JOURNAL OF ECONOMIC THEORY 63, 299–325 (1994)

Farsighted Coalitional Stability*

MICHAEL SUK-YOUNG CHWE

Department of Economics, University of Chicago, 1126 East 59th Street, Chicago, Illinois 60637

Received September 9, 1992; revised May 6, 1993

I define the largest consistent set, a solution concept which applies to situations in which coalitions freely form but cannot make binding contracts, act publicly, and are fully "farsighted" in that a coalition considers the possibility that, once it acts, another coalition might react, a third coalition might in turn react, and so on, without limit. I establish weak nonemptiness conditions and apply it to strategic and coalitional form games and majority rule voting. I argue that it improves on the von Neumann–Morgenstern stable set as it is usually defined but is consistent with a generalization of the stable set as in the theory of social situations. *Journal of Economic Literature* Classification Numbers: C70, C71.

Two problems have occupied theories of strategic stability for nearly a century or more. The problem of "myopia" is expressed in Fisher's ([16]; see Scherer [46]) criticism of Cournot's duopoly model: "No business man assumes either that his rival's output or price will remain constant.... On the contrary, his whole thought is to forecast what move the rival will make in response to one of his own." The problem of emptiness is typified by Condorcet's ([14]; see Black [11]) "contradictory" case, in which each of three candidates loses by majority rule to another. The core (Gillies [17]) assumes that a coalition will deviate regardless of possible further deviations and is often empty.

* I am grateful for suggestions from many people, including Byoung-Song Chwe, Peter DeMarzo, Joseph Greenberg, Charlie Kahn, Ehud Kalai, Michael Maschler, Aki Matsui, Steve Matthews, Roger Myerson, Andy Newman, Charlie Plott, Stan Reiter, Jim Snyder, Alexander Sotskov, Robert Weber, Steve Williams, Asher Wolinsky, and anonymous referees. I am also thankful for the hospitality and comments of participants in seminars at the University of California, Davis, the University of Chicago, Korea University, the University of Minnesota, Ohio State University, the University of Pittsburgh, Rutgers University, the University of Western Ontario, and of conference participants at the International Conference on Game Theory and Economics, State University of New York at Stony Brook, 1991, and at the MEDS Summer Game Theory Conference, Northwestern University, 1992. The most severe audience, however, was my Spring 1993 game theory class at the University of Chicago, who discovered two mistakes in the penultimate version.

0022-0531/94 $6.00

The "largest consistent set" defined here solves these two problems simultaneously: it takes "farsightedness" fully into account and is non-emtpy in a wide range of environments. It applies to situations in which coalitions freely form but cannot make binding contracts, act publicly, and are fully "farsighted" in that a coalition considers the possibility that, once it acts, another coalition might react, a third coalition might in turn react, and so on, whithout limit.

The concept aims to be weak: not so good at picking out, but ruling out with confidence. Like rationalizability (Bernheim [8], Pearce [37]), it does not determine what will happen but what can possibly happen. Like the von Neumann–Morgenstern [52] solution, also called the stable set, it is not defined by conditions on individual outcomes but by a condition on a set of outcomes. It improves on the stable set as it is usually defined but is consistent with a generalization of the stable set as in the theory of social situations (Greenberg [19]). I apply it to strategic form games, coalitional form games, and majority rule voting.

Motivation

To illustrate the problem of myopia, we can see how it applies to strong Nash equilibrium (Aumann [3]), the analogue of the core in the context of strategic form games. A strategy profile is a strong Nash equilibrium if no set of players can jointly deviate and make all of its members better off. For example, in the Prisoners' Dilemma, the only Nash equilibrium, (Defect, Defect), is not a strong Nash equilibrium since the two players could deviate to (Cooperate, Cooperate) and both become better off (Table I). The problem of myopia can be expressed in three ways. First, strong Nash equilibrium assumes that players do not consider the future. In the Prisoners' Dilemma, (Defect, Defect) is ruled out because the players see that (Cooperate, Cooperate) would be better. But they do not consider what might happen once they actually decide to play (Cooperate, Cooperate). Second, it involves dubious counterfactuals. The implicit assumption is that if the players were to deviate to (Cooperate, Cooperate), there would be no further deviations, an assumption which is unreasonable. Third, it is not consistent. The outcome (Defect, Defect) is not a strong Nash equilibrium because of the deviation to (Cooperate, Cooperate). But could this deviation actually take place if (Cooperate, Cooperate) is itself not a strong Nash equilibrium?

The bargaining set (Aumann and Maschler [6], Maschler [32]), deals with this by going one step further, considering deviations from deviations ("counterobjections") as well as deviations ("objections"). But fully rational players would surely consider deviations from deviations from deviations,

TABLE I

The Prisoners' Dilemma

	Cooperate	Defect
Cooperate	3, 3	0, 4
Defect	4, 0	1, 1

and so on. To fully address the problem of myopia, a solution concept should allow players to look arbitrarily far ahead. In the context of extensive form games, subgame perfect Nash equilibrium makes this assumption. What would be the analogue of subgame perfect Nash equilibrium in other contexts?

The stable set of von Neumann and Morgenstern [52] is defined to be self-consistent. But all concepts based on stable sets, such as coalition-proof Nash equilibrium (Bernheim *et al.* [9]), are subject to a criticism made by Harsanyi [21]. In stable sets, a deviation is invalidated if there is a further deviation to some stable outcome. But a coalition might deviate knowing full well that there will be a further deviation; it might like the further deviation even better. The largest consistent set defined here incorporates this insight.

Definition of the Game

A *game* Γ is defined as $\Gamma = (N, Z, \{\prec_i\}_{i \in N}, \{\rightarrow_S\}_{S \subset N, S \neq \varnothing})$, where N is the set of players, Z is the set of outcomes (N and Z are nonempty), $\{\prec_i\}$ are the players' strong preference relations defined on Z, and $\{\rightarrow_S\}$ are "effectiveness relations" defined on Z. The relation $\rightarrow_S$ represents what coalition S can do: $a \rightarrow_S b$ means that if a is the status quo, coalition S can make b the new status quo. It does not mean that coalition S can enforce b no matter what anyone else does; after S "moves to" b from a, another coalition T might move to c, where $b \rightarrow_T c$. No restrictions are placed on the effectiveness relations $\{\rightarrow_S\}$: $\rightarrow_S$ can be empty, $a \rightarrow_S a$ is possible, and $a \rightarrow_S b$ need not imply $b \rightarrow_S a$. This way of defining a game is similar to but less general than Rosenthal's [41] "effectiveness form" and Greenberg's [19] "inducement correspondence" (see also Moulin and Peleg [36] and Wilson [56]).

This game is "played" in the following manner. When the game begins (in fact at any given time) there is a status quo outcome, say a. If the members of a coalition S decide to change the status quo to outcome b, where $a \rightarrow_S b$, then the new status quo becomes b. This changing of the status quo we call a coalition's "move" or "deviation" "from" a "to" b. From this new status quo b, other coalitions might move, and so forth, without limit. All actions are public. If a status quo c is reached and no coalition decides to move from c, then c is a "stable" outcome and the game is over; then (and only then) the players receive their payoffs from c. Finally, there are no time preferences: players care only about the end outcome and not how many moves it takes to get there.

This game definition is of a cooperative and not noncooperative spirit in that many details are left out. From a status quo a, typically many different coalitions will be able to move from a. Coalitions do not move in a specified order: it is not the case that first a particular coalition S_1 gets to move from a, and if it does not move, then a particular coalition S_2 gets to move from a, and so on. The game well specifies what happens if coalition S_1 moves from a to b but not what happens if it does not move (although it does clearly specify what happens if no coalition moves from a). Issues such as preemptory moves (coalition S_1 moving from a to b to prevent coalition S_2 from moving from a to c) arise. Also, typically a given player will be a member of several coalitions, each of which has the ability to move. How does a player decide which coalition to "join?" Typically, a coalition will be able to move to several different outcomes. Which outcome will the coalition actually move to? These are related questions; in this game, the mechanisms by which players form a coalition and collectively decide which outcome to move to are not specified in any detail. However, the advantages of leaving out details such as these are those characteristic of cooperative approaches: relative simplicity and possibly wider applicability than a particular detailed (and thus perhaps necessarily arbitrary) specification. These issues will be further considered in the conclusion.

More Definitions

If $a \prec_i b$ for all $i \in S$, we write $a \prec_S b$. We say that a is *directly dominated* by b, or $a < b$, if there exists an S such that $a \rightarrow_S b$ and $a \prec_S b$. The logic behind the core is that if $a < b$, then a cannot be stable because the coalition S is capable of moving to b and all of its members prefer b to a.

Indirect dominance (Harsanyi's [21] "indirect dominance" relation is slightly different, but he introduces the one here (p. 1494) without giving it an explicit name) captures the idea that coalitions can anticipate other coalitions' actions.

Definition. We say a is *indirectly dominated* by b, or $a \ll b$, if there exist $a_0, a_1, a_2, ..., a_m$ (where $a_0 = a$ and $a_m = b$) and $S_0, S_1, S_2, ..., S_{m-1}$ such that $a_i \rightarrow_{S_i} a_{i+1}$ and $a_i \prec_{S_i} b$ for $i = 0, 1, 2, ..., m-1$.

For example, say every member of coalition S_0 prefers a_2 to a_0 ($a_0 \prec_{S_0} a_2$) but S_0 cannot move from a_0 to a_2 ($a_0 \not\rightarrow_{S_0} a_2$). According to the logic of the core, S_0 is stuck at a_0. However, say S_0 can move from a_0 to an outcome a_1 ($a_0 \rightarrow_{S_0} a_1$) and another coalition S_1 can move from a_1 to a_2 ($a_1 \rightarrow_{S_1} a_2$), and all members of S_1 prefer a_2 over a_1 ($a_1 \prec_{S_1} a_2$). Then coalition S_0 might move from a_0 to a_1, anticipating that S_1 would then move to a_2. So even though a_0 might not be directly dominated by a_2, it is *indirectly* dominated by a_2, and hence a_0, which might even be in the core, need not be stable.

Note that if $a < b$, then $a \ll b$. The indirect dominance relation $\ll$ can also be defined by a consistency criterion: if relations are thought of as subsets of $Z \times Z$, it is the smallest (with respect to set inclusion) relation which contains $<$ and satisfies the property ($a \rightarrow_S b$ and $b \ll c$ and $a \prec_S c) \Rightarrow a \ll c$. Note that $\ll$ is not the transitive closure of $<$, that is, the smallest transitive relation which contains $<$ (see Kalai *et al.* [23], Kalai and Schmeidler [24], Sen [47, p. 56], and Suzumura [48]).

