game $\Gamma^\delta(\Sigma(S), v^S)$. For simplicity of notation, I omit δ in v_i^δ and $v_i^{S,\delta}$ when no confusion arises.

Theorem 1: *In every non-cooperative solution $\sigma^* = (\sigma_1^*, \ldots, \sigma_n^*)$ of the bargaining game $\Gamma^\delta(\Sigma, v)$, every player $i \in N$ proposes in round 1 an optimal solution (S_i, y^{S_i}) of the maximization problem:*

$$\max_{S,y} \left(v(S) - \sum_{j \in S, j \neq i} y_j + \delta v_i^S \right)$$

$$\text{s.t.} \quad (i) \;\; i \in S \in \Sigma, \; y \in X(S),$$

$$(ii) \;\; y_j + \delta v_j^S \geqslant \delta v_j \qquad \text{for all } j \in S \text{ with } j \neq i. \tag{8}$$

Moreover, the proposal (S_i, y^{S_i}) is accepted.

Proof. Let m_i be the maximum value of (8). The objective function of (8) represents the total payoff of player i when his proposal (S, y) is accepted in round 1. The theorem is proved in three steps.

(1) Since the rule of $\Gamma^\delta(\Sigma, v)$ implies $v_k \geqslant 0$ for all $k = 1, \ldots, n$, the pair (N, y) such that

$$y_i = v(N) - \sum_{j \neq i} v_j \qquad \text{and} \qquad y_j = v_j \qquad \text{for all } j \in N \text{ with } j \neq i$$

is a feasible solution of the maximization problem (8). Note that $v_k^N = 0$ for all k in N because the game ends in round 1 when $S = N$. It follows from the rule of $\Gamma^\delta(\Sigma, v)$ and the monotonicity of v that $\sum_{j=1}^n v_j \leqslant v(N)$. Then, since $y_i \geqslant v_i$, we obtain $m_i \geqslant v_i$.

(2) It is now clear that $\delta v_i \leqslant v_i \leqslant m_i$. I will prove $\delta v_i < m_i$. Suppose, on the contrary, that $\delta v_i = m_i$. This yields $\delta v_i = v_i = m_i$, and thus $v_i = m_i = 0$ since $0 \leqslant \delta < 1$. As there exists a feasible solution (N, y) with $y_j = \delta v_j$ for all $j \in N$ with $j \neq i$, we have

$$m_i \geqslant v(N) - \delta \sum_{j \in N, j \neq i} v_j.$$

Since $m_i = 0$, we have

$$\delta \sum_{j \in N, j \neq i} v_j \geqslant v(N) \geqslant \sum_{j \in N, j \neq i} v_j.$$

This yields $v(N) = 0$, which contradicts $v(N) > 0$.

(3) Let (S^*, y^*) be an optimal solution of (8). Then, it must hold that

$$y_j^* + \delta v_j^{S^*} = \delta v_j \qquad \text{for all } j \in S^* \text{ with } j \neq i.$$

For sufficiently small $\varepsilon > 0$, define $z \in X(S^*)$ such that $z_j = y_j^* + \varepsilon/(|S^*| - 1)$ for all $j(\neq i)$ in S^*. Suppose that player i proposes (S^*, z). Since $z_j + \delta v_j^{S^*} > \delta v_j$ for all $j(\neq i)$ in S^*, the subgame perfection of σ^* implies that this proposal is accepted. Thus, player i can obtain the total payoff $m_i - \varepsilon$. Since $\delta v_i < m_i$ from step (2), we can select $\varepsilon > 0$ sufficiently small so that $\delta v_i < m_i - \varepsilon$. On the other hand, if player i proposes some (S, y) such that his total payoff (given by the objective function of (8)) is strictly larger than m_i, there exists at least one member j of S such that constraint (ii) of (8) does not hold true, because m_i is the maximum value of (8). The subgame perfection of σ^* implies that the proposal is rejected, and thus player i obtains payoff δv_i from the subgame consistency of σ^*. These arguments imply that on the equilibrium play of σ^* player i proposes an optimal solution (S^*, y^*) of (8), and that it must be accepted. ■

The maximization problem (8) can be interpreted as follows. Suppose that player i proposes (S, y) in round 1. If the proposal is accepted by all other members in S, then the agreement (S, y) is reached in round 1 and the renegotiation subgame $\Gamma^{\delta,2}(\Sigma, v|\alpha_2)$ with $\alpha_2 = ((S, y))$ is played in round 2. In this case, the total (expected) discounted payoff for each responder j is given by $y_j + \delta v_j^S$. On the other hand, if player i's proposal is rejected, the same game $\Gamma^\delta(\Sigma, v)$ is repeated in round 2. In this case of disagreement, each responder receives the discounted payoff δv_j. Therefore, the constraints in the maximization problem (8) produce the conditions necessary for responders to accept the proposal. Subject to these incentive constraints, player i makes a proposal optimal to him.

