Higher-Order Quantification and the Elimination of Abstract Objects

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1 *Higher-Orderese*

From first order logic to higher order logic, in four steps.

Step Zero: types.

- We can keep track of syntactic categories in familiar formal languages using special strings called *types*. An expression of type e is an individual constant; an expression of type t is a sentence; an expression of type $\sigma_1 \to \cdots \to \sigma_n \to t$ is something that combines with something of type $\sigma_1, \ldots,$ and something of type σ_n to make a sentence.[1]
- In the language of standard first-order logic, we have: individual constants (type e); formulae (type t); n-place predicate constants (type $e \to \cdots \to e \to t$); $\neg$ of type $t \to t$; $\wedge, \vee, \to, \leftrightarrow$ of type $t \to t \to t$. Modal logic adds $\Box : t \to t \ldots$.

[1] Definition: e is a type; t is a terminal type; if σ is a type and τ is a terminal type, $(\sigma \to \tau)$ is a terminal type; that's all. et abbreviates $(e \to t)$.
Some versions of first-order logic also have function symbols, which would require types like $e \to e$, $e \to e \to e$, etc. I won't need these with these, since their interpretation raises messy questions.

Step One: complex predicates.

- When v is a (first-order) variable and F is an n-place first-order predicate[2], $\lambda v.F$ is a $n+1$-place predicate.
- When F is an $n+1$-place predicate and a is a (first-order) term, Fa is an n-place predicate.
- It's natural to treat $\forall v\, P$ and $\exists v\, P$ as shorthand for $\forall(\lambda v.P)$ and $\exists(\lambda v.P)$, where $\forall$ and $\exists$ are of type $(e \to t) \to t$.[3]
- We'll need a basic logical schema for manipulating λ-terms:

$$(\beta) \qquad P[(\lambda v.F)a] \leftrightarrow P[F[a/v]]$$

[2] A.k.a. term of type $\overbrace{e \to \cdots \to e}^{n \text{ times}} \to t$

[3] The insight that first-order quantifiers are higher-order predicates is due to Frege (1879).

For example:
$\neg((\lambda x.\, \mathsf{red}\, x \wedge \mathsf{round}\, x)a) \;\leftrightarrow\; \neg(\mathsf{red}\, a \wedge \mathsf{round}\, a)$

Step Three: complex higher-order predicates.

- A higher-order predicate is just an expression of type $\sigma_1 \to \cdots \to \sigma_n \to t$ where at least one σ_i is not e. We already have some *simple* examples: $\neg : t \to t$; $\wedge : t \to t \to t$; $\forall : (e \to t) \to t$. We could add more: e.g. if we are interested in formalizing reasoning about meaning, we might want a constant means_{et} of type $e \to et \to t$ that would let us say things like 'red'means_{et} red.

- We can make *complex* predicates in these types by adding a supply of variables in each type which can be bound by λ. Examples:

$$\lambda p^t.\lambda q^t.\neg p \wedge \neg q$$
$$\lambda X^{et}.\forall z(\mathsf{red}\, z \to Xz)$$
$$\lambda X.X\, \mathsf{mars}$$

- Extend the β schema to cover these.[4]

Step Three: higher-order quantifiers.

- For each type σ, we have a new logical constant $\forall_\sigma$ of type $(\sigma \to t) \to t$.[5]
- We extend the standard classical quantifier rules[6] to the new quantifiers.
- For some types σ we can find close natural language analogues: 'He is something I am not—kind'; 'However he says things are, thus they are' (Prior 1971; Rayo and Yablo 2001). But we shouldn't hold the intelligibility of the formal language hostage to our ability to translate each of its sentences into a natural language (Frege 1879; Williamson 2003).
- We can also add higher-order identity constants $=_\sigma$ of type $\sigma \to \sigma \to t$, governed by the standard identity rules.[7]
- For some types σ we can find close natural language analogues: 'To be a water molecule is to a molecule consisting of two hydrogen atoms and one oxygen atoms'; 'For there to be vixens is for there to be female foxes' (Dorr 2016).

2 *Higher-Orderese and 'property' talk*

In *pronouncing* sentences of Higher-Orderese, it can be pedagogically helpful to use words like 'property', 'relation', 'proposition', etc.[8]

But we can *understand* Higher-Orderese without relying on these "translations"—fortunately, since the workings of 'property' talk in English are murky.[9]

Instead, we can use our independent understanding of Higher-Orderese to shed light on how words like 'property' work in English, and on metaphysical debates conducted using these words.