One could object that just because $a \ll b$, it does not follow that the coalitions $S_0, S_1, ..., S_{m-1}$ will actually move from a to b: each coalition wants to reach b, but would not coalition S_0, for example, worry that the coalitions $S_1, S_2, ..., S_{m-1}$ might move toward some other outcome c? Also, would the coalitions move to b if b itself is not stable? We interpret indirect dominance this way: if $a \ll b$ and b is presumed stable, then it is possible, not certain, that the coalitions $S_0, S_1, S_2, ..., S_{m-1}$ will move from a to b.

To check if an outcome a is stable, consider a deviation by coalition S to d. There might be further deviations which end up at e, where $d \ll e$. There might not be any further deviations, in which case the ending outcome $e = d$. In either case the ending outcome e should itself be stable. If some member of coalition S does not prefer e to the original outcome a, then the deviation is deterred. An outcome is stable if every deviation is deterred. Since whether an outcome is stable depends on whether other outcomes are stable, the set of stable outcomes should satisfy a consistency conditon.

DEFINITION. A set $Y \subset Z$ is *consistent* if $a \in Y$ if and only if $\forall d, S$ such that $a \rightarrow_S d$, $\exists e \in Y$, where $d = e$ or $d \ll e$, such that $a \not\prec_S e$.

Again, the intention here is to define a weak concept, one which eliminates with confidence. A deviation is deterred as long as there is *some* stable ending outcome which might be reached which the deviating coalition does not prefer. If Y is consistent and $a \in Y$, the interpretation is not that a *will be* stable but that it is *possible* for a to be stable. If an outcome b is not contained in any consistent Y, the interpretation is that b cannot possibly be stable: there is no consistent story in which b is stable.

Although there can be many consistent sets, there uniquely exists a "largest" consistent set, that is, a consistent set which contains all others. Thus, if an outcome is not in the largest consistent set, it cannot possibly be stable. The largest consistent set is the set of all outcomes which can possibly be stable.

PROPOSITION 1. *Say* $\Gamma = (N, Z, \{<_i\}_{i \in N}, \{\to_S\}_{S \subset N, S \neq \varnothing})$. *Then there uniquely exists a* Y *such that* Y *is consistent and* (Y' *consistent* $\Rightarrow Y' \subset Y$). *The set* Y *is called the largest consistent set of* Γ, *written* LCS(Γ).

Proof. (see Tarski [49]). Define a function $f: 2^Z \to 2^Z$, where $f(X) = \{a \in Z: \forall d, S$ such that $a \to_S d$, $\exists e \in X$, where $d = e$ or $d \ll e$, such that $a \not<_S e\}$. A set Y is consistent if and only if Y is a fixed point of f; that is, $Y = f(Y)$. Note that f is isotonic; that is, $X \subset Y \Rightarrow f(X) \subset f(Y)$.

Let $\Sigma = \{X \subset Z: f(X) \supset X\}$, which is nonempty since $f(\varnothing) \supset \varnothing$. Let $Y = \bigcup_{X \in \Sigma} X$. Since f is isotonic, $f(Y) \supset f(X)$ for all $X \in \Sigma$, and hence $f(Y) \supset \bigcup_{X \in \Sigma} f(X) \supset \bigcup_{X \in \Sigma} X = Y$. So $f(Y) \supset Y$. Since f is isotonic, $f(f(Y)) \supset f(Y)$. Hence $f(Y) \in \Sigma$, and thus $f(Y) \subset Y$. So $f(Y) = Y$; that is, Y is consistent. To show that it contains all other consistent sets, let $f(Y') = Y'$. Then $f(Y') \supset Y'$ and hence $Y' \subset Y$. ∎

This proof is very similar to that of Roth [42, 44]: characterize the solution concept as a fixed point of an isotonic function and use Tarski's argument to show that a fixed point exists.

NONEMPTINESS

Strictly speaking, existence and nonemptiness are different. For example, the core always exists, even though it can be the empty set. The von Neumann–Morgenstern solution sometimes does not exist: sometimes no set, including the empty set, satisfies the definition. Although the largest consistent set always uniquely exists, it can be empty, as in the following game: $Z = \{1, 2, 3, \ldots\}$, $N = \{1\}$, $i \to_{\{1\}} i+1$, and $i <_1 j$ if $i < j$. Here it makes sense that no outcome is stable, since Player 1 would always want to move to the next outcome.

When Z is finite or countably infinite, a sufficient condition for nonemptiness is that there are no anomalies of this sort; that is, there do not exist infinite $\ll$-chains: there is no $a_1, a_2, a_3, \ldots$ such that $i < j \Rightarrow a_i \ll a_j$. This condition also gives us the "external stability" property: for any outcome a, either a is in the largest consistent set or there is a stable b in the largest consistent set such that $a \ll b$. Starting from any initial status quo, the largest consistent set makes a prediction about which stable outcomes might be reached.

PROPOSITION 2. *Say* $\Gamma = (N, Z, \{\prec_i\}_{i \in N}, \{\to_S\}_{S \subset N, S \neq \varnothing})$, *where* Z *is finite or countably infinite. Say that there are no infinite* $\ll$*-chains; that is, there is no* $a_1, a_2, a_3, \ldots$ *such that* $i < j \Rightarrow a_i \ll a_j$. *Then the largest consistent set* $\mathrm{LCS}(\Gamma)$ *is nonempty and has the external stability property*: $\forall a \in Z \backslash \mathrm{LCS}(\Gamma)$, $\exists b \in \mathrm{LCS}(\Gamma)$ *such that* $a \ll b$.

Proof. Say $X \subset Z$ is nonempty and define a relation $\lhd_X$ on Z:

$$a \lhd_X b \qquad \text{if} \quad (b \in X \Rightarrow a \ll b) \text{ and } ((c \in X \text{ and } b \ll c) \Rightarrow a \ll c).$$

Note that if $W \supset X$, then $a \lhd_W b \Rightarrow a \lhd_X b$. Show that $\lhd_X$ is transitive: say $a \lhd_X b$ and $b \lhd_X c$. First, say $c \in X$ and show $a \ll c$. Since $c \in X$ and $b \lhd_X c$, $b \ll c$. Since $c \in X$ and $b \ll c$, from $a \lhd_X b$ we know $a \ll c$. Second, say $d \in X$ and $c \ll d$ and show $a \ll d$. Since $b \lhd_X c$, $b \ll d$. Since $b \ll d$ and $a \lhd_X b$, $a \ll d$.

Given a nonempty $X \subset Z$ and relation $\lhd$, let $M(X, \lhd) = \{a \in X : \nexists b \in X$ such that $a \lhd b\}$. Show that for all $a \in X$, either $a \in M(X, \lhd_X)$ or $\exists b \in M(X, \lhd_X)$ such that $a \lhd_X b$ (hence $M(X, \lhd_X)$ is nonempty). Say $a \in X \backslash M(X, \lhd_X)$. Then $\exists a_1 \in X$ such that $a \lhd_X a_1$. If $a_1 \in M(X, \lhd_X)$, we are done. If not, then $\exists a_2 \in X$ such that $a_1 \lhd_X a_2$, and since $\lhd_X$ is transitive, $a \lhd_X a_2$. If $a_2 \in M(X, \lhd_X)$, we are done. Continuing in this manner, the only case we must check is if $\exists a_1, a_2, \ldots \in X$ such that $a_1 \lhd_X a_2 \lhd_X \cdots$. But then since $\lhd_X$ is transitive, $i < j \Rightarrow a_i \lhd_X a_j \Rightarrow a_i \ll a_j$, a contradiction.

Assume now that Z is finite. If we define a function $m: 2^Z \to 2^Z$, where $m(X) = M(X, \lhd_X)$, then let $Z_1 = m(Z)$, $Z_2 = m(m(Z))$, and in general let $Z_i = m^i(Z)$, where $Z_0 = Z$. So we have a nested sequence $Z_0 \supset Z_1 \supset Z_2 \supset \cdots$. Since Z is finite, $\exists j$ such that $Z_j = Z_{j+1}$. Let $V = Z_j$, and so $V = M(V, \lhd_V)$. Since Z_i nonempty $\Rightarrow Z_{i+1}$ nonempty, we know Z_i is nonempty for all i, and hence V is nonempty.

Show by induction that for all i, if $a \in Z \backslash Z_{i+1}$, then $\exists b \in Z_{i+1}$ such that $a \lhd_{Z_i} b$. This is true when $i = 0$ since $Z_1 = M(Z, \lhd_Z)$. Assume it is true for i and show it is true for $i + 1$. Say $a \in Z \backslash Z_{i+2}$. If $a \in Z_{i+1} \backslash Z_{i+2}$, we are done since $Z_{i+2} = M(Z_{i+1}, \lhd_{Z_{i+1}})$. If $a \in Z \backslash Z_{i+1}$, by the induction assumption, $\exists b \in Z_{i+1}$ such that $a \lhd_{Z_i} b$. Since $Z_i \supset Z_{i+1}$, $a \lhd_{Z_{i+1}} b$. If $b \in Z_{i+2}$, we are done; if $b \in Z_{i+1} \backslash Z_{i+2}$, $\exists c \in Z_{i+2}$ such that $b \lhd_{Z_{i+1}} c$, and since $\lhd_{Z_{i+1}}$ is transitive, $a \lhd_{Z_{i+1}} c$.

So if $a \in Z \backslash V$, then $\exists b \in V$ such that $a \lhd_V b$, and hence $a \ll b$. Therefore V has the external stability property. Show also that $V \subset M(Z, \lhd_V)$. Say $a \in V$ and $a \notin M(Z, \lhd_V)$; that is, $\exists b \in Z$ such that $a \lhd_V b$. If $b \in V$, this contradicts $a \in V = M(V, \lhd_V)$. If $b \in Z \backslash V$, we know from above that $\exists c \in V$ such that $b \lhd_V c$, and hence $a \lhd_V c$, which also contradicts $a \in V = M(V, \lhd_V)$.

Now show $f(V) \supset V$, where f is defined in the proof of Proposition 1. Say $a \in V$ and $a \notin f(V)$. Then $\exists d, S$, where $a \to_S d$, such that $\forall e \in V$ such that $d = e$ or $d \ll e$, $a \prec_S e$. But then $d \in V \Rightarrow a \ll d$ and $\forall e \in V$ such that $d \ll e$,

$a \ll e$. Thus $a \lhd_V d$, a contradiction since $a \in V \subset M(Z, \lhd_V)$. Since $f(V) \supset V$ and V is nonempty, by the proof of Proposition 1 LCS(Γ) is nonempty; it contains V and hence has the external stability property.

