Disputed principles in the logic of conditionals

Conditionals: Cian Dorr & John Hawthorne

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1. Metarules

Conditional Proof:*	If $(\Gamma, A) \vdash B$, then $\Gamma \vdash A \rightarrow B$
No-premise Conditional Proof:	If $A \vdash B$, then $\vdash A \rightarrow B$
"Conditional Necessitation" (CN[1]):	If $\vdash B$, then $\vdash A \rightarrow B$
Single-premise closure (RCM):	If $B \vdash C$, then $A \rightarrow B \vdash A \rightarrow C$
Full closure (RCK):	If $B_1, \ldots, B_n \vdash C$, then $A \rightarrow B_1, \ldots A \rightarrow B_n \vdash A \rightarrow C$ $\quad (n \leq 0)$
Substitution of antecedents (RCEA):	If $\vdash A \equiv B$, then $\vdash (A \rightarrow C) \equiv (B \rightarrow C)$
Substitution of consequents (RCEC):	If $\vdash B \equiv C$, then $\vdash (A \rightarrow B) \equiv (A \rightarrow C)$

2. Rules/axioms

CK:	$A \rightarrow B, A \rightarrow (B \supset C) \vdash A \rightarrow C$
Finite Conjunction (CC):	$A \rightarrow B, A \rightarrow C \vdash A \rightarrow (B \wedge C)$
Infinite Conjunction:	$A \rightarrow B_1, A \rightarrow B_2, A \rightarrow B_3 \ldots \vdash A \rightarrow (B_1 \wedge B_2 \wedge B_3 \wedge \ldots)$
Identity (ID):	$\vdash A \rightarrow A$
Modus Ponens (MP):	$A, A \rightarrow B \vdash B$ $\qquad$ or: $\vdash (A \rightarrow B) \supset (A \supset B)$
Modus Tollens (MT):	$\neg B, A \rightarrow B \vdash \neg A$
Contraposition:*	$A \rightarrow B \vdash \neg B \rightarrow \neg A$
Transitivity:*	$A \rightarrow B, B \rightarrow C \vdash A \rightarrow C$
Antecedent Strengthening:*	$A \rightarrow C \vdash (A \wedge B) \rightarrow C$
Simplification:*	$(A \vee B) \rightarrow C \vdash A \rightarrow C \wedge B \rightarrow C$
Weakened Transitivity (CSO):	$A \rightarrow B, B \rightarrow C, B \rightarrow A \vdash A \rightarrow C$
Limited Transitivity (RT):	$A \rightarrow B, (A \wedge B) \rightarrow C \vdash A \rightarrow C$
Limited Antecedent Strengthening (CV):	$\neg(A \rightarrow \neg B), A \rightarrow C \vdash (A \wedge B) \rightarrow C$
Very Limited A.S. (CMon):	$A \rightarrow B, A \rightarrow C \vdash (A \wedge B) \rightarrow C$
Disjunction (CA):	$A \rightarrow C, B \rightarrow C \vdash (A \vee B) \rightarrow C$

[1] The alphabetic labels are drawn from Horacio Arlo-Costa's SEP entry on 'The Logic of Conditionals'; many are originally due to Brian Chellas (*Modal Logic*).

'Reverse disjunction[2]':	$(A{\vee}B){\rightarrow}C \vdash A{\rightarrow}C \vee B{\rightarrow}C$
MOD:	$\neg A{\rightarrow}A \vdash B{\rightarrow}A$
'S4 for crashing':	$A{\rightarrow}\bot \supset B{\rightarrow}(A{\rightarrow}\bot)$
'B for crashing':	$A \supset (A{\rightarrow}\bot){\rightarrow}\bot$
Conditional Excluded Middle (CEM):	$\vdash (A{\rightarrow}B) \vee (A{\rightarrow}\neg B)$
And-to-if (CS):	$A{\wedge}B \vdash A{\rightarrow}B$
Or-to-if:*	$A{\vee}B \vdash \neg A{\rightarrow}B$
Falsity:*	$\neg(A{\rightarrow}B) \vdash A{\wedge}\neg B$
Import-Export:*	$\vdash (A{\wedge}B) \rightarrow C \equiv A{\rightarrow}(B{\rightarrow}C)$
Weakened Import-Export:	$\vdash (A{\wedge}B) \rightarrow C \equiv A{\rightarrow}((A{\wedge}B){\rightarrow}C)$

- As usual, '$\Phi \supset \Psi$' abbreviates '$\neg\Phi \vee \Psi$', and '$\Phi \equiv \Psi$' abbreviates '$(\Phi{\supset}\Psi) \wedge (\Psi{\supset}\Phi)$'.
- A 'normal conditional logic' is one closed under classical consequence and RCEA + RCK (or equivalently RCEA+CN+CK, or equivalently RCEA+RCM+CC).
- The weakest system studied in Lewis's *Counterfactuals* is V, which is the smallest normal conditional logic that validates ID, CSO, CV and CA. (V also validates CK,CC RT, CMon, CA and MOD). The logic Lewis actually endorses (for counterfactuals) is VC, which we get from V by adding MP and CS.
- Stalnaker's favoured logic C2 is the smallest normal conditional logic that validates ID, MP, CSO and CEM. (We can also get C2 by adding CEM to VC.) Stalnaker also endorses Infinite Conjunction, whereas Lewis rejects it.
- The rules marked with an asterisk are ones that are valid for material conditionals, but denied to be valid (at least for some natural-language conditionals) by Stalnaker, Lewis and many others in that tradition.

3. Interaction with modals

(NB: one might want to say different things about different kinds of modals—e.g., epistemic or metaphysical—and different kinds of conditionals.)

Necessity of consequents:	$\Box B \vdash A{\rightarrow}B$
Modal Or-to-if:	$\Box(A{\supset}B) \vdash A{\rightarrow}B$
Possibility preservation:	$\Diamond A \wedge A{\rightarrow}B \vDash \Diamond B$
Crashing is impossibility:	$\neg A{\rightarrow}A \vdash \Box A$
Antecedent possibility:	$A{\rightarrow}B \vdash \Diamond A$

[2] Lee Walters suggested including this attractive rule.

Necessity of conditionals: $A \rightarrow B \vdash \Box(A \rightarrow B)$

Importing and exporting necessity: $\Box(A \rightarrow B) \equiv A \rightarrow \Box B$

Importing and exporting possibility: $A \rightarrow \Diamond B \equiv \Diamond(A \wedge B)$

4. Interaction with 'actually'

Rigidity: Actually B $\vdash$ A$\rightarrow$Actually B

Vacuity: $A \rightarrow B \vdash$ A$\rightarrow$Actually B