Theorem 1 shows that no delay of agreements holds in the bargaining game $\Gamma^\delta(\Sigma, v)$ with renegotiations. Note that an analogue of Theorem 1 holds true for every bargaining game $\Gamma^\delta(\Sigma(S), v^S)$ whenever $v(S) < v(N)$. In view of Lemma 1, this means that the equilibrium coalition is expanded through renegotiations and that it eventually becomes an efficient coalition. The following theorem and proof demonstrate this.

Theorem 2: *In every non-cooperative solution σ^* of the n-person bargaining game $\Gamma^\delta(\Sigma, v)$, the agreement of an efficient coalition S satisfying $v(S) = v(N)$ is reached in most $n - 1$ rounds.*

Proof. It follows from Theorem 1 that an initial agreement (S_1, x^{S_1}) is reached in round 1 on the equilibrium play of σ^*. If $v(S_1) = v(N)$, then the theorem is proved. Otherwise the game goes on to round 2, and the renegotiation subgame $\Gamma^{\delta,2}(\Sigma, v|\alpha_2)$ with $\alpha_2 = (S_1, x^{S_1})$ starts in round 2. By Lemma 1, $\Gamma^{\delta,2}(\Sigma, v|\alpha_2)$ is isomorphic to the bargaining game $\Gamma^\delta(\Sigma(S_1), v^{S_1})$, where $\Sigma(S_1)$ and v^{S_1} are defined in (5) and (6), respectively. It can be seen that Theorem 1 remains true for the game $\Gamma^\delta(\Sigma(S_1), v^{S_1})$ if we replace (Σ, v) with $(\Sigma(S_1), v^{S_1})$. Since the restriction of σ^* on $\Gamma^{\delta,2}(\Sigma, v|\alpha_2)$ is a non-cooperative solution of it, Theorem 1 again implies that the second agreement (S_2, x^{S_2}) with $S_1 \subset S_2$ is reached in round 2. By repeating the same argument, it can be shown that a new agreement is reached in every round of negotiations on every play of σ^* as long as the value of the agreed coalition is less than $v(N)$. Thus, the equilibrium coalition is expanded in each round of negotiations, and the game ends in most $n-1$ rounds with an efficient coalition S. ■

When the discount factor δ of future payoffs is strictly less than one, the theorem shows that, in general, the coalition of players gradually expands through a repetition of renegotiations, and that an efficient coalition eventually forms. In the limit where the discount factor δ converges to one, the final payoff allocation is Pareto-efficient. In this way, the efficiency principle of the n-person coalitional bargaining holds true through renegotiations in the limiting case. Seidmann and Winter (1998) prove a result similar to Theorem 2 in the fixed-order model with renegotiations. However, note that, unlike in the random-proposers model, delay of agreement may occur in the fixed-order model, as shown by Chatterjee *et al.* (1993).

When the discount factor of future payoffs is not very large, one may argue that the efficiency of a payoff allocation diminishes to a certain extent by the gradual formation of coalitions. Suppose that an agreement history of coalitions, $(S_1, \ldots, S_m)$ with $v(S_m) = v(N)$, is realized in a non-cooperative solution of $\Gamma^\delta(\Sigma, v)$. The total discounted payoff of n players is given by

$$(1-\delta)\left(v(S_1) + \delta v(S_2) + \ \ldots\ + \delta^{m-2} v(S_{m-1}) + \frac{\delta^{m-1}}{1-\delta} v(N)\right).$$

Therefore, efficiency loss by the gradual formation of coalitions is evaluated by

$$(1-\delta)\left(\frac{1-\delta^{m-1}}{1-\delta} v(N) - v(S_1) - \delta v(S_2) - \ \ldots\ - \delta^{m-2} v(S_{m-1})\right).$$

Furthermore, if the cost of renegotiations is not negligible, the efficiency loss becomes greater as the coalition becomes larger in the process of renegotiation.