Imagine we are alien field linguists (from the planet Hol), whose native language is Higher-Orderese.[10] We are trying to figure out what

[4] E.g., $(\lambda X^{et}.\forall z(\mathsf{red}\, z \to Xz))\, \mathsf{round} \leftrightarrow \forall z(\mathsf{red}\, z \to \mathsf{round}\, z)$

[5] $\forall_\sigma v\, P$ abbreviates $\forall_\sigma(\lambda v.P)$.
$\exists_\sigma$ abbreviates $\lambda X^{\sigma\to t}.\neg\forall_\sigma y \neg Xy$.

[6] **UI**: $\vdash \forall_\sigma v\, P \to P[a/v]$ (where $a : \sigma$
GEN: If $\vdash P$ then $\vdash \forall_\sigma v\, P$

[7] **Ref:** $\vdash a =_\sigma a$
LL: $\vdash a =_\sigma b \to (Fa \to Fb)$

[8] For example, one might pronounce $\neg\forall_{et} X(X\, \mathsf{mars} \to X\, \mathsf{venus})$ as 'Not every property of Mars is a property of Venus', and $\exists p.\neg p$ as 'Some proposition is not true' or 'Some state of affairs does not obtain'.

[9] Consider the property-theoretic version of Russell's paradox; debates about nominalism; etc.

[10] I mean a language with the syntax of Higher-Orderese and a rich supply of non-logical constants, e.g. $\mathsf{animal} : et$, $\mathsf{charged} : et$, $\mathsf{love} : e \to et$, $\mathsf{between} : e \to e \to et \ldots$

English-speakers are saying with various sentences, including those involving words like 'property'.

Fleshing out the thought experiment a bit more, we can imagine that the following thesis is quite popular among Hollian philosophers:

Materialism$_e$ $\forall_e x$ $\mathsf{material}\, x$

'material' is a bit of a placeholder; we could cash it out in terms of having a shape, size, mass, parts, location. . . .

This contrasts with the situation facing philosophers on Earth who propound claims like

Materialism Everything is a material object.

Materialism is *not* so popular, since it is apparently refuted by arguments like the following:

> Redness isn't a material object; so not everything is a material object.
>
> The fact that there are dogs isn't a material object; so not everything is a material object.
>
> The number two isn't a material object; so not everything is a material object.
>
> The word 'cat' isn't a material object; so not everything is a material object.

While proponents of First Order Materialism on Hol still have to deal with arguments concerning gods, ghosts, souls, etc., they face no arguments analogous to these, since Hollian has no type-e terms that are at all analogous to 'redness', 'the fact that there are dogs', or 'the number two'. The closest analogues are terms of higher types.[11]

So let's imagine that our field linguists are (at least initially) inclined towards Materialism.

[11] E.g. in Hollian, two is a term of type $et \to t$ equivalent to $\lambda X.\exists_y \exists_z (Xz \wedge Yz \wedge y \neq z)$, and number is a term of type $(et \to t) \to t$.
Quotes in Hollian are used to make terms of type et: e.g., 'cat'x is true when x is a blob of ink shaped like this: cat.

3 *Doing natural-language semantics in Higher-Orderese*

In English we have a bewildering variety of words for talking about semantics—'means', 'refers', 'denotes', 'expresses'. . . . In Hollian it's a bit cleaner: for each type σ, they have a predicate E_σ, of type $et \to \sigma \to t$.[12]

If they have appropriate other terms in their language, they can formulate hypotheses about us like the following:[13]

$$\text{'dances'}\ \mathsf{E}_{et}\ \mathsf{dances}$$
$$\text{'bachelor'}\ \mathsf{E}_{et}\ \lambda x^e.\, \mathsf{man}\, x \wedge \neg\, \mathsf{married}\, x$$
$$\text{'some bachelor dances'}\ \mathsf{E}_t\ \exists_e x(\mathsf{man}\, x \wedge \neg\, \mathsf{married}\, x \wedge \mathsf{dances}\, x)$$

[12] Truths of the form $a\ \mathsf{E}_\sigma\ x$ are contingent; they would be false the relevant speakers had used a differently. Strictly there should be a third argument for a community of speakers, but I'll suppress it for clarity.

[13] Maybe there aren't any dogs on Hol, so they don't have any simple closed term c for which 'dog' E_{et} c would be plausible.

Of course they don't want to just make long lists; they'd like to identify general patterns. Doing this properly requires an integrated theory of semantics and syntax. But we can try to bracket syntactic details by helping ourselves to a predicate comb (of type $et \to et \to et \to t$), where $\mathsf{comb}\,cab$ intuitively means 'c is the well-formed result of combining a with b'.