We sketch the proof when Z is countably infinite. Define a family of sets $\{Z_{n_1, n_2, n_3, \dots}\}$ recursively, where $Z_{0,0,0,\dots} = Z$, $Z_{n_1+1, n_2, n_3, \dots} = m(Z_{n_1, n_2, n_3, \dots})$, $Z_{0, n_2+1, n_3, \dots} = \bigcap_{i=0}^{\infty} Z_{i, n_2, n_3, \dots}$, $Z_{0,0,n_3+1,\dots} = \bigcap_{i=0}^{\infty} Z_{0, i, n_3, \dots}$, and so on. It is easy to see that if $l_1, l_2, l_3, \dots$ precedes $n_1, n_2, n_3, \dots$ in the "backward lexicographic" ordering, then $Z_{l_1, l_2, l_3, \dots} \supset Z_{n_1, n_2, n_3, \dots}$.

Given that there are no infinite $\ll$-chains, we can show that $Z_{n_1, n_2, n_3, \dots}$ is nonempty for all $n_1, n_2, n_3, \dots$. If $Z_{n_1+1, n_2, n_3, \dots} \neq Z_{n_1, n_2, n_3, \dots}$ for all n_1, n_2, $n_3, \dots$, then given $n_1, n_2, n_3, \dots$ we can uniquely assign an $a_{n_1, n_2, n_3, \dots} \in Z_{n_1, n_2, n_3, \dots} \setminus Z_{n_1+1, n_2, n_3, \dots}$. But then $\{a_{n_1, n_2, n_3, \dots}\} \subset Z$ is uncountably infinite, a contradiction. So there exists a nonempty V such that $V = M(V, \lhd_V)$, and we similarly show that $V \subset M(Z, \lhd_V)$ and has external stability. ▮

This proof is very similar to a construction of Gillies [17], who takes the direct dominance relation $<$ and defines the "majorization" relation: b majorizes a if $a < b$ and for all $c \in Z$ such that $b < c$, $a < c$. Call Z^* the unmajorized elements of Z. Then a new majorization relation can be defined on Z^*: b majorizes a if $a < b$ and for all $c \in Z^*$ such that $b < c$, $a < c$. Call Z^{**} the unmajorized elements of Z^*, and similarly generate Z^{***}, Z^{****}, ..., each of which can be shown to contain all von Neumann–Morgenstern solutions of the original game. Gillies's goal was to simplify finding von Neumann–Morgenstern solutions. My proof starts with the indirect instead of the direct dominance relation and shows that the nonempty limit of the sequence is contained in the largest consistent set.

When the set of outcomes Z is finite, a simple way to satisfy the no infinite $\ll$-chains condition is to require that preferences be irreflexive; that is, no player prefers an outcome to itself.

COROLLARY. *Say* $\Gamma = (N, Z, \{\prec_i\}_{i \in N}, \{\rightarrow_S\}_{S \subset N, S \neq \varnothing})$, *where* Z *is finite and preferences are irreflexive*: $\forall i \in N$, $a \in Z$, $a \not\prec_i a$. *Then the largest consistent set* LCS(Γ) *is nonempty and has the external stability property.*

Proof. Say that there exists an infinite $\ll$-chain: $a_1, a_2, a_3, \dots$ such that $i < j \Rightarrow a_i \ll a_j$. Since Z is finite, $\exists i, j$ such that $i < j$ and $a_i = a_j$. Thus $a_i \ll a_i$, a contradiction since $a_i \prec_S a_i$ is impossible. ▮

HOW TO FIND THE LARGEST CONSISTENT SET WHEN THE SET OF OUTCOMES IS FINITE

Finding the largest consistent set of a game with a finite set of outcomes is easy. For example, say we have two players and four outcomes: $N = \{1, 2\}$ and $Z = \{a, b, c, d\}$. Say preferences are given by $a \prec_1 b \prec_1 c \prec_1 d$ and

$d \prec_2 a \prec_2 b \prec_2 c$ (when written this way, assume preferences are transitive), and the effectiveness relations are $a \rightarrow_{\{1\}} b$, $b \rightarrow_{\{2\}} c$, $c \rightarrow_{\{1\}} d$, $d \rightarrow_{\{2\}} a$, and $a \rightarrow_{\{1,2\}} c$. We compute the indirect dominance relation iteratively (remember $\ll$ is the smallest relation which contains the direct dominance relation $<$ and satisfies the property ($a \rightarrow_S b$ and $b \ll c$ and $a \prec_S b) \Rightarrow a \ll c$) to find that $a \ll b$, $a \ll c$, $b \ll c$, $c \ll d$, $d \ll a$, $d \ll b$, and $d \ll c$ exactly define the indirect dominance relation.

To find the largest consistent set, we employ another iterative procedure. Take the function f as defined in the proof of Proposition 1—remember a set Y is consistent if and only if $f(Y) = Y$. We know $Z \supset f(Z)$. Since f is isotonic, we can apply it to both sides to get $f(Z) \supset f(f(Z))$. We can apply it to both sides again to get $f(f(Z)) \supset f(f(f(Z)))$, and we have a nested sequence $Z \supset f(Z) \supset f(f(Z)) \supset f(f(f(Z))) \supset \cdots$. Since Z is finite, $f^j(Z) = f^{j+1}(Z)$ for some j. Thus $f^j(Z)$ is consistent, and it is easy to see that it must be the largest consistent set.

So start with $Z = \{a, b, c, d\}$. It turns out that $f(Z) = \{b, c\}$. Note that if Player 1 moves from a to b, there are only two possibilities (look at the indirect dominance relation): either no one will move from b or Player 2 will move from b to c. Either way, Player 1 is better off, so he will move from a to b. So a cannot be stable. Also, note that if Player 2 moves from d to a, there are three possibilities: either no one will move from a, Player 1 will move from a to b and no one will further move, or c will be reached (either the coalition $\{1, 2\}$ will move from a to c directly or Player 1 will move from a to b and then Player 2 will move from b to c). All of these cases are better for Player 2 than remaining at d. So d cannot be stable. The outcome b is (provisionally) stable since Player 2's deviation to c is deterred by the further deviation of Player 1 to d. The outcome c is (provisionally) stable since Player 1's deviation to d is deterred by, for example, Player 2's further deviation to a.

Now compute $f(f(Z)) = f(\{b, c\})$. It turns out that $f(\{b, c\}) = \{c\}$. Since b and c are now the only possible stable outcomes, if Player 2 deviates from b to c, the only possibility is that no one will move from c. Before this deviation was deterred by Player 1's further deviation to d. But we just showed that d cannot be stable; there will surely be further deviations. The outcome c is still stable: Player 1 would not deviate to d because $d \ll c$; the players will move from d right back to c.

It is easy to check that $f(\{c\}) = \{c\}$; therefore $\{c\}$ is the largest consistent set. This process is the spirit behind the largest consistent set: first throw out outcomes which cannot be stable; given this, throw out outcomes which cannot be stable; repeat until you cannot throw out any more. The mechanical nature of this algorithm lends itself to mechanical computation (the computer program I use is documented in Chwe [13] and is available from the author).

A Comparison with the Stable Set

The stable set of von Neumann and Morgenstern [52] is one of the earliest concepts of game theory (see Lucas [30]). As Harsanyi [21] argues, however, the stable set as usually defined does not capture the assumption of fully farsighted players because of a conceptual flaw: namely, a further deviation need not deter but can actually encourage a deviation. The largest consistent set takes this insight into account and improves on the stable set as it is usually defined. However, the largest consistent set and the stable set can be interestingly reconciled within the theory of social situations (Greenberg [19]).

Given Z and a relation $\lhd$ defined on Z, we say V is a *stable set* of $(Z, \lhd)$ if (1) $\nexists a, b \in V$ such that $a \lhd b$ (internal stability); and (2) $\forall b \in Z \backslash V$, $\exists a \in V$ such that $b \lhd a$ (external stability). Equivalently, if we define the "dominion" of V to be $\mathrm{Dom}(V) = \{a \in Z\colon \exists b \in V$ such that $a \lhd b\}$, the set of outcomes dominated by outcomes in V, then V is a stable set of $(Z, \lhd)$ if and only if $V = Z \backslash \mathrm{Dom}(V)$.

von Neumann and Morgenstern argue that stable sets of $(Z, <)$, where $<$ is the direct dominance relation, are solutions to the game Γ. Stable sets do not always exist. Take for example the Condorcet "paradox," in which $N = \{1, 2, 3\}$, $Z = \{a, b, c\}$, preferences are $a \prec_1 b \prec_1 c$, $c \prec_2 a \prec_2 b$, $b \prec_3 c \prec_3 a$, and effectiveness relations are $a \rightarrow_{\{1,2\}} b$, $b \rightarrow_{\{1,3\}} c$, and $c \rightarrow_{\{2,3\}} a$. Then $a < b$, $b < c$, and $c < a$, and no stable set exists. The largest consistent set, however, is $\{a, b, c\}$.

When stable sets do exist, they can make predictions quite different from the largest consistent set. For example, let $N = \{1, 2\}$, $Z = \{a, b, c, d\}$, $a \prec_1 b \prec_1 c \prec_1 d$, $d \prec_2 a \prec_2 b \prec_2 c$, and $a \rightarrow_{\{1\}} b$, $b \rightarrow_{\{2\}} c$, $c \rightarrow_{\{1\}} d$, $d \rightarrow_{\{2\}} a$, and $a \rightarrow_{\{1,2\}} c$ (the example we computed in the previous section). Then $a < b$, $b < c$, $c < d$, $d < a$, and $a < c$. The only stable set is $\{b, d\}$, while the largest consistent set is $\{c\}$.

In this game, according to the logic of the stable set (von Neumann and Morgenstern [52, p. 265]), the reason that Player 2 will not deviate from d to a is that a is "unsound," being dominated by b, an element of the stable set. But Player 2 prefers b over d; in fact, she likes it even better than a. The fact that a is dominated by b does not deter Player 2's deviation; if anything, it encourages it.

To make this clearer, consider a game in which $N = \{1, 2\}$, $Z = \{a, b, c\}$, $a \prec_1 c \prec_1 b$, $a \prec_2 b \prec_2 c$, and $a \rightarrow_{\{1\}} b$ and $b \rightarrow_{\{2\}} c$. Then $a < b$ and $b < c$, and the only stable set is $\{a, c\}$. Again, the fact that b is dominated by c does not deter Player 1's deviation from a to b, since Player 1 prefers c over a. Player 2 would surely move from b to c, and hence Player 1 would surely move from a to b; the only stable outcome should be c. The largest consistent set is $\{c\}$, making this prediction.