We next investigate when an efficient coalition can be formed immediately, that is in the first round. To simplify the analysis, assume that

$$0 \leqslant v(S) < v(N) \text{ for any } S \in \Sigma, \qquad \text{and } 0 < v(S) \text{ for some } S \in \Sigma \text{ with } S \neq N. \quad (9)$$

Thus, only the grand coalition N can produce an efficient payoff allocation, and the unanimous game where $v(S) = 0$ for every subcoalition $S \subset N$ is excluded.[4)]

Definition 3: A non-cooperative solution $\sigma^* = (\sigma_1^*, \ldots, \sigma_n^*)$ of the bargaining game $\Gamma^\delta(\Sigma, v)$ is termed *renegotiation-proof* if the grand coalition N is formed in the first round of $\Gamma^\delta(\Sigma, v)$ and also in the first round of every renegotiation subgame of $\Gamma^\delta(\Sigma, v)$, regardless of who is selected as a proposer.

In a renegotiation-proof solution, all players agree to form the grand coalition immediately, i.e. in the first round of $\Gamma^\delta(\Sigma, v)$, and also after every possible agreement history. That is, renegotiation does not take place either on or off equilibrium plays

4) When v is an n-person unanimous game, the bargaining game $\Gamma^\delta(\Sigma, v)$ is simply reduced to the random-proposers model without renegotiations. In this case, for any $\delta < 1$, there exists a unique solution in which the grand coalition is formed in the first round.

and thus no efficiency loss is realized by the gradual formation of coalitions in the bargaining game $\Gamma^{\delta}(\Sigma, v)$.

When the discount factor δ is almost zero, the bargaining game $\Gamma^{\delta}(\Sigma, v)$ is reduced to the "ultimatum" bargaining game, where all players receive zero payoffs if no agreement is made in the first round. Therefore, in such a case every proposer demands the whole value $v(N)$ of the grand coalition in equilibrium and this is accepted by all the others. This equilibrium is clearly renegotiation-proof.

The following theorem presents a necessary and sufficient condition for a renegotiation-proof solution of the bargaining game $\Gamma^{\delta}(\Sigma, v)$ to exist.

Theorem 3: *A renegotiation-proof solution of the bargaining game $\Gamma^{\delta}(\Sigma, v)$ exists if and only if*

$$\delta \leqslant \frac{v(N) - v(S)}{v(N) - (s/n)v(S)} \qquad \text{for any } S \in \Sigma \text{ with } 0 < v(S) < v(N), \tag{10}$$

where s is the number of players in S and, moreover, the same inequality as (10) holds when the characteristic function v on Σ is replaced with the characteristic function v^T on $\Sigma(T)$ for every $T \in \Sigma$, where $v^T(S) = v(S) - v(T)$ for $S \in \Sigma(T)$.[5)]

Proof (only-if part). Let $v_i = v_i^{\delta}$ be the expected payoff of player $i (\in N)$ for the bargaining game $\Gamma^{\delta}(\Sigma, v)$ in a renegotiation-proof solution σ^*. From Theorem 1, we have

$$v_i = \frac{1}{n}\left(v(N) - \delta \sum_{j: j \in N, j \neq i} v_j\right) + \frac{n-1}{n}\delta v_i \qquad \text{for every } i = 1, \ldots, n. \tag{11}$$

It is easy to see that (11) has a unique solution $v_1 = \ldots = v_n = v(N)/n$ for any $\delta < 1$. For every $S \in \Sigma$, let v_i^S be the expected payoff of player i for the bargaining game $\Gamma^{\delta}(\Sigma(S), v^S)$ in σ^*. Since the grand coalition is formed in $\Gamma^{\delta}(\Sigma(S), v^S)$ if σ^* is played, an equation similar to (11) yields $v_i^S = v^S(N)/n = (v(N) - v(S))/n$ for all $i \in N$. Then, since it is optimal for player i to propose the grand coalition N in σ^*, Theorem 1 imposes, for every $S \in \Sigma$,

$$\begin{aligned} v(N) - \sum_{j \in N, j \neq i} \delta \frac{v(N)}{n} &\geqslant v(S) - \sum_{j \in S, j \neq i}\left(\delta \frac{v(N)}{n} - \delta \frac{v(N) - v(S)}{n}\right) \\ &\quad + \delta \frac{v(N) - v(S)}{n} = v(S) - \sum_{j \in S, j \neq i} \delta \frac{v(N)}{n} + \delta s \frac{v(N) - v(S)}{n}. \end{aligned} \tag{12}$$

This yields

$$(1 - \delta)v(N) \geqslant (1 - \delta s/n)v(S),$$

which is equivalent to (10). By Definition 3, the renegotiation-proof solution σ^* of $\Gamma^{\delta}(\Sigma, v)$ induces a renegotiation-proof solution to the bargaining game $\Gamma^{\delta}(\Sigma(T), v^T)$ for every $T \in \Sigma$. Accordingly, the same argument as above yields the last part of the condition.