Then we can formulate the following hypothesis:

Functional Composition (FC)

$$\forall_{et}a\forall_{et}b\forall_{et}c\forall_{\sigma\to\tau}x\forall_{\sigma}y(a\ \mathsf{E}_{\sigma\to\tau}\ x \wedge b\ \mathsf{E}_{\sigma}\ y \wedge \mathsf{comb}\,cab \to c\ \mathsf{E}_{\tau}\ xy)$$

FC is a *schema*, not a single *sentence* of higher-orderese. To formulate a single sentence that implies every instance of such a schema, our linguists can avail themselves of a *disquotational truth predicate* $\mathsf{true} : et \to t$ for (the true-free fragment of) their own language.

FC lets us derive many plausible results about the semantics of complex expressions, including sentences from plausible hypotheses about the semantics of individual words. For example:

$$\dfrac{\begin{array}{c}\text{'some' } \mathsf{E}_{et\to et\to t}\\ \lambda X^{et}.\lambda Y^{et}.\exists_e z(Xz \wedge Yz)\end{array} \qquad \begin{array}{c}\text{'bachelor' } \mathsf{E}_{et}\\ \lambda x^{e}.\,\mathsf{man}\,x \wedge \neg\,\mathsf{married}\,x\end{array}}{\begin{array}{c}\text{'some bachelor' } \mathsf{E}_{et\to t}\\ (\lambda X^{et}.\lambda Y^{et}.\exists_e z(Xz \wedge Yz))(\lambda x^{e}.\,\mathsf{man}\,x \wedge \neg\,\mathsf{married}\,x)\end{array}}\ \text{FC}$$

$$\dfrac{\begin{array}{c}\text{'some bachelor' } \mathsf{E}_{et\to t}\\ (\lambda X^{et}.\lambda Y^{et}.\exists_e z(Xz \wedge Yz))(\lambda x^{e}.\,\mathsf{man}\,x \wedge \neg\,\mathsf{married}\,x)\end{array}}{\begin{array}{c}\text{'some bachelor' } \mathsf{E}_{et\to t}\\ \lambda Y^{et}.\exists_e z(\mathsf{man}\,z \wedge \neg\,\mathsf{married}\,z \wedge Yz)\end{array}}\ \beta \times 2$$

$$\dfrac{\begin{array}{c}\text{'some bachelor' } \mathsf{E}_{et\to t}\\ \lambda Y^{et}.\exists_e z(\mathsf{man}\,z \wedge \neg\,\mathsf{married}\,z \wedge Yz)\end{array} \qquad \begin{array}{c}\text{'dances' } \mathsf{E}_{et}\\ \mathsf{dances}\end{array}}{\begin{array}{c}\text{'some bachelor dances' } \mathsf{E}_{t}\\ (\lambda Y^{et}.\exists_e z(\mathsf{man}\,z \wedge \neg\,\mathsf{married}\,z \wedge Yz))\,\mathsf{dances}\end{array}}\ \text{FC}$$

$$\dfrac{\begin{array}{c}\text{'some bachelor dances' } \mathsf{E}_{t}\\ (\lambda Y^{et}.\exists_e z(\mathsf{man}\,z \wedge \neg\,\mathsf{married}\,z \wedge Yz))\,\mathsf{dances}\end{array}}{\begin{array}{c}\text{'some bachelor dances' } \mathsf{E}_{t}\\ \exists_e z(\mathsf{man}\,z \wedge \neg\,\mathsf{married}\,z \wedge \mathsf{dances}\,z)\end{array}}\ \beta$$

Such examples suggests a couple of tempting schemas:

Since Montague (1974), semantics has often looked very like this, though often coupled with some set -theoretic glosses on what the basic assertions mean.

Uniformity

$$\forall_{et}a\forall_{et}b(\mathsf{sameSyntax}\,ab \wedge \mathsf{meaningful}\,b \wedge \exists_{\sigma}x(a\ \mathsf{E}_{\sigma}\ x) \to \exists_{\sigma}y(b\ \mathsf{E}_{\sigma}\ y))$$

'Every English syntactic category is semantically uniform'

Type-Uniqueness

$$\forall_{et}a\forall_{\sigma_1}x\forall_{\sigma_2}y(a\ \mathsf{E}_{\sigma_1}\ x \to \neg(a\ \mathsf{E}_{\sigma_2}\ y)) \quad \text{where } \sigma_1 \neq \sigma_2$$

'Each expression has a unique semantic type'[14]

[4] Ordinary examples of ambiguity/polysemy like 'tank' and 'book' challenge the schema

$$a\ \mathsf{E}_{\sigma}\ x \wedge a\ \mathsf{E}_{\sigma}\ y \to x = y$$

but not Type-Uniqueness.