What matters is not *whether* a deviation is vulnerable to further deviations but *to where* a deviation will ultimately lead. Not all further deviations will deter the deviation; some might encourage it. Consider a game exactly like the above, but in which Player 1's preferences are $c \prec_1 a \prec_1 b$ instead. Then $a < b$ and $b < c$: the domination relation is the same, and hence the only stable set is again $\{a, c\}$. But in this game Player 1 does not prefer c over a, and hence Player 2's move from b to c deters Player 1's move from a to b, making a stable. The largest consistent set is $\{a, c\}$. The dominance relation, and hence the stable set, treats these two games identically, ignoring relevant information.

The two games are different in that c indirectly dominates a in the first but not in the second. It turns out that the most immediate way to deal with farsightedness, by considering stable sets of $(Z, \ll)$ instead of $(Z, <)$, is not bad. The logic goes like this: say V is a stable set of $(Z, \ll)$ and let $a \in V$. Say there is a deviation $a \to_S d$. If $d \in V$, then $a \nprec_S d$, because otherwise $a \ll d$, violating internal stability, and so coalition S is deterred. So let $d \in Z \backslash V$. By external stability, there is an $e \in V$ such that $d \ll e$. But since a and e are in V, by internal stability, $a \not\ll e$. Hence it must be that $a \nprec_S e$, and so coalition S is deterred by the further deviations to e. Hence a is stable.

The reason that this argument works with indirect but not direct dominance is because if $a \to_S d$ and $d \ll e$, then $a \not\ll e$ tells us something about the preferences of coalition S. If $a \to_S d$ and $d < e$, then $a \not< e$ does not tell us anyting. It could be that $a \nprec_S e$, in which case the deviation is deterred, or it could be that $a \not\to_S e$ and $a \prec_S e$, in which case the deviation is encouraged.

So if players are farsighted, stable sets of $(Z, \ll)$ are good and stable sets of $(Z, <)$ are not so good. It is easy to show that any stable set of $(Z, \ll)$ is contained in the largest consistent set.

PROPOSITION 3. *Say* $\Gamma = (N, Z, \{\prec_i\}_{i \in N}, \{\to_S\}_{S \subset N, S \neq \varnothing})$. *If* V *is a stable set of* $(Z, \ll)$, *then* $V \subset \mathrm{LCS}(\Gamma)$.

Proof. Say V is a stable set of $(Z, \ll)$. By the proof of Proposition 1, it suffices to show that $f(V) \supset V$. Say $a \in V$ and $a \notin f(V)$. Then $\exists d, S$, where $a \to_S d$, such that for all $e \in V$ such that $d = e$ or $d \ll e$, $a \prec_S e$. So $d \in V \Rightarrow a \ll d$ and $\forall e \in V$ such that $d \ll e$, $a \ll e$. Say $d \in V$. Then $a \ll d$, violating internal stability. So let $d \in Z \backslash V$. From external stability, $\exists e \in V$ such that $d \ll e$. But then $a \ll e$, violating inner stability. ∎

The situation is similar with the core: the largest consistent set has no relationship with the core of $(Z, <)$, but when it has the external stability property (as in Proposition 2) it is easy to see that it contains the core of $(Z, \ll)$.

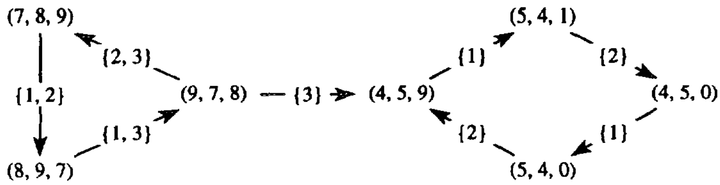

FIG. 1. The stable set is arbitrary.

The familiar disadvantages of the stable set of $(Z, \ll)$ are that it sometimes does not exist, and when it does it need not be unique (but see Asilis and Kahn [1]). But there are two further disadvantages. First, it is sometimes arbitrary, as in the following three-player game, represented as a directed graph (Fig. 1).

Here there are seven outcomes, each represented by a utility profile, and the effectiveness relations are represented by labeled directed arcs. In this game the indirect and direct dominance relations are equivalent, and the only stable set is $\{(8, 9, 7), (4, 5, 9), (4, 5, 0)\}$. Note that the four rightmost outcomes, $\{(4, 5, 9), (5, 4, 1), (4, 5, 0), (5, 4, 0)\}$, form a game in itself in the following sense: Players 1 and 2 are the only players; once in this set, Player 3 can do nothing, and Players 1 and 2 cannot move outside this set. In this smaller game, both $\{(5, 4, 1), (5, 4, 0)\}$ and $\{(4, 5, 9), (4, 5, 0)\}$ are stable sets, and in fact they are completely symmetric with respect to Players 1 and 2. But only (4, 5, 9) and (4, 5, 0) are considered stable in the larger game because of the three leftmost outcomes. Whether an outcome is stable depends on outcomes which cannot possibly be reached. The largest consistent set of this game is the set of all outcomes, which is not much of a prediction but at least does not rule out arbitrarily.

Secondly, the stable set cannot make "obvious" predictions, as in the following modification of the Condorcet "paradox" (Fig. 2). Here Player 3 will surely move from (0, 0, 0) because she would be better off no matter what happens. But no stable set of $(Z, \ll)$, or of $(Z, <)$, exists. The largest consistent set, however, is $\{(9, 7, 8), (7, 8, 9), (8, 9, 7)\}$.

In summary, replacing direct dominance with indirect improves the stable set, but it still has disadvantages when compared to the largest

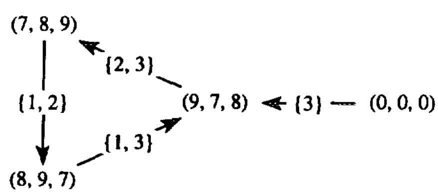

FIG. 2. The stable set cannot make an "obvious" prediction.

consistent set. The main disadvantage of the largest consistent set seems to be its lack of internal stability, as in the Condorcet "paradox." But there are two ways to respond this criticism.

First, the largest consistent set, like rationalizability, does not try to say what *will* happen but what can *possibly* happen. Saying that all three outcomes of the Condorcet "paradox" *are* stable might be self-contradictory, but saying that all three outcomes are *possibly* stable is not. In the example above, we cannot say that a given outcome in $\{(9, 7, 8), (7, 8, 9), (8, 9, 7)\}$ *is* stable, but at least we can say that $(0, 0, 0)$ cannot *possibly* be stable.

Second, some games have so much domination (direct or indirect) that almost no nontrivial solution concept can satisfy internal stability. For example, if an odd number of voters choose by majority rule over a finite set of alternatives and each voter has complete strong preferences (discussed in a following section), then in any pair of outcomes one outcome dominates the other. If one outcome exists which dominates all other outcomes (a "Condorcet winner") then a stable set exists. Otherwise, no stable set exists because no set of two or more outcomes can possibly satisfy internal stability. Here internal stability works against not just the largest consistent set, but against any solution concept which does not always yield singletons or the empty set.

For the sake of comparison I have emphasized the differences between the largest consistent set and the stable set. With the theory of social situations (Greenberg [19]), however, they can be brought together. Here I deploy the theory not in its full generality, but just enough for the purpose of demonstration.

So let each $a \in Z$ be a *position*. Each position a has associated outcomes $X(a) = \{a\} \cup \{b \in Z : a \ll b\}$; that is, from the position a the final outcome will either be a itself or an outcome b which indirectly dominates a. The set $X(a)$ denotes which outcomes are feasible from the position a. Say that $\sigma(a) \subset X(a)$ is the set of outcomes which might actually "happen." Both X and σ can be considered correspondences from Z to Z; we call σ a *standard of behavior*.

Not all standards of behavior are sensible: a standard of behavior should satisfy some notion of consistency. To define what we mean by this, let the *conservative dominion* of σ, $\text{CDom}(\sigma)$, be a correspondence from Z to Z defined by $\text{CDom}(\sigma)(a) = \{b \in X(a) : \exists S, d \text{ such that } b \rightarrow_S d,\ \sigma(d) \neq \emptyset, \text{ and } b \prec_S e\ \forall e \in \sigma(d)\}$. The idea here is as follows: given a standard of behavior σ, we can conclude that some of the outcomes in $X(a)$ will not happen. Take an outcome $b \in X(a)$ and say there exists some coalition S which can deviate to a position d. From this position d, according to the standard of behavior, $\sigma(d)$ is the set of outcomes which might happen. If the coalition S prefers every outcome in $\sigma(d)$ to b (note that $\sigma(d) \neq \emptyset$ prevents this from being satisfied trivially), then the coalition, even acting conservatively, would deviate to position d. So the outcome $b \in X(a)$ cannot happen.

We say that σ is a *conservative stable standard of behavior*, or CSSB, if $\sigma \equiv X \backslash \text{CDom}(\sigma)$; that is, $\sigma(a) = X(a) \backslash \text{CDom}(\sigma)(a)$ for all $a \in Z$. In other words, for every position a, outcomes outside $\sigma(a)$ are ruled out by σ, and no outcome in $\sigma(a)$ is ruled out by σ. The spirit of the stable set is here apparent: the only difference is that instead of a coalition deviating from an outcome to another outcome, a coalition deviates from an outcome to a position, from which several outcomes might occur. Coalitions are assumed to deviate conservatively, only if all possible outcomes make them better off. The connecting result (I am indebted to Joseph Greenberg and Benyamin Shitovitz for this) is as follows.

PROPOSITION 4. *Say* $\Gamma = (N, Z, \{\prec_i\}_{i \in N}, \{\rightarrow_S\}_{S \subset N, S \neq \varnothing})$.

(a) *If* σ *is a CSSB, then* $\exists Y \subset Z$ *such that* $\sigma(a) = X(a) \cap Y$ *for all* $a \in Z$. *This* Y *is consistent.*

(b) *If the condition of Proposition* 2 *holds, then the standard of behavior* σ *defined by* $\sigma(a) = X(a) \cap \text{LCS}(\Gamma)$ *is a CSSB.*

Proof. Straightforward; details can be obtained from the author. Note that in (b), the external stability property of LCS(Γ) guarantees that $\sigma(a) \neq \varnothing \ \forall a \in Z$. ▮

Result (a) says that if σ is a CSSB, then there is a consistent Y which makes exactly the same predictions: from a position $a \in Y$, the outcome a might happen ($a \in \sigma(a)$); from a position $a \in Z \backslash Y$, the set of outcomes which might happen are those outcomes in Y which indirectly dominate a ($\sigma(a) = \{b \in Z: a \ll b\} \cap Y$). Result (b) says that the largest consistent set thought of as a standard of behavior is actually a CSSB.

