Informal proofs using conditionals

- Basic valid steps:
  - Modus ponens: from P and $P \rightarrow Q$, infer Q.
  - Biconditional elimination: from P and $P \leftrightarrow Q$ or $Q \leftrightarrow P$, infer Q.
- Other valid steps:
  - Modus tollens: from $P \rightarrow Q$ and $\neg Q$, infer $\neg P$

- Useful equivalences:
  - *Contraposition*: $P \rightarrow Q$ and $\neg Q \rightarrow \neg P$ are logically equivalent.
  - $P \leftrightarrow Q$ and $\neg Q \leftrightarrow \neg P$ are also logically equivalent.

Conditional proof

- To derive a conclusion of the form P → Q from some premises, assume that P is true (in addition to those premises), and derive Q subject to that assumption.

- An example:
  - Given the premises $(\text{Tet}(a) \wedge \text{Small}(a)) \rightarrow \text{Small}(b)$ and $\text{Tet}(a)$, we want to prove $\text{Small}(a) \rightarrow \text{Small}(b)$.
  - Proof: Suppose that $\text{Small}(a)$ is true. Then $\text{Tet}(a) \wedge \text{Small}(a)$ by the second premise, and so by the first premise, $\text{Small}(b)$. So by conditional proof we conclude that $\text{Small}(a) \rightarrow \text{Small}(b)$.

- Another example: 8.4
  - Premises: (1) The unicorn, if horned, is elusive and dangerous. (2) If elusive or mythical, the unicorn is rare. (3) If a mammal, the unicorn is not rare. Conclusion: The unicorn, if horned, is not a mammal.
  - Argument. Suppose that the unicorn is horned, and assume for *reductio* that it is a mammal. By (1) it is elusive, so by (2) it is rare. But by (3) it is not rare: contradiction. Hence, if the unicorn is horned, it is not a mammal.

- Proving biconditionals.
  - Biconditional introduction: If we can derive Q from the assumption that P (plus our premises), and we can derive P from the assumption that Q (plus our premises), then we can derive P ↔ Q from our premises.
  - Circles of proofs.

For next week:

- Read: chapters 7 and 8; optionally, chapter 9.
- Do: exercises 7.6 - 7.8 (10% each); 7.11 (10%); 7.12 and 7.13 (20%); 8.3, 8.5, 8.6, 8.9 (10% each).