5) If there is no $S \in \Sigma(T)$ such that $0 < v^T(S) < v^T(N)$, then no condition is imposed.

(if-part). We define the following strategy for every player i in $\Gamma^\delta(\Sigma, v)$. Given every round t and every agreement (T, x),

(a) when (T, x) is the current agreement at the beginning of round t, (i) propose (N, x^t) such that

$$x_i^t = x_i + \left(1 - \frac{n-1}{n}\delta\right)v^T(N) \qquad \text{and}$$

$$x_j^t = x_j + \frac{v^T(N)}{n}\delta, \qquad \text{for every } j \neq i,$$

where $v^T(N) = v(N) - v(T)$, and (ii) accept any new proposal (S, y) with $S \supset T$ if and only if $y_i + v^S(N)\delta/n \geqslant x_i + v^T(N)\delta/n$;

(b) when no agreement is reached before round t, emply the same strategy as in (a) by setting $(T, x) = (\emptyset, 0)$ (the null agreement) and $v^T = v$.

Let σ^* be the strategy for $\Gamma^\delta(\Sigma, v)$ defined by (a) and (b). When σ^* is employed, every player receives the expected payoff $v(N)/n$ in $\Gamma^\delta(\Sigma, v)$. It is clear that σ^* is renegotiation-proof and subgame-consistent. I will show that σ^* prescribes every player's locally optimal choice at his every move in $\Gamma^\delta(\Sigma, v)$ if the condition of the theorem holds. For notational simplicity, we consider case (b) only. The same arguments can be applied to case (a) in view of lemma 1. Without loss of generality, we can assume $t = 1$ in case (b). In σ^*, every player i proposes the grand coalition N and receives the payoff $\{1 - (n-1)\delta/n\}v(N)$. On the other hand, if he proposes a subcoalition S, player i obtains either (at most)

$$v(S) - \sum_{j \in S, j \neq i}\left(\delta\frac{v(N)}{n} - \delta\frac{v^S(N)}{n}\right) + \delta\frac{v^S(N)}{n}$$

or $v(N)\delta/n$, in the case of no agreement. Noting that

$$[1 - (n-1)\delta/n]v(N) > v(N)\delta/n,$$

(12) implies that σ^* prescribes a locally optimal choice of the proposer under (10). When a proposal (S, y) is made, every responder j receives $y_j + \delta v^S(N)/n$ if he and all remaining responders accept it, and $\delta v(N)/n$ otherwise. Thus, σ^* prescribes a locally optimal choice of responder j. Finally, it is known that local optimality of a strategy implies global optimality in an infinite-length perfect-information game, such as the bargaining game $\Gamma^\delta(\Sigma, v)$, in which a player's evaluation is given by the sum of discounted payoffs (see Fudenberg and Tirole, 1991, theorem 4.2). This fact can also be proved by the same method as Selten (1981, p. 137). Therefore, σ^* is a subgame-perfect equilibrium point of $\Gamma^\delta(\Sigma, v)$. ■

Theorem 3 shows that a renegotiation-proof solution of the game $\Gamma^\delta(\Sigma, v)$ exists if and only if the discount factor is not very large. More precisely, we can prove the following theorem from Theorem 3.

Theorem 4: *There exists some* $0 < \delta^* < 1$ *such that a renegotiation-proof solution of the game* $\Gamma^\delta(\Sigma, v)$ *exists if and only if* $0 \leqslant \delta \leqslant \delta^*$.

Proof. Let Σ^* be the class of all coalitions $T \in \Sigma$ such that there exists some $S \in \Sigma$

satisfying $T \subset S$ and $0 < v^T(S) < v^T(N)$. From (9), Σ^* includes the null coalition $\emptyset$. For every T in Σ^*, define

$$\delta(T) = \min\left\{ \frac{v^T(N) - v^T(S)}{v^T(N) - (s/n)v^T(S)} \middle| S \in \Sigma(T),\ 0 < v^T(S) < v^T(N) \right\},$$

$$\delta^* = \min\{\delta(T) | T \in \Sigma^*\}.$$

We can prove that $0 < \delta(T) < 1$ for each $T \in \Sigma^*$, and that the same inequality holds for δ^* since Σ^* is a finite set. Then the theorem follows from Theorem 3. ■

Theorem 4 shows that there exists some critical value δ^*, strictly less than one, of the discount factor δ such that a renegotiation-proof solution of $\Gamma^\delta(\Sigma, v)$, in which the grand coalition is formed in the first round, exists if and only if the discount factor is below the critical value. The payoff allocation in such a solution is in favour of the proposer and he can exploit the $[1 - (n-1)\delta/n]$ portion of the whole value $v(N)$.