This connection shows not only how the largest consistent set and stable set are similar but also exactly how they differ. If instead we let $X(a) = \{a\}$ for all $a \in Z$, then a CSSB of this simpler setup corresponds exactly to a stable set of $(Z, <)$; that is, if V is a stable set of $(Z, <)$, then $\sigma(a) = \{a\} \cap V$ is a CSSB, and if σ is a CSSB, then there exists a stable set V such that $\sigma(a) = \{a\} \cap V$ for all $a \in Z$. The difference between the largest consistent set and the stable set as originally defined by von Neumann and Morgenstern is how $X(a)$ is defined. For the largest consistent set, an outcome is not just an outcome but a position from which a set of outcomes might occur.

Strategic Form Games

Say we have a strategic form game with a finite, nonempty set of players N, each individual $i \in N$ having a finite, nonempty, strategy set C_i and utility function $u_i: C \rightarrow \Re$, where $C = \times_{i \in N} C_i$, the set of strategy profiles (given $S \subset N$, let $C_S = \times_{i \in S} C_i$, and given $c \in C$ and $S \subset N$, let $c_S = (c_i)_{i \in S}$). There are at least four ways to apply the largest consistent set to this strategic form game.

First, the largest consistent set can predict which of several pure strategy Nash equilibria will be played if there is unlimited public preplay communication. Let Z be the set of pure strategy Nash equilibria, let $a \prec_i b$ if $u_i(a) < u_i(b)$, and let $a \rightarrow_S b$ if $a_{N \setminus S} = b_{N \setminus S}$. For example, consider the following four-person game (Table II). There are three pure strategy Nash equilibria: $(1a, 2a, 3a, 4a)$, $(1a, 2a, 3b, 4b)$, and $(1b, 2b, 3a, 4a)$. There is no strong Nash equilibrium. But if we let $Z = \{(1a, 2a, 3a, 4a), (1a, 2a, 3b, 4b), (1b, 2b, 3a, 4a)\}$ and define $\prec_i$ and $\rightarrow_S$ as above, then the largest consistent set is $\{(1b, 2b, 3a, 4a)\}$. So the unique prediction is that if the players can freely, but publicly, communicate before playing the game, then they will play $(1b, 2b, 3a, 4a)$.

This is perhaps the most natural application of the largest consistent set to strategic form games. Since individuals must play simultaneously and cannot commit themselves, the outcome of the game is presumably a Nash equilibrium. Which Nash equilibrium they will actually play is a matter of coalitional bargaining. In the example, Players 1 and 2 might propose to play $(1a, 2a, 3a, 4a)$. But then Players 3 and 4 might propose to play $(1a, 2a, 3b, 4b)$, and so forth. Since communication is public, any coalition can "further respond" and preplay negotiations do not exogenously end. If players are assumed to be farsighted, however, the largest consistent set can make a prediction.

The second application is to Greenberg's [19, p. 98] "individual contingent threats situation," in which the strategic form game is not "played"

TABLE II

Of the Three Nash Equilibria, the LCS Selects One

	2a	2b		2a	2b	
1a	3, 3, 1, 1	0, 0, 0, 0	1a	0, 0, 0, 0	0, 0, 0, 0	4a
1b	0, 0, 0, 0	2, 2, 3, 3	1b	0, 0, 0, 0	0, 0, 0, 0	
	2a	2b		2a	2b	
1a	0, 0, 0, 0	0, 0, 0, 0	1a	1, 1, 2, 2	1, 0, 1, 1	4b
1b	0, 0, 0, 0	0, 0, 0, 0	1b	0, 1, 1, 1	0, 0, 0, 0	
	3a			3b		

in the sense of simultaneous moves. Rather, each individual, facing a proposed strategy profile $c \in C$, can declare: "If all you other players stick to playing $c_{N \setminus \{i\}}$, I will play $d_i \in C_i$ instead of c_i." Each player can make such contingent threats, and realizes that other players can make contingent threats in turn. Players can revise their threats; no one is committed to anything.

So let $Z = C$, let $a \prec_i b$ if $u_i(a) < u_i(b)$, and let $a \rightarrow_S b$ if $S = \{i\}$ and $a_{N \setminus \{i\}} = b_{N \setminus \{i\}}$. Greenberg's solution is the stable sets of $(Z, <)$, where $<$ is the direct dominance relation. Stable sets of $(Z, <)$ always exist if there are one or two players (Greenberg [19, p. 100]) but not if there are three or more, as shown in the following game (Table III, adapted from Greenberg [19, p. 101]). Here seven outcomes dominate each other in a cycle. The other five outcomes do not dominate anything and each are dominated by one or more of these seven. Since seven is an odd number, no stable set exists. However, the largest consistent set is $\{(1a, 2b, 3b)\}$, making a unique prediction.

The largest consistent set in the individual contingent threats situation contains the strict Nash equilibria. A strategy profile $c \in C$ is a *strict Nash equilibrium* if for all $i \in N$, and for all $d_i \in C_i \setminus \{c_i\}$, $u_i(c_i, c_{N \setminus \{i\}}) > u_i(d_i, c_{N \setminus \{i\}})$. A strategy profile is a strict Nash equilibrium if each player is playing his unique best response, given the other players' strategies (Harsanyi [20] uses the word "strong" instead of "strict").

PROPOSITION 5. *Say $(N, (C_i)_{i \in N}, (u_i)_{i \in N})$ is a strategic form game in which N is finite and nonempty and the strategy sets C_i are finite and nonempty. Let $\Gamma = (N, Z, \{\prec_i\}_{i \in N}, \{\rightarrow_S\}_{S \subset N, S \neq \varnothing})$ be a game corrsponding to the individual contingent threats situation, where $Z = \times_{i \in N} C_i$, $a \prec_i b$ if $u_i(a) < u_i(b)$, and $a \rightarrow_S b$ if $S = \{i\}$ and $a_{N \setminus \{i\}} = b_{N \setminus \{i\}}$. If c is a strict (pure strategy) Nash equilibrium of the strategic form game, then $c \in \mathrm{LCS}(\Gamma)$.*

Proof. Say c is a strict (pure strategy) Nash equilibrium. By the proof of Proposition 1, it suffices to show that $f(\{c\}) \supset \{c\}$. Say $c \rightarrow_S d$. If $d = c$, then $c \nprec_S c$. If $d \neq c$, by the definition of Γ, we know that $S = \{i\}$ for some i and that $c_{N \setminus \{i\}} = d_{N \setminus \{i\}}$, and hence $d \rightarrow_{\{i\}} c$. Since c is a strict Nash equilibrium, $u_i(c) > u_i(d)$ and hence $d \prec_i c$. Hence $d \ll c$ and $c \nprec_i c$. ∎

TABLE III

No Stable Set Exists but the LCS Predicts a Unique Outcome

	2a	2b		2a	2b
1a	1, 1, 4	0, 0, 0	1a	0, 7, 1	6, 6, 5
1b	2, 2, 6	3, 3, 7	1b	0, 0, 0	0, 0, 0
1c	0, 0, 0	4, 4, 2	1c	0, 0, 0	5, 5, 3
	3a			3b	

TABLE IV

Strict, Not All, Nash Equilibria Are Contained in the LCS

	2a	2b
1a	1, 1	0, 0
1b	1, 1	2, 2

For example, in the following game (Table IV), the largest consistent set is $\{(1b, 2b)\}$. Here $(1a, 2a)$, a Nash equilibrium but not a strict Nash equilibrium, is not stable because the deviation to $(1b, 2a)$ cannot be deterred.

The third application is to coalitional analysis of the strategic form game. The traditional solution concepts are strong Nash equilibrium and the alpha-core and beta-core (Aumann [3, 4, 5], Aumann and Peleg [7]; see also Laffond and Moulin [26], Li [28], and Ray and Vohra [39]). To apply the largest consistent set we can use Greenberg's [19, p. 102] "coalitional contingent threats situation": not only each individual but each coalition S can declare: "If all you other players stick to playing $c_{N\setminus S}$, we will play $d_S \in C_S$ instead of c_S." So let $Z = C$, let $a \prec_i b$ if $u_i(a) < u_i(b)$, and let $a \rightarrow_S b$ if $a_{N\setminus S} = b_{N\setminus S}$. The set of strong Nash equilibria is the core of $(Z, <)$, and Greenberg's solution is the stable sets of $(Z, <)$ (Chakravorti and Kahn [12] also define an appropriate dominance relation and use a stable set approach).

The largest consistent set in the coalitional contingent threats situation contains the "strict" strong Nash equilibria. Here "strict" strong Nash equilibrium is defined similarly to strict Nash equilibrium: a strategy profile $c \in C$ is a *strict strong Nash equilibrium* if for all $S \subset N$, and for all $d_S \in C_S \setminus \{c_S\}$, there exists $j \in S$ such that $u_j(c) > u_j(d_S, c_{N\setminus S})$. *Strong Nash equilibrium* can be defined similarly by changing $>$ to $\geqslant$; hence all strict strong Nash equilibria are strong Nash equilibria. Also, it is easy to see that just as all strong Nash equilibria are Nash equilibria, all strict strong Nash equilibria are strict Nash equilibria.

PROPOSITION 6. *Say $(N, (C_i)_{i\in N}, (u_i)_{i\in N})$ is a strategic form game in which N is finite and nonempty and the strategy sets C_i are finite and nonempty. Let $\Gamma = (N, Z, \{\prec_i\}_{i\in N}, \{\rightarrow_S\}_{S\subset N, S\neq\varnothing})$ be a game corresponding to the coalitional contingent threats situation, where $Z = \times_{i\in N} C_i$, $a \prec_i b$ if $u_i(a) < u_i(b)$, and $a \rightarrow_S b$ if $a_{N\setminus S} = b_{N\setminus S}$. If c is a strict strong Nash equilibrium of the strategic form game, then $c \in \mathrm{LCS}(\Gamma)$.*

Proof. Say c is a strict strong Nash equilibrium. Show by induction that for all $d \neq c$, $d \ll c$. First, show that for all $d \in (C_i \setminus \{c_i\}) \times \{c_{N\setminus\{i\}}\}$, we have $d \ll c$. Since c is a strict strong Nash equilibrium, $u_i(c) > u_i(d)$, and hence $d \prec_i c$. Since $d \rightarrow_{\{i\}} c$, we have $d \ll c$. Next, assume that for all

TABLE V

The LCS and Strong Nash Equilibrium Differ

	2a	2b		2a	2b
1a	2, 2, 4	0, 2, 0	1a	5, 5, 3	0, 2, 0
1b	0, 0, 0	1, 4, 1	1b	0, 0, 0	3, 3, 0
	3a			3b	

$d \in \times_{i \in S}(C_i \backslash \{c_i\}) \times \{c_{N \backslash S}\}$, where $|S| = k$ and $k \geqslant 1$, we have $d \ll c$. Show that for all $d \in \times_{i \in S}(C_i \backslash \{c_i\}) \times \{c_{N \backslash S}\}$, where $|S| = k+1$, we have $d \ll c$. Let $d = (d_S, c_{N \backslash S})$, where $d_S \in \times_{i \in S}(C_i \backslash \{c_i\})$ and $|S| = k+1$. Since c is a strict strong Nash equilibrium, there exists some $j \in S$ such that $u_j(c) > u_j(d)$, and so $d \prec_j c$. Also, if we let $d' = (d_{S \backslash \{j\}}, c_j, c_{N \backslash S}) = (d_{S \backslash \{j\}}, c_{N \backslash (S \backslash \{j\})})$, we have $d \to_{\{j\}} d'$. But $d_{S \backslash \{j\}} \in \times_{i \in S \backslash \{j\}}(C_i \backslash \{c_i\})$ and $|S \backslash \{j\}| = k$ since $j \in S$. Hence, by assumption $d' \ll c$. Since $d \prec_j c$, $d \to_{\{j\}} d'$, and $d' \ll c$, we know $d \ll c$ by the definition of $\ll$.