4 *The challenge of property-talk*

Since 'instantiates' is meaningful and syntactically like 'loves' and 'redness' is meaningful and syntactically like 'Mars' [15], Uniformity re-

[15] Not exactly. But we could use 'the property of being red', which is more like 'Mars', and very like 'the city of Vancouver'.

quires

(1) $\exists_{e\to et} R(\text{'instantiates'}\ \mathsf{E}_{e\to et}\ R)$

(2) $\exists_e x(\text{'redness'}\ \mathsf{E}_e\ x)$

But what could this R and x be? Our field linguists' materialist$_e$ assumptions don't suggest any good candidates, given how speakers treat 'Mars instantiates redness' as equivalent to 'Mars is red', and more generally treat 'Mars instantiates the property of VP+ing' as equivalent to 'Mars VP'.

Some options:

- Take English-speakers to be deeply in the grip of some extravagant "Platonist" view, wildly inconsistent with Materialism$_e$, which posits a realm of "universals", including some fit to be referred to 'redness'.
- Take what's expressed$_t$ by sentences like 'Mars instantiates redness' and 'Mars instantiates some property' to conflict with Materialism$_e$, but deny that English-speakers mean them literally (Button and Trueman, forthcoming).
- Give up Uniformity and endorse the premises of derivations like this one:

⇐ My favourite!

$$\dfrac{\text{'Mars'}\ \mathsf{E}_e\ \mathsf{mars} \qquad \dfrac{\dfrac{\text{'instantiates'}\ \mathsf{E}_{et\to et}\ \lambda X.X \qquad \text{'redness'}\ \mathsf{E}_{et}\ \mathsf{red}}{\text{'instantiates redness'}\ \mathsf{E}_{et}\ (\lambda X.X)\,\mathsf{red}}\,\text{FC}}{\text{'instantiates redness'}\ \mathsf{E}_{et}\ \mathsf{red}}\,\beta}{\text{'Mars instantiates redness'}\ \mathsf{E}_t\ \mathsf{red\,mars}}\,\text{FC}$$

The difference between 'redness' and 'is red' is *merely* syntactic.[16]

[16] Peter of Spain likewise thinks that 'animal' has the same *significatio* in 'Animal is a genus', 'Every animal is mortal', 'Some animal is mortal', and 'Every man is an animal'. However he does seem to posit some kind of semantic difference: the first and fourth 'animal' involve *suppositio simplex*, the second *suppositio personalis confusa*, and the third *suppositio personalis determinata*.

5 *Natural-languages as type-ambiguous*

If we reject Uniformity, we'll also want to reject Type-Uniqueness. For example:

- 'interesting' will have to both express something of type et (to handle 'Mars is interesting') and something of type $et \to t$ ('Redness is interesting'). Also something of type $(et \to t) \to t$ ('The property of being a property of Mars is interesting').[17]
- 'thinks about' will, likewise, have to express something of type $e \to et$; something of type $et \to et$; something of type $(et \to t) \to et$;. . . .

[17] Our field linguists might perfectly well have appropriate constants $\mathsf{interesting}_e$, $\mathsf{interesting}_{et}$, $\mathsf{interesting}_{et\to t}$. . .

- 'some' will have to not only express $\lambda X^{et}.\lambda Y^{et}.\exists_e z(Xz \wedge Yz)$ of type $et \to et \to t$, but also $\lambda X^{et\to t}.\lambda Y^{et\to t}.\exists_{et} z(Xz \wedge Yz)$ of type $(et \to t) \to (et \to t) \to t$, to handle 'Mary thinks about something instantiated by Mars'.

In fact, type-ambiguity will need to be quite pervasive, if we don't want to have to count some OK-seeming sentences are meaningless.[18]

Consider 'Redness is not a material object'. Even though it's the sort of sentence only a philosopher would utter, it seems meaningful. Indeed, the dominant philosopher's reactions suggest that we should interpret it as saying something true. To do so, we'll need a type-$et \to t$ meaning M for 'material object'; plausibly, one for which $\Box\forall_{et} X\, \neg MX$ is true.

One simple hypothesis is that 'material object' $\mathsf{E}_{et\to t}$ $\lambda X^{et}.\bot$, so that 'redness is a material object' E_t $\bot$.[19]

- *Objection:* 'Redness is a material object' differs in *cognitive value* from any explicit contradiction, since it is not *obviously* false.
- *Response:* semantics (in our sense) is not a theory of cognitive value, since

  $\forall p$('Hesperus is Phosphorus' E_t $p \leftrightarrow$ 'Hesperus is Hesperus' E_t p)[20]

- Similarly, 'instantiates' will have to not only express something of type $et \to et$ (namely $\lambda X.X$) but something of type $e \to et$, to explain the meaningfulness of 'Mars instantiates Venus' and the truth of its negation. Again, a simple option is to say that 'instantiates' $\mathsf{E}_{e\to et}$ $\lambda x.\lambda y.\bot$.