4. The first-mover rent in renegotiations

Theorem 4 shows that there is no renegotiation-proof solution of the bargaining game $\Gamma^\delta(\Sigma, v)$ if and only if the discount factor is higher than the critical value. In every non-cooperative solution of the game $\Gamma^\delta(\Sigma, v)$, players may propose subcoalitions instead of the grand coalition in the first round if they are patient enough. This is in contrast to the efficiency result in the model without renegotiations: for any sufficiently large discount factor, there exists an efficient solution in which the grand coalition is formed in the first round if and only if $v(N)/n \geqslant v(S)/s$ for every $S \in \Sigma$ (see Okada, 1996, theorem 3).

Why do players have an incentive to propose an inefficient subcoalition first in the process of renegotiations? An answer to this question can be found if we examine the optimal proposals of players characterized by Theorem 3. It can be seen from the optimality condition in (8) that every player i proposes coalition S to maximize his total discounted payoff,

$$v(S) - \sum_{j \in S, j \neq i} \delta v_j + \delta \sum_{k \in S} v_k^S. \tag{13}$$

Here, note that all constraints (ii) of the maximization problem (8) must be binding at an optimal solution. The last term of (13) shows that the proposer can "exploit" the sum of expected payoffs that all other members in his coalition S can gain in future renegotiations. This term is missing in the proposer's total payoff in the bargaining game without renegotiation, and we may call it the *first-mover rent* in renegotiations. When the discount factor δ is close to one, the first-mover rent becomes large enough to give players an incentive to propose subcoalitions first in negotiations.

Theorem 2 shows that the grand coalition can be eventually formed in the process of renegotiation. When players anticipate this result of a renegotiation, they may be tempted to propose a subcoalition first and to exploit the first-mover rent. If this is the case, the final agreement may result in an unequal payoff allocation. By the following example of a three-person symmetric game, it can be shown that renegotiations actually distort the equity of a payoff allocation when players are patient.

Example 2

$N = \{1, 2, 3\}$, $\Sigma = \{\text{all subsets of } N\}$, $v(\{i\}) = 0$ for $i = 1, 2, 3$; $v(\{1, 2\}) = v(\{2, 3\}) = v(\{1, 3\}) = a(0 < a < 1)$; $v(\{1, 2, 3\}) = 1$.

As all three players are symmetric in this example, the equal allocation (1/3, 1/3, 1/3) can be regarded as the only efficient payoff allocation satisfying equity. In the following analysis, we assume that the discount factor δ of future payoffs is almost equal to one. It is known that the equal allocation (1/3, 1/3, 1/3) is agreed on, regardless of a proposer, in the random-proposers model without renegotiations if and only if $0 \leqslant a \leqslant 2/3$ (Okada, 1996). The condition is equivalent to inclusion of equal allocation in the core of the game. How does this result change when players are allowed to renegotiate their agreements?

Consider the following strategies for players in the bargaining game $\Gamma^1(\Sigma, v)$:[6]

(*a*) When no previous agreement has been reached,
  (i) player 1 proposes $(\{1, 2\}, (2a/3, a/3, 0))$, and accepts any proposal whenever he is offered at least 1/3 in the 3-person coalition and at least $a/3$ in a two-person coalition;
  (ii) player 2 proposes $(\{2, 3\}, (0, 2a/3, a/3))$, and employs the same response rule as player 1;
  (iii) player 3 proposes $(\{1, 3\}, (a/3, 0, 2a/3))$, and employs the same response rule as player 1.

(*b*) After an agreement (S, x) has been reached for any two-person coalition S,
  (i) every player $i = 1, 2, 3$ proposes $(\{1, 2, 3\}, (x_i + (1 - a)/3\colon i = 1, 2, 3)$, and accepts any proposal whenever he can get at least $x_i + (1 - a)/3$.