To show that $c \in \mathrm{LCS}(\Gamma)$, by the proof of Proposition 1 it suffices to show that $f(\{c\}) \supset \{c\}$. Say $c \to_S d$. If $d = c$, then $c \not\prec_S c$. If $d \neq c$, we know that $d \ll c$ (from the above) and $c \not\prec_S c$. ∎

To see why only "strict" strong Nash equilibria are necessarily contained in the largest consistent set, consider the following game (Table V). The only strong Nash equilibrium is $(1a, 2a, 3a)$ but the largest consistent set is $\{(1a, 2a, 3b)\}$. From $(1a, 2a, 3a)$, Players 1 and 2 will move to $(1b, 2b, 3a)$ anticipating the move by all three players to $(1a, 2a, 3b)$.

However, "generically" the largest consistent set contains the strong Nash equilibria. It also makes a prediction when no strong Nash equilibria exist, as in the following game (Table VI). Here there is no strong Nash equilibrium, and there is no stable set of $(Z, <)$ or of $(Z, \ll)$. However, the largest consistent set is $\{(1a, 2a, 3a), (1a, 2b, 3b), (1b, 2b, 3a)\}$.

Finally, outcomes in the largest consistent set can be Pareto dominated by other outcomes in the largest consistent set (Table VII). In this game the largest consistent set is $\{(1a, 2a), (1b, 2b), (1b, 2c), (1c, 2b), (1c, 2c)\}$, even though $(1b, 2b)$ Pareto dominates $(1a, 2a)$.

In the fourth application, it is assumed that coalitions are concerned with the further deviations not of all coalitions but only of coalitions

TABLE VI

No Strong Nash Equilibria Exist

	2a	2b		2a	2b
1a	1, 2, 3	0, 0, 0	1a	4, 0, 4	3, 1, 2
1b	0, 4, 4	2, 3, 1	1b	0, 0, 0	4, 4, 0
	3a			3b	

TABLE VII

An Outcome in the LCS Is Pareto Dominated by Another Outcome in the LCS

	2*a*	2*b*	2*c*
1*a*	2, 4	0, 0	0, 0
1*b*	0, 0	3, 5	1, 7
1*c*	0, 0	7, 1	5, 3

composed of a subset of its members, "subcoalitions." Chakravorti and Kahn [12] call this the "nestedness" assumption: the coalition $\{1, 2, 3\}$ anticipates the further deviations of, say, $\{1, 2\}$ (who must in turn anticipate the further deviations of $\{1\}$ and $\{2\}$), but does not anticipate the further deviations of $\{4, 5\}$ or $\{1, 4\}$ (see also Maschler [31] and Thomson [50]). On this assumption Bernheim *et al.* [9] recursively define "coalition-proof Nash equilibrium." The idea is that a coalition's deviation can be invalidated by a subcoalition's valid further deviation; a further deviation is valid if there are no valid further deviations by "subsubcoalitions," and so forth, recursing all the way down to one-player coalitions, whose deviations are always valid.

For example, in the following game (Table VIII), $(1b, 2b, 3b)$ is the only coalition-proof Nash equilibrium. The deviation by $\{1, 2\}$ to $(1c, 2c, 3b)$ is invalidated by the further deviation of $\{1\}$ to $(1a, 2c, 3b)$. But here is the familiar problem: Players 1 and 2 are encouraged, not deterred, by this further deviation.

The largest consistent set is designed to apply to situations in which deviations are public and hence any coalition, not just subcoalitions, can further deviate. For the sake of comparison, however, we can adapt it to the nestedness assumption by letting $Z = C \times (2^N \backslash \{\varnothing\})$ and defining $(a, R) \prec_i (b, T)$ if $u^i(a) < u^i(b)$ and $(a, R) \rightarrow_S (b, T)$ if $R \supset S = T$ and $a_{N \backslash S} = b_{N \backslash S}$. Greenberg [18] shows that a stable set V of $(Z, <)$ exists uniquely, and a is a coalition-proof Nash equilibrium if and only if $(a, N) \in V$ (see also Kahn and Mookherjee [22]).

With this adaptation, we can compute the largest consistent set in this example. It turns out that $(1a, 2a, 3a)$ is the only outcome such that $(\cdot, N)$

TABLE VIII

The LCS and Coalition-Proof Nash Equilibrium Differ

	2*a*	2*b*	2*c*		2*a*	2*b*	2*c*
1*a*	1, 1, 1	0, 0, 0	0, 0, 0	1*a*	0, 5, 0	0, 0, 0	4, 4, 0
1*b*	0, 0, 0	0, 0, 0	0, 0, 0	1*b*	0, 0, 0	2, 2, 2	0, 0, 0
1*c*	0, 0, 0	0, 0, 0	0, 0, 0	1*c*	0, 0, 0	0, 0, 0	3, 3, 0
		3*a*				3*b*	

is in the largest consistent set. Player 3 will not move with the other players to $(1b, 2b, 3b)$ because of the further deviations which will end up at $(1a, 2c, 3b)$.

To summarize, the largest consistent set usefully applies to strategic form games in a variety of ways, and can provide interesting predictions, even under the nestedness assumption, for which it was not designed.

Coalitional Form Games

Most previous work on farsightedness has been on coalitional form games. It is in this context, however, that the largest consistent set is least satisfactory: proving nonemptiness is difficult and all but simple examples are hard to compute. Nevertheless, we can make some comparisons.

A (transferable utility) coalitional form game is defined by a characteristic function $v: 2^N \setminus \{\varnothing\} \to \Re$, which for each coalition S gives the total amount of utility $v(S)$ they can distribute among themselves without the help of players outside of S (we normalize $v(N) = 1$ and $v(\{i\}) = 0$ for all i). If we let $Z = \{a \in \Re^n : \sum_{i=1}^n a_i = 1 \text{ and } a_i \geqslant 0 \ \forall i\}$ be the set of imputations, and let $a \to_S b$ if $\sum_{i \in S} b_i \leqslant v(S)$, then we can apply the largest consistent set (nontransferable utility games can be adapted similarly).

In three-person games, the direct and indirect dominance relations are equivalent (see Harsanyi [21] and Weber [55], who define indirect dominance slightly differently). Hence all von Neumann–Morgenstern solutions (stable sets of $(Z, <)$) are stable sets of $(Z, \ll)$, and since stable sets of $(Z, \ll)$ are contained in the largest consistent set, the largest consistent set contains all von Neumann–Morgenstern solutions. Since von Neumann–Morgenstern solutions exist for all three-person games, the largest consistent set is nonempty for all three-person games.

For example, if $N = \{1, 2, 3\}$ and $v(\{1, 2\}) = v(\{1, 3\}) = v(\{2, 3\}) = 1$ (the "divide the dollar by majority" game), the largest consistent set is the whole set Z. If $v(\{1, 2\}) = 1/5$, $v(\{1, 3\}) = 3/5$, and $v(\{2, 3\}) = 3/5$, then the largest consistent set is the shaded region in Fig. 3.

Of all the solution concepts dealing with farsightedness, the various bargaining sets (Aumann and Maschler [6], Maschler [32]) have been most developed (see also Asscher [2], Billera [10], and Dutta *et al.* [15]). They are quite different in spirit: a counterobjection need not actually deter an objection (see Wilson [56, p. 260]). A counterobjection is understood not as a deterrent but more like a "counteroffer."

In Vickrey's [51] self-policing pattern, only deviations outside the pattern, "heresies," and not all deviations, are considered. For a heresy to be deterred, it must be "suicidal": in *all* possible further deviations, some member of the deviating coalition must be made worse off, as opposed to

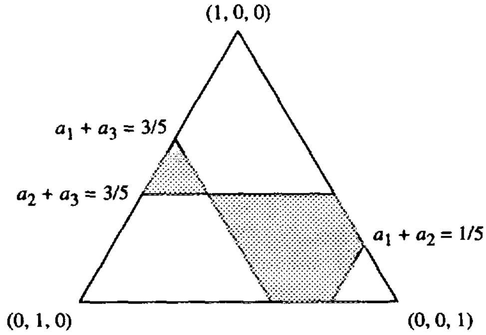

FIG. 3. The LCS in a three-person coalitional form game.

there being *some* further deviation which does not make some member of the deviating coalition better off.

In Roth's [42, 43, 44] "subsolution," outcomes can rule out each other; in contrast, the external stability property tells us that all outcomes not in the largest consistent set are indirectly dominated by outcomes in the set. A generalization of the von Neumann–Morgenstern solution, it is subject to the same criticisms: since it is defined using the direct dominance relation, it does not take into account relevant information.

Finally, Harsanyi [21] and Weber [54] (see also Perry and Reny [38]) explicitly model the intuitive basis for the largest consistent set, an extensive form bargaining game. In Harsanyi's bargaining game, there is in addition to the regular players a "chairman" who decides at each stage which coalition can make a counterproposal to the current proposal, and who wants to make the bargaining process last as long as possible. In Weber's bargaining game, a particular coalition is "given the floor" by some perhaps probabilistic rule. To keep coalitions from making suggestions indefinitely, the game either has an exogenously detemined maximum length or an exogenous probability of ending at any given stage.