It looks like type-ambiguity is quite rampant, e.g.:

$$\mathsf{verbPhrase}\, a \to \exists_{\sigma\to t} x(a\ \mathsf{E}_{\sigma\to t}\ x)$$
$$\mathsf{transitiveVerb}\, a \to \exists_{\sigma_1\to\sigma_2\to t} x(a\ \mathsf{E}_{\sigma_1\to\sigma_2\to t}\ x)$$

6 *Independent reasons to reject Type-Uniqueness*

Failures of Type-Uniqueness are well-motived quite independent of the special puzzles of property-talk. For example:

- It's plausible that all of the following are true:[21]

  (a) 'or' $\mathsf{E}_{t\to t\to t}$ $\lambda p.\lambda q.p \vee q$

  (b) 'or' $\mathsf{E}_{et\to et\to et}$ $\lambda X.\lambda Y.\lambda z.Xz \vee Yz$

  (c) 'or' $\mathsf{E}_{(et\to t)\to(et\to t)\to et\to t}$ $\lambda X.\lambda Y.\lambda z.Xz \vee Yz$

[18] Prior did just this: 'For "Percy is a fact" (which would mean "It is the case that Percy", if it meant anything), "Percy is a falsehood" (="It is not the case that Percy"), 'Percy is neither a fact nor a falsehood (="It neither is nor is not the case that Percy") are all of them senseless, ungrammatical.' (**PriorOP**)

[19] $\bot$ is some arbitrarily chosen contradiction.

[20] *Objection:* 'Quine once suggested that redness is a material object' seems true, whereas 'Quine once suggested that snow is white and not white' seems false. *Response:* either our intuitions about the truth values of sentences with intensional operators like 'suggested that' are systematically misleading, or else they are weird (in something like the way quotation is weird) and don't conform to FC.

[21] Cf. Partee and Rooth 1983, Dowty 1988.

(a) and (b) explain why ‘Mars is red or Mars is round’ and ‘Mars is red or is round’ are both meaningful (and equivalent). (a) and (c) do the same for ‘Every dog barks or every cat barks’ and ‘Every dog or every cat barks’.

- It’s plausible that both of the following are true:

(d) $$\text{‘mars’ } \mathsf{E}_e \ \mathsf{mars}$$

(e) $$\text{‘mars’ } \mathsf{E}_{et\to t} \ \lambda X.X\,\mathsf{mars}$$

Combining (c) and (e) explains why ‘Mars or Venus is red’ means what it does:

$$\dfrac{\dfrac{\dfrac{\begin{array}{c}\text{‘Mars’ } \mathsf{E}_{et\to t}\\ \lambda X.X\,\mathsf{mars}\end{array} \quad \dfrac{\begin{array}{c}\text{‘or’ } \mathsf{E}_{(et\to t)\to(et\to t)\to et\to t}\\ \lambda UVX.VX\vee UX\end{array} \quad \begin{array}{c}\text{‘Venus’ } \mathsf{E}_{et\to t}\\ \lambda X.X\,\mathsf{venus}\end{array}}{\begin{array}{c}\text{‘or Venus’ } \mathsf{E}_{(et\to t)\to et\to t}\\ \lambda VX.VX\vee X\,\mathsf{venus}\end{array}}\,\text{FC}}{\text{‘Mars or Venus’ } \mathsf{E}_{et\to t}\ \lambda X.X\,\mathsf{mars}\vee X\,\mathsf{venus}}\,\text{FC} \quad \text{‘red’ } \mathsf{E}_{et}\ \mathsf{red}}{\text{‘Mars or Venus is red’ } \mathsf{E}_t\ (\lambda X.X\,\mathsf{mars}\vee X\,\mathsf{venus})\,\mathsf{red}}\,\text{FC}}{\text{‘Mars or Venus is red’ } \mathsf{E}_t\ \mathsf{red\,mars}\vee\mathsf{red\,venus}}\,\beta$$

- It’s plausible that both of the following are true:

(f) $$\text{‘barks’ } \mathsf{E}_{et}\ \mathsf{barks}$$

(g) $$\text{‘barks’ } \mathsf{E}_{(et\to t)\to t}\ \lambda X.X\,\mathsf{barks}$$

Combining (c) and (g) above, this explains why ‘Every dog barks or meows’ has the ‘…I forget which’ ($\vee > \forall$) reading as well as the obvious one.