When the strategy defined by (*a*) and (*b*) is employed, the final payoff allocations are given by $((1 + a)/3, 1/3, (1 - a)/3)$, $((1 - a)/3, (1 + a)/3, 1/3)$, $(1/3, (1 - a)/3, (1 + a)/3)$, respectively, when players 1, 2 and 3, respectively, are selected as the proposer in round 1. The expected payoff of every player is 1/3. It is easy to see that the strategy given by (*b*) composes a subgame-perfect equilibrium point in every renegotiation subgame after a two-person coalition is formed.

We next check the optimality of the response rule of every player i given by (*a*): if he rejects an offer x_i in the three-person coalition, negotiations go to the next round, and his expected payoff will be 1/3. Thus, it is optimal for him to accept any offer in the three-person coalition if he gets at least 1/3. Similarly, if player i accepts an offer x_i in a two-person coalition, he can receive the total payoff $x_i + (1 - a)/3$ where $(1 - a)/3$ is his expected gain in the renegotiation subgame. If he rejects it, he can obtain the expected payoff 1/3. Thus, it is optimal for him to accept the offer in any two-person coalition if $a/3 \leqslant x_i$.

Finally, we check the optimality of every player's proposal given by (*a*). Given the response rules of other players, he can get the total payoff $2a/3 + (1 - a)/3 =$

6) $\Gamma^1(\Sigma, v)$ represents the limit version of $\Gamma^\delta(\Sigma, v)$ where the discount factor δ converges to one.

$(1 + a)/3$ by proposing a two-person coalition, and at most $1/3$ by proposing the three-person coalition. Thus, it is optimal for him to propose a two-person coalition with the demand $2a/3$. Therefore, the strategy constructed above is a non-cooperative solution of $\Gamma^1(\Sigma, v)$ for any value $(0 < a < 1)$ of the two-person coalition. As Theorem 4 shows, there exists no renegotiation-proof solution of $\Gamma^1(\Sigma, v)$ in which the three-person coalition is immediately formed with equal payoff allocation.

To summarize the analysis of a three-person symmetric game in Example 2, the possibility of renegotiations can yield an efficient payoff allocation regardless of the value of two-person coalitions; the final allocation, however, is asymmetric in favour of the proposer in the first round. Thus, renegotiations may distort the equal allocation that can be agreed on under the bargaining rule without renegotiations when the game has a non-empty core.

4. Concluding remarks

I have examined the efficiency principle supporting the well-known Coase Theorem in the framework of non-cooperative coalitional bargaining theory. The main conclusions of the paper are two-fold. First, the efficiency principle holds true under successive renegotiations when threat-points of renegotiations are given by prevailing agreements. Second, the possibility of renegotiations may have a negative effect in distorting the equity of allocations by creating “vested interests” in the process of recontracting agreements. We have seen that the prospect of renegotiations itself motivates bargainers to propose subcoalitions first and to exploit the first-mover rent if they are sufficiently patient.

Finally, I conclude the paper with two remarks about future works.

(1) An empirical question of interest related to these theoretical results is whether and how an inefficient payoff allocation is agreed on among actual decision-makers. Okada and Riedl (1999) conducted experiments on three-person “ultimatum” bargaining with coalition formation. The game played in the experiments is a one-shot version of the present bargaining model with a predetermined proposer, where negotiation takes place in one and only one round (and where, alternatively, the discount factor for future payoffs is zero). In the case of no agreement, all three subjects receive zero payoffs. When the values of two-person coalitions were high (about 93% of that of the grand coalition), we observed a high frequency (about 83% of the total 368 observations) of two-person inefficient coalitions, in contrast to the equilibrium prediction (0%) assuming the monetary payoff maximization as the only motivational force of subjects. In our experiments, the formation of an inefficient subcoalition was caused by the monetary payoff maximization subject to negative reciprocity. (If you are unkind to me, I will be unkind to you.) When proposers made unequal offers, responders punished them by rejection. Anticipating such high acceptance levels of responders, a huge majority of proposers chose two-person inefficient coalitions.

(2) This paper shows that a full achievement of efficiency and equity in a payoff allocation is not always possible in voluntary bargaining among rational agents when there exists neither a centralized mechanism of arbitration nor a restriction on coalition formation. Nevertheless, some partial cooperation, even if inefficient or unequal (or both), can be realized through negotiations and may contribute to development of

a society. In turn, development may affect the possibility and the form of cooperation. In Okada (1998), I consider a dynamic interaction of cooperation and development in the framework of an *n*-person prisoners' dilemma with institutional arrangements, and investigate dynamic patterns of social development. Much work is left to future research towards a game theory of cooperation and development.

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