These models are similar in spirit to the largest consistent set, but differ in how they treat the problems of what happens if coalitions object indefinitely and which coalition of several gets to further respond. In the largest consistent set, there is no need for either an exogenous stopping of the game or a no-agreement payoff. As long as the largest consistent set is nonempty, this problem is endogenized. In the largest consistent set, players think that any coalition capable of responding might be the one that responds, and make either a very optimistic (when deciding to move) or conservative (when deciding not to) choice, instead of an expected value calculation (see also Klingaman [25]). The largest consistent set might be considered a compromise, keeping the spirit of subgame perfect equilibrium without the complexity of an explicit extensive form bargaining game.

MAJORITY RULE VOTING

Which alternative a group of people will choose by majority rule was perhaps the first question of coalitional stability. The Condorcet "paradox" ranks, along with the Prisoners' Dilemma, as one of the archetypes of modern social theory, believed important enough by Riker [40] to rule out certain theories of democracy. Perhaps it is not a paradox of voting but a paradox in assuming only myopic rationality.

In the simplest environment, the set of alternatives (outcomes) is finite and nonempty and there are an odd number of voters (players), each with complete, transitive, and asymmetric strong preferences. At least three solution concepts are always nonempty: the top cycle (Ward [53], Miller [34]), the uncovered set (Miller [35], McKelvey [33]), and the stability set (Rubinstein [45], Le Breton and Salles [27]; see also Li [29]). To compare the largest consistent set with these three, let $a \to_S b$ for all a and b if S is a majority.

Here we write $a \prec b$ if $a \prec_S b$ for some majority S (it is easy to see that $\prec$, $<$, and $\ll$ are equivalent, and, because there are an odd number of players, complete and asymmetric). The *top cycle* is a nonempty $P \subset Z$ such that (a) for all $b \in P$ and for all $a \in Z \backslash P$, $a \prec b$, and (b) there is no nonempty proper subset of P which satisfies (a). The top cycle is the smallest subset such that every outcome inside beats every outcome outside.

Say *b covers a* if $b \neq a$ and for all $c \in Z$ such that $c \prec a$, $c \prec b$. In other words, b covers a if everything beaten by a is also beaten by b. The *uncovered set* is $\{a \in Z$: there is no $b \in Z$ which covers $a\}$.

The idea behind the stability set is that even if S is a majority and $a \prec_S b$, Player $i \in S$ might not vote for b over a if there is a possibility that another majority would then implement an outcome c which is worse for i than a. So the members of S would vote for b over a only if $a \prec\!\!\prec_S b$; that is, $a \prec_S b$ and there does not exist $i \in S$ and $c \in Z$ such that $b \prec c$ and $c \prec_i a$ (note that c itself need not be stable; players are not fully farsighted). The *stability set* is $\{a \in Z$: there is no $b \in Z$ and majority S such that $a \prec\!\!\prec_S b\}$.

If a beats all other outcomes by majority, then the three concepts and the largest consistent set all agree in predicting $\{a\}$. The general relationships between them are as follows (Fig. 4).

PROPOSITION 7. *Let $\Gamma = (N, Z, \{\prec_i\}_{i \in N}, \{\to_S\}_{S \subset N, S \neq \varnothing})$, where Z is finite, $|N|$ is finite and odd, preferences $\prec_i$ are complete, transitive, and asymmetric, and $a \to_S b$ if and only if $|S| > |N|/2$. Then*

(a) $\mathrm{LCS}(\Gamma) \subset$ *stability set*;

(b) *uncovered set* $\subset$ *stability set*;

(c) *uncovered set* $\subset$ *top cycle*.

(d) $\mathrm{LCS}(\Gamma) \cap$ *uncovered set* $\neq \varnothing$.

Proof. (a) Say $a \in \text{LCS}(\Gamma)$ and $a \prec\!\!\prec_S b$, where S is a majority. Then $a \prec_S b$. Then $\exists c \in \text{LCS}(\Gamma)$ such that $b \ll c$ and $a \not\prec_S c$. Since preferences are complete and symmetric, $\exists i \in S$ such that $c \prec_i a$. Since $b \ll c$, there is some majority T such that $b \prec_T c$. So $\exists i \in S$ and c such that $b \prec c$ and $c \prec_i a$, a contradiction of $a \prec\!\!\prec_S b$.

(b) Say a is uncovered. Miller [35, p. 73] shows that for all $b \neq a$, either $b \prec a$ or $\exists c$ such that $b \prec c \prec a$. Now say $a \prec\!\!\prec_S b$. Then $a \prec_S b$ and hence it cannot be that $b \prec a$. So $\exists c$ such that $b \prec c \prec_T a$, where T is a majority. Since S and T are majorities, they intersect and hence $\exists i \in S$ such that $c \prec_i a$. Since $b \prec c$, we have a contradiction of $a \prec\!\!\prec_S b$.

(c) Proved in Miller [35].

(d) It is easy to show that b covers a if and only if $a \lhd_Z b$, where $\lhd_Z$ is defined in the proof of Proposition 2. We defined V to be the limit of $Z_0 \supset Z_1 \supset Z_2 \supset \cdots$, where $Z_0 = Z$ and $Z_{i+1} = \{a \in Z_i : \nexists b \in Z_i \text{ such that } a \lhd_{Z_i} b\}$, and showed that V is nonempty and $V \subset \text{LCS}(\Gamma)$. Since the uncovered set is $\{a \in Z : \nexists b \in Z \text{ such that } a \lhd_Z b\}$, the uncovered set is simply Z_1. Since V is nonempty and $V \subset Z_1$ and $V \subset \text{LCS}(\Gamma)$, $\text{LCS}(\Gamma) \cap$ uncovered set $\neq \varnothing$. ∎

The two competitors in terms of specificity of prediction are the largest consistent set and the uncovered set. However, the uncovered set can be too "small." Say $|N| = 3$ and $Z = \{(1, 3, 4), (3, 4, 1), (4, 1, 3), (2, 2, 2)\}$, in which each outcome is represented by a utility vector. Then the uncovered set (and top cycle) is $\{(1, 3, 4), (3, 4, 1), (4, 1, 3)\}$ while the largest consistent set (and stability set) is the entire set Z. Since (2, 2, 2) does not beat any alternative, it is not in the uncovered set. But, for example, Player 2 might not join with Player 1 in moving to (3, 4, 1) because of the possibility that Players 1 and 3 would then move to (4, 1, 3). So the uncovered set (and even the top cycle, which is larger) might not contain alternatives which are stable from a farsighted point of view.

But the uncovered set can also be too "large." Say $|N| = 3$ and $Z = \{(1, 3, 4), (4, 2, 3), (5, 4, 1), (2, 5, 2), (3, 1, 5)\}$. Then the uncovered set (and stability set) is $\{(4, 2, 3), (5, 4, 1), (2, 5, 2), (3, 1, 5)\}$, the top cycle is Z, and the largest consistent set is $\{(4, 2, 3), (5, 4, 1), (2, 5, 2)\}$, the smallest

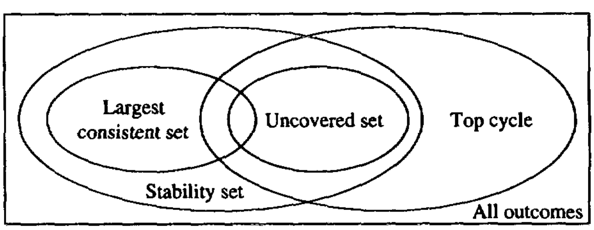

FIG. 4. The LCS, top cycle, uncovered set, and stability set.

of all. It makes sense that (1, 3, 4) should be thrown out, since $\{1, 2\}$ would certainly move to (5, 4, 1) because either (5, 4, 1) would be stable or $\{2, 3\}$ would from there move to (2, 5, 2), which $\{1, 2\}$ also prefers. But once (1, 3, 4) is thrown out, then $\{1, 2\}$ would surely move from (3, 1, 5) to (4, 2, 3), since either (4, 2, 3) would be stable or $\{1, 2\}$ would from there move to (5, 4, 1), which $\{1, 2\}$ also prefers. So (3, 1, 5) should be thrown out too.

In other words, if we accept the logic of the uncovered set, we throw out (1, 3, 4) since it is covered by (5, 4, 1). But in the "reduced game" in which $Z = \{(4, 2, 3), (5, 4, 1), (2, 5, 2), (3, 1, 5)\}$, (3, 1, 5) is covered by (4, 2, 3), and hence (3, 1, 5) should also be thrown out. A similar criticism applies to the stability set. Since $(1, 3, 4) \lll_{\{1,2\}} (5, 4, 1)$, (1, 3, 4) is thrown out. However, the only reason that $(3, 1, 5) \lll_{\{1,2\}} (4, 2, 3)$ is not true is that $(4, 2, 3) \prec (1, 3, 4)$ and hence Player 1 would hesitate. But since (1, 3, 4) is not in the stability set, Player 1 should be able to ignore it. In the reduced game, $(3, 1, 5) \lll_{\{1,2\}} (4, 2, 3)$, which eliminates (3, 1, 5).

As opposed to the uncovered set and the stability set, the largest consistent set has the property that after throwing out the nonstable outcomes, reapplying it gives the same answer. Compared to the stability set, it shows how assuming full farsightedness can tighten the prediction.

Concluding Remarks

Earlier I discussed how many details are left out of our definition of a game, thus blurring important issues. Here I illustrate some of these issues with examples.

Say $N = \{1\}$, $Z = \{a, b, c\}$, $a \rightarrow_{\{1\}} b$ and $a \rightarrow_{\{1\}} c$, and $a \prec_1 b \prec_1 c$. The largest consistent set is $\{b, c\}$. Since $a \ll b$ and $a \ll c$, starting from the status quo of a, the prediction is that Player 1 will either move to b or to c. But Player 1 will surely move to c since he prefers it to b. This example illustrates how the largest consistent set does not incorporate any idea of "best response": coalitions will move to any, not just the best, of the outcomes which are better than the status quo.

Say $N = \{1, 2\}$, $Z = \{a, b, c\}$, $a \rightarrow_{\{1,2\}} b$ and $a \rightarrow_{\{2\}} c$, and $c \prec_1 a \prec_1 b$ and $a \prec_2 b \prec_2 c$. The largest consistent set is $\{b, c\}$. Since $a \ll b$ and $a \ll c$, starting from the status quo of a, the prediction is that either the coalition of both players will move to b or Player 2 will move to c. But surely Player 2 will not join with Player 1 and move to b, since he could do better all by himself by moving to c. It seems that subcoalitions should be able to veto coalitional moves. Also, for example, if $a \rightarrow_S b$ and $a \rightarrow_T c$, perhaps the members of $S \cap T$ should be able to decide which move might be made. The largest consistent set does not say anything about these issues.