However in these cases there are natural general laws which could predict the failures of Type-Uniqueness:

Raising $a\ \mathsf{E}_{\tau\to\tau\to\tau}\ U \to a\ \mathsf{E}_{(\sigma\to\tau)\to(\sigma\to\tau)\to\sigma\to\tau}\ \lambda XYz.U(Xz)(Yz)$

Lifting $a\ \mathsf{E}_{\sigma}\ y \to a\ \mathsf{E}_{(\sigma\to t)\to t}\ \lambda X.Xy$

By contrast, there is no operation that could generate the higher-type meanings for ‘interesting’, ‘thinks about’, ‘some’, etc. out of their lowest-type meaning.

7 *The problem of mixed predication*

The standard objection to a higher-order treatment of property-talk in natural language[22] involves sentences like

[22] Chierchia 1982,Bealer 1993,Button and Trueman, forthcoming.

(3) [Either] Mars or redness is interesting.[23]

We'd like this to turn out to be interchangeable with

(4) Mars is interesting or redness is interesting.

We can explain why (4)E_t $\mathsf{interesting}_e\,\mathsf{mars} \vee \mathsf{interesting}_{et}\,\mathsf{red}$, by invoking two differently-typed meanings for the two occurrences of 'interesting'. But how can we arrange this for (3), where we only have one occurrence to work with?[24]

8 Sum-types and their reduction

I'll take a sightly indirect approach: (i) describe a syntactically richer variant of Higher-Orderese, such that English mixed predication would not be a big semantic challenge for native speakers of that dialect, and (ii) show how to translate systematically from that dialect to the original simple Higher-Orderese.

The type-system for the extended language adds *sum-types* like $e + et$. These behave intuitively like "disjoint unions" of their ingredients.[25]

We have three new ways of forming terms:

- When A is a term of type α, $\mathsf{inleft}_\beta\, A$ is a term of type $\alpha + \beta$.[26]
- When A is a term of type α, $\mathsf{inright}_\beta\, A$ is a term of type $\beta + \alpha$.[27]
- When F is a term of type $\alpha \to \gamma$ and G is a term of type $\beta \to \gamma$, $\mathsf{case}\, FG$ is a term of type $(\alpha + \beta) \to \gamma$.[28]

Our logic for the extended language that lets us manipulate these terms in the following ways:

Inleft $\quad P[\mathsf{case}\, FG\, \mathsf{inleft}_\beta\, A] \leftrightarrow P[FA]$

Inright $\quad P[\mathsf{case}\, FG\, \mathsf{inright}_\alpha\, A] \leftrightarrow P[GA]$

Case $\quad P[\mathsf{case}(\lambda v^\alpha . F\, \mathsf{inleft}_\beta\, v)(\lambda u^\beta . F\, \mathsf{inright}_\alpha\, u)] \leftrightarrow P[F] \quad (u, v \text{ not free in } F)$

Prima facie, sum-types seem metaphysically suspect. But it turns out that we can specify a systematic translation from $\mathcal{L}^+$ to our original higher order language $\mathcal{L}$. Here I'll just give the intuitive idea.

The translation of a term $F : \alpha$ will be a sequence of terms $\langle F_1, \ldots, F_n \rangle$; when α is a simple type, $n = 1$. Some examples:[29]

- The translation of a term $A : \sigma + \sigma'$ is a length-one sequence $\langle A_0 \rangle$. If A is of the form $\mathsf{inleft}_{\sigma'}\, B$ or $\mathsf{inleft}_\sigma\, B$ for an $\mathcal{L}$-term B, the translation of A is $\langle B \rangle$.

[23] We could make the same point using 'and'; 'or' is a little easier because it doesn't require plural marking.

[24] 'When I was young, the things I cared most about were things that I could see or feel, but now they are things I can know to be true'. Etc. Bealer 1993: 9, n. 8

[25] (i) e is a type. (ii) t is a terminal type. (iii) Whenever α is a type and β is a terminal type, $\alpha \to \beta$ is a terminal type. (iv) Whenever α and β are types, $\alpha + \beta$ is a type. (v) These are the only types and terminal types.

[26] $\mathsf{inleft}_\beta\, x$ is the "left injection" of x; intuitively, its representative in type $\alpha + \beta$.

[27] $\mathsf{inright}_\beta\, x$ is the "right injection" of x.

[28] Intuitively: the function that applies F or G according as an element of type $\alpha + \beta$ is represents something of a type-α or type-β.

[29] Here σ, σ' are simple types and τ, τ' are simple terminal types

- The translation of any term $F : (\sigma + \sigma') \to \tau$ is a pair $\langle F_0, F_1 \rangle$ where $F_0 : \sigma \to \tau$ and $F_1 : \sigma' \to \tau$. If F is of the form $\mathsf{case}\, GH$ for $\mathcal{L}$-terms $G : \sigma \to \tau$ and $H : \sigma' \to \tau$ are simple terms, the translation of F is $\langle G, H \rangle$.
- The translation of a term $G : ((\sigma + \sigma') \to \tau) \to \tau'$ is a length-one sequence $\langle G_0 \rangle$, where $G_0 : (\sigma \to \tau) \to (\sigma' \to \tau) \to \tau'$.
- The translation of GF is $\langle G_0 F_0 F_1 \rangle$.

9 *Sum-types and mixed predication*

The problem posed by mixed predications like (3) looks quite straightforward when we have sum-types. First, we can introduce general (schematic) laws that lets us freely lift anything into a sum-type:

Sum-Lift
$$\forall_{et} a \forall_\alpha x (a\ \mathsf{E}_\alpha\ x \to a\ \mathsf{E}_{\alpha+\beta}\ \mathsf{inleft}_\beta\ x)$$
$$\forall_{et} a \forall_\alpha x (a\ \mathsf{E}_\alpha\ x \to a\ \mathsf{E}_{\beta+\alpha}\ \mathsf{inright}_\beta\ x)$$

Second, we'll require meaningful count nouns and verb-phrases to express things of type $(\alpha + \beta) \to t$. For example, plausibly

$$\text{'is interesting'}\ \mathsf{E}_{(e+et)\to t}\ \mathsf{case}\,\mathsf{interesting}_e\ \mathsf{interesting}_{et}$$

With all this in place, we can straightforwardly adapt the same approach we would use for 'Mars or Venus is red' to this setting.[30]

[30] Here, J and Jt abbreviate the types $((e + et) \to t$ and $(((e + et) \to t) \to t$ respectively.

$$\dfrac{\dfrac{\text{'Mars'}\ \mathsf{E}_e\ \mathsf{mars}}{\text{'Mars'}\ \mathsf{E}_{e+et}\ \ \mathsf{inleft}_{et}\,\mathsf{mars}}\ \scriptstyle\text{Sum-Lift}}{\text{'Mars'}\ \mathsf{E}_{Jt}\ \ \lambda X.X\,\mathsf{inleft}_{et}\,\mathsf{mars}}\ \scriptstyle\text{Lift}$$

$$\dfrac{\dfrac{\text{'or'}\ \mathsf{E}_{t\to t\to t}\ (\vee)}{\text{'or'}\ \mathsf{E}_{Jt\to Jt\to Jt}\ \ \lambda UVX.VX \vee UX}\ \scriptstyle\text{Raise} \qquad \dfrac{\dfrac{\text{'redness'}\ \mathsf{E}_{et}\ \mathsf{red}}{\text{'redness'}\ \mathsf{E}_{e+et}\ \ \mathsf{inright}_e\,\mathsf{red}}\ \scriptstyle\text{Sum-Lift}}{\text{'redness'}\ \mathsf{E}_{Jt}\ \ \lambda X.X\,\mathsf{inright}_e\,\mathsf{red}}\ \scriptstyle\text{Lift}}{\text{'or redness'}\ \mathsf{E}_{Jt\to Jt}\ \ \lambda V.\lambda X.VX \vee X\,\mathsf{inright}_e\,\mathsf{red}}\ \scriptstyle\text{FC}$$

$$\dfrac{\text{'Mars'}\ \mathsf{E}_{Jt} \qquad \text{'or redness'}\ \mathsf{E}_{Jt\to Jt}}{\text{'Mars or redness'}\ \mathsf{E}_{Jt}\ \ \lambda X.X\,\mathsf{inleft}_{et}\,\mathsf{mars} \vee X\,\mathsf{inright}_e\,\mathsf{red}}\ \scriptstyle\text{FC}$$

$$\dfrac{\text{'Mars or redness'}\ \mathsf{E}_{Jt}\ \ \lambda X.X\,\mathsf{inleft}_{et}\,\mathsf{mars} \vee X\,\mathsf{inright}_e\,\mathsf{red} \qquad \text{'is interesting'}\ \mathsf{E}_J\ \ \mathsf{case}\,\mathsf{interesting}_e\,\mathsf{interesting}_{e\to t}}{\text{'Mars or redness is interesting'}\ \mathsf{E}_t\ \ (\lambda X.X\,\mathsf{inleft}_{et}\,\mathsf{mars} \vee X\,\mathsf{inright}_e\,\mathsf{red})\,\mathsf{case}\,\mathsf{interesting}_e\,\mathsf{interesting}_{e\to t}}\ \scriptstyle\text{FC}$$

$$\dfrac{\text{'Mars or redness is interesting'}\ \mathsf{E}_t\ \ (\lambda X.X\,\mathsf{inleft}_{et}\,\mathsf{mars} \vee X\,\mathsf{inright}_e\,\mathsf{red})\,\mathsf{case}\,\mathsf{interesting}_e\,\mathsf{interesting}_{e\to t}}{\text{'Mars or redness is interesting'}\ \mathsf{E}_t\ \ \mathsf{case}\,\mathsf{interesting}_e\,\mathsf{interesting}_{e\to t}\,\mathsf{inleft}_{et}\,\mathsf{mars} \vee \mathsf{case}\,\mathsf{interesting}_e\,\mathsf{interesting}_{e\to t}\,\mathsf{inright}_e\,\mathsf{red}}\ \scriptstyle\beta$$

$$\dfrac{\text{'Mars or redness is interesting'}\ \mathsf{E}_t\ \ \mathsf{case}\,\mathsf{interesting}_e\,\mathsf{interesting}_{e\to t}\,\mathsf{inleft}_{et}\,\mathsf{mars} \vee \mathsf{case}\,\mathsf{interesting}_e\,\mathsf{interesting}_{e\to t}\,\mathsf{inright}_e\,\mathsf{red}}{\text{'Mars or redness is interesting'}\ \mathsf{E}_t\ \ \mathsf{interesting}_e\,\mathsf{mars} \vee \mathsf{interesting}_{et}\,\mathsf{red}}\ \scriptstyle\text{case}$$

We *could* derive the new meaning for 'interesting' from a general law:
Combination

$$\forall_{et} a \forall_{\alpha\to\gamma} x \forall_{\beta\to\gamma} y (a \ \mathsf{E}_{\alpha\to\gamma} \ x \wedge a \ \mathsf{E}_{\beta\to\gamma} \ y \to a \ \mathsf{E}_{(\alpha+\beta)\to\gamma} \ \mathsf{case}\, xy)$$

although we'll need to restrict this if we stop bracketing ordinary ambiguity.[31]

[31] Otherwise, we'll get a false reading for 'Either Barclays or Willowy is a bank' equivalent to '$\mathsf{riverbank\ barclays} \vee \mathsf{moneybank\ willowy}$

10 *Sum-types and cross-type quantification*

Sum-types also help with a related problem. Plausibly,

(5) Everything Mary mentioned is interesting

has a reading where it both entails 'If Mary mentioned Mars, Mars is interesting' and 'If Mary mentioned redness, redness is interesting'. We can get this by extending the type-ambiguity of the quantifiers to cover the sum-types, e.g.:

$$\text{'every'} \ \mathsf{E}_{((e+et)\to t)\to((e+et)\to t)\to t} \ \lambda X.\lambda Y.\forall_{e+et} z.Xz \to Yz$$

This provides true interpretations for some characteristic utterances of anti-nominalists, like 'Some but not all things are material, since Mars is material and redness is not'.[32]

[32] We can also take nominalists who say 'Everything is material' to be speaking truly using a first-order quantifier: like other kinds of ambiguity, type-ambiguity generates possibilities for merely verbal disagreements.

Note however that this will only give meanings of (5) equivalent to

$$\bigwedge_{1\le i\le n} \forall_{\sigma_i} x(\mathsf{mentioned}_{\sigma_i} \, x \, \mathsf{mary} \to \mathsf{interesting}_{\sigma_i} \, x)$$

for any *finitely many* types $\sigma_1 \ldots \sigma_n$.

11 *Fully type-neutral generality?*

It's tempting to think that (5) also admits a reading stronger than all of these.

It's very natural for philosophers to want to use it in this way, although it's not obvious that this should be written into its literal semantics.[33]

[33] Is this temptation just a symptom of the bad metaphysics that can flow from speaking a language rife with type-ambiguity? No: even native Higher-Orderese speakers face a real communicative need here, which they discharge using their disquotational truth predicate
Here, $\mathsf{Rigid}\, X$ means (or at least implies) that X is modally rigid in the way that sets are: $\forall p(Xp \to \Box Xp)$ and $\forall Y(\forall p(Xp \to \Box Yp) \to \Box\forall p(Xp \to Yp))$: see Dorr, Hawthorne, and Yli-Vakkuri 2021: §1.5.

We could achieve this using some new basic principles that generates meanings for certain quantified sentences *not* in accordance with FC. For example:

$$\mathsf{comb}\, abc \wedge \mathsf{comb}\, b\text{'every'}d \wedge \mathsf{Rigid}\, X \wedge \forall p(Xp \to a \ \mathsf{E}_t \ p) \\ \to a \ \mathsf{E}_t \ \forall p(Xp \to p)$$

In effect, this allows any 'every'-sentence to express the conjunction of any (including infinitary) collection of things it expresses.

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