Say $N=\{1,2\}$, $Z=\{a,b,c\}$, $a \to_{\{1\}} b$ and $a \to_{\{2\}} c$, and $a \prec_1 b \prec_1 c$ and $a \prec_2 b \prec_2 c$. The largest consistent set is $\{b,c\}$. Since $a \ll b$ and $a \ll c$, starting from the status quo of a, the prediction is that either Player 1 will move to b or Player 2 will move to c. But since both players like c best, Player 1 would not move to b but would "wait" and let Player 2 move to c. This example shows that in the largest consistent set, a coalition believes that *if it does not move, no other coalition will. This is clearly not consistent.* A coalition is farsighted enough to consider what further moves other coalitions will make once it moves, but does not consider what other coalitions will do if it does not move.

If we take this into account, we have the possibility of coalitions moving to preempt the moves of other coalitions. For example, let $N=\{1,2\}$, $Z=\{a,b,c\}$, $a \to_{\{1\}} b$ and $a \to_{\{2\}} c$, and $a \prec_1 c \prec_1 b$ and $b \prec_2 c \prec_2 a$. Here the largest consistent set is $\{b,c\}$, and since $a \ll b$ but $a \not\ll c$, starting from the status quo of a, the prediction is that Player 1 will move to b. But knowing this, Player 2 might preempt Player 1 by moving to c: even though it is worse than a, at least it is better than b. A coalition might move from a good outcome to a bad outcome to keep another coalition from moving to a worse outcome.

Without further development, it is unclear whether in response to these issues we should require more from the game definition or more from the solution concept. In any case, I think that these and other issues might first be settled in a myopic framework, and then extended to a farsighted framework such as in this paper. I hope I have shown that this can be a feasible and interesting project.

References

1. C. Asilis and C. M. Kahn, Semi-stability in game theory: A survey of ugliness, *in* "Game Theory and Economic Applications: Proceedings of the International Conference Held at the Indian Statistical Institute, New Delhi, India, December 18–22, 1990" (B. Dutta *et al.*, Eds.), Springer-Verlag, Berlin, 1992.
2. N. Asscher, An ordinal bargaining set for games without side payments, *Math. Oper. Res.* **1** (1976), 381–389.
3. R. J. Aumann, Acceptable points in general cooperative *n*-person games, *in* "Contributions to the Theory of Games" (A. W. Tucker and R. D. Luce, Eds.), Vol. 4, Princeton Univ. Press, Princeton, NJ, 1959.
4. R. J. Aumann, The core of a cooperative game without side payments, *Trans. Amer. Math. Soc.* **98** (1961), 539–552.
5. R. J. Aumann, A survey of cooperative games without side payments, *in* "Essays in Mathematical Economics in Honor of Oskar Morgenstern" (M. Shubik, Ed.), Princeton Univ. Press, Princeton, NJ, 1967.
6. R. J. Aumann and M. Maschler, The bargaining set for cooperative games, *in* "Advances in Game Theory" (M. Dresher, L. S. Shapley, and A. W. Tucker, Eds.), Princeton University Press, Princeton, NJ, 1964.

7. R. J. AUMANN AND B. PELEG, von Neumann–Morgenstern solutions to cooperative games without side payments, *Bull. Amer. Math. Soc.* **60** (1960), 173–179.
8. B. D. BERNHEIM, Rationalizable strategic behavior, *Econometrica* **52** (1984), 1007–1028.
9. B. D. BERNHEIM, B. PELEG, AND M. D. WHINSTON, Coalition-proof Nash equilibria. I. Concepts, *J. Econ. Theory* **42** (1987), 1–12.
10. L. J. BILLERA, Existence of general bargaining sets for cooperative games without side payments, *Bull. Amer. Math. Soc.* **76** (1970), 375–379.
11. D. BLACK, "The Theory of Committees and Elections," Cambridge Univ. Press, Cambridge, 1958.
12. B. CHAKRAVORTI AND C. M. KAHN, Universal coalition-proof equilibrium, mimeograph, University of Illinois at Urbana-Champaign, 1991.
13. M. S.-Y. CHWE, "Farsighted Coalitional Stability," Ph.D. thesis, Northwestern University, 1992.
14. CONDORCET, "Essai sur l'Application de l'Analyse à la Probabilité des Décisions Rendues à la Pluralité des Voix," Paris, 1785.
15. B. DUTTA, D. RAY, K. SENGUPTA, AND R. VOHRA, A consistent bargaining set, *J. Econ. Theory* **49** (1989), 93–112.
16. I. FISHER, Cournot and mathematical economics, *Quart. J. Econ.* **12** (1898), 119–138.
17. D. B. GILLIES, Solutions to general non-zero-sum games, *in* "Contributions to the Theory of Games" (A. W. Tucker and R. D. Luce, Eds.), Vol. 4, Princeton University Press, Princeton, NJ, 1959.
18. J. GREENBERG, Deriving strong and coalition-proof Nash equilibria from an abstract system, *J. Econ. Theory* **49** (1989), 195–202.
19. J. GREENBERG, "The Theory of Social Situations: An Alternative Game-Theoretic Approach," Cambridge Univ. Press, Cambridge, 1990.
20. J. C. HARSANYI, Oddness of the number of equilibrium points: A new proof, *Int. J. Game Theory* **2** (1973), 235–250.
21. J. C. HARSANYI, An equilibrium-point interpretation of stable sets and a proposed alternative definition, *Management Sci.* **20** (1974), 1472–1495.
22. C. M. KAHN AND D. MOOKHERJEE, The good, the bad and the ugly: Coalition proof equilibrium in infinite games, *Games Econ. Behav.* **4** (1992), 101–121.
23. E. KALAI, E. A. PAZNER, AND D. SCHMEIDLER, Collective choice correspondences as admissible outcomes of social bargaining processes, *Econometrica* **44** (1976), 233–240.
24. E. KALAI AND D. SCHMEIDLER, An admissible set occuring in various bargaining situations, *J. Econ. Theory* **14** (1977), 402–411.
25. D. KLINGAMAN, A note on a cyclical majority problem, *Public Choice* **6** (1969), 99–101.
26. G. LAFFOND AND H. MOULIN, Stability by threats and counterthreats in normal form games, *in* "Mathematical Techniques of Opimization, Control and Decision" (J. P. Aubin, A. Bensoussan, and I. Ekeland, Eds.), Birkhäuser, Boston, 1981.
27. M. LE BRETON AND M. SALLES, The stability set of voting games: Classification and genericity results, *Int. J. Game Theory* **19** (1990), 111–127.
28. S. LI, Far-sighted strong equilibrium and oligopoly, *Econ. Lett.* **40** (1992), 39–44.
29. S. LI, Stability of voting games, *Soc. Choice Welfare*, **10** (1993), 51–56.
30. W. F. LUCAS, Developments in stable set theory, *in* "Game Theory and Applications" (T. Ichiishi, A. Neyman, and Y. Tauman, Eds.), Academic Press, San Diego, 1990.
31. M. MASCHLER, Consistency, *in* "Game Theory and Applications" (T. Ichiishi, A. Neyman, and Y. Tauman, Eds.), Academic Press, San Diego, 1990.
32. M. MASCHLER, The bargaining set, kernel, and nucleolus: A survey, mimeograph, Hebrew University of Jerusalem, 1991.
33. R. D. MCKELVEY, Covering, dominance, and institution-free properties of social choice, *Amer. J. Polit. Sci.* **30** (1986), 283–314.

34. N. R. MILLER, Graph-theoretical approaches to the theory of voting, *Amer. J. Polit. Sci.* **21** (1977), 769–803.
35. N. R. MILLER, A new solution set for tournaments and majority voting: Further graph-theoretical approaches to the theory of voting, *Amer. J. Polit. Sci.* **24** (1980), 68–96.
36. H. MOULIN AND B. PELEG, Cores of effectivity functions and implementation theory, *J. Math. Econ.* **10** (1982), 115–145.
37. D. G. PEARCE, Rationalizable strategic behavior and the problem of perfection, *Econometrica* **52** (1984), 1029–1050.
38. M. PERRY AND P. J. RENY, A non-cooperative view of coalition formation and the core, mimeograph, University of Western Ontario, 1991.
39. D. RAY AND R. VOHRA, Equilibrium binding agreements, Working Paper 92-8, Brown University, Department of Economics, 1992.
40. W. H. RIKER, "Liberalism against Populism: A Confrontation between the Theory of Democracy and the Theory of Social Choice," Freeman, San Francisco, 1982.
41. R. W. ROSENTHAL, Cooperative games in effectiveness form, *J. Econ. Theory* **5** (1972), 88–101.
42. A. E. ROTH, A lattice fixed-point theorem with constraints, *Bull. Amer. Math. Soc.* **81** (1975), 136–138.
43. A. E. ROTH, Subsolutions and the supercore of cooperative games, *Math. Oper. Res.* **1** (1976), 43–49.
44. A. E. ROTH, A fixed point approach to stability in cooperative games, *in* "Fixed Points: Algorithms and Applications" (Stephan Karamardian, Ed.), Academic Press, New York, 1977.
45. A. RUBINSTEIN, Stability of decision systems under majority rule, *J. Econ. Theory* **23** (1980), 150–159.
46. F. M. SCHERER, "Industrial Market Structure and Economic Performance," Rand-McNally, Chicago, 1980.
47. A. SEN, Social choice theory: A re-examination, *Econometrica* **45** (1977), 53–89.
48. K. SUZUMURA, "Rational Choice, Collective Decisions, and Social Welfare," Cambridge Univ. Press, Cambridge, 1983.
49. A. TARSKI, A lattice-theoretical fixpoint theorem and its applications, *Pac. J. Math.* **5** (1955), 285–309.
50. W. THOMSON, The consistency principle, *in* "Game Theory and Applications" (T. Ichiishi, A. Neyman, and Y. Tauman, Eds.), Academic Press, San Diego, 1990.
51. W. VICKREY, Self-policing properties of certain imputation sets, *in* "Contributions to the Theory of Games" (A. W. Tucker and R. D. Luce, Eds.), Vol. 4, Princeton Univ. Press, Princeton, NJ, 1959.
52. J. VON NEUMANN AND O. MORGENSTERN, "Theory of Games and Economic Behavior," 3rd ed., Princeton Univ. Press, Princeton, NJ, 1953.
53. B. WARD, Majority rule and allocation, *J. Conflict Res.* **5** (1961), 379–389.
54. R. J. WEBER, "Bargaining Solutions and Stationary Sets in N-Person Games," Technical Report 223, Department of Operations Research, College of Engineering, Cornell University, 1974.
55. R. J. WEBER, Absolutely stable games, *Proc. Amer. Math. Soc.* **55** (1976), 116–118.
56. R. WILSON, Stable coalition proposals in majority-rule voting, *J. Econ. Theory* **3** (1971), 